VOL. 14, NO. 6, MARCH 2019 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

DISTRIBUTION OF TEMPERATURE IN A SPATIALLY ONE-DIMENSIONAL OBJECT AS A RESULT OF THE ACTIVE POINT SOURCE

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ABSTRACT

This article presents the concept of systems with distributed parameters and investigates the method of distributed controller synthesis and a homogeneous control object. This article analyzes main methods of transferring heat energy. On the basis of the heat-transfer equation, the function of initial heating has been obtained, and the process has been mathematically simulated, and the results obtained have been analyzed. The practical results of this research make it possible to draw a conclusion about the possibility of building a silicon-carbide heating element made in the shape of an isotropic rod.

Keywords: function grinna, mathematical model, portioned systems, analysis.

INTRODUCTION

In continuous automatic control systems, information is represented as signals described by continuous functions. Along with continuous methods of signal transmission and conversion, discrete signals are widely used in which signals are quantized. Quantization or sampling consists in the representation of a continuous signal by its discrete values. Depending on the type of quantization, the automatic control systems with lumped parameters are divided into discrete (relay), discrete (pulse), discrete in level and time (relay-pulse) in level.

The implementation of the input effect in systems with distributed parameters is carried out by discretizing it over spatial coordinates. For example, the realization of the heat flux field is carried out by means of a sectional heater, and the number of sections can be arbitrarily large. Thus, to the specified types of discretization it is necessary to add discretization with respect to spatial coordinates, both the input action and the sensors for observing.

The field of application of discrete systems is very diverse. There are two main categories of such systems.

Systems that are discrete in their physical nature, i.e. information in them exists only at discrete instants of time. Examples of this category of discrete systems are the radar detection and tracking systems of the target. There are numerous physical and biological phenomena, processes in social and economic systems, the dynamics of which can adequately be described only by discrete models.

Systems in which information exists continuously but is intentionally quantized to obtain some new properties as compared to continuous systems. These properties can be: ease of implementation, increased reliability, increased accuracy, smaller overall dimensions and cost. Rapid progress in computer technology, the widespread use of microprocessors in control systems further increase interest in discrete systems.

MATERIALS AND METHODS

Analysis of the management system**:** as a spatial object, we consider a homogeneous cylindrical rod. We will assume that the controlling effect is the heat flux generated by the sources, realized as sections of a sectional heater, distributed along the boundary of the lateral surface of the cylinder. The sources are switched on using relay elements. The zero temperature is maintained at the ends of the rod. Let us set the problem of temperature stabilization at a level of a certain value $T_{зад}$ (Ilyushin, Pervukhin, Afanasieva, Klavdiev, Kolesnichenko, 2014).

RESULTS AND DISCUSSIONS

Consider a cylindrical rod of radius R length l, presented in the picture 1. The mathematical model of the process of heat propagation has the form (Ilyushin, Pervukhin, Afanasieva, Klavdiev, Kolesnichenko, 2015):

$$\frac{\ddot{a}T}{\ddot{a}t} = a^2\left(\frac{\ddot{a}^2T}{\ddot{a}r^2} + \frac{1}{r}\frac{\ddot{a}T}{\ddot{a}r} + \frac{\ddot{a}^2T}{\ddot{a}x^2}\right);\ 0 < r < R; \qquad (1)$$

$T(x,r,t)-$ temperature field of a cylindrical rod; $a-$ preset factor; $R,l-$ given numbers; $x,r-$ spatial coordinates; $t-$ time.

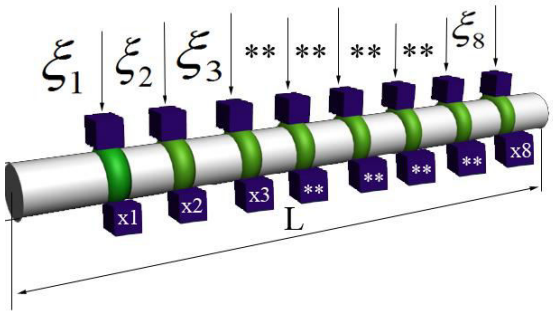

Figure-1. Cylindrical rod.

The boundary conditions of equation (1) have the form:

$$T(0,r,t) = T(l,r,t) = 0; \quad (2)$$

$$T(x,R,t) = u(x,t); \quad (3)$$

$$\frac{\ddot{a}T(x,0,\tau)}{\ddot{a}r} = 0. \quad (4)$$

The function of the output is the function $T(x,R^*,t)$, R^* – specified number $(0 < R^* < R)$. Condition (2) indicates that the ends of the rod have a temperature equal to zero. Condition (4) is a symmetry condition for the thermal fields. The input effect is distributed along the boundary, which reflects the condition (3).

Suppose that the rod is thin enough that at any point in time the temperature at all points of the cross section could be considered the same. In other words, we assume that the cylinder is spatially one-dimensional. In this case, we keep the boundary condition (3), assuming that the boundary is not only the ends of the rod, but also its lateral surface. We introduce this boundary condition into the right-hand side of the basic equation. The necessity of the boundary condition (4) disappears. We add a zero initial condition. With all the assumptions made, the mathematical model of the process of heat propagation will take the form (Galkin, 2017):

$$\frac{\partial T}{\partial t} = a^2 \frac{\partial^2 T}{\partial x^2} + u(x,t); \; 0 < x < l; \; t > 0 \quad (5)$$

Boundary and initial conditions:

$$T(0,t) = T(l,t) = 0; \quad (6)$$

$$T(x,0) = 0. \quad (7)$$

The control effect $u(x,t)$ is created by sources that are switched on by means of relay elements. The output variable of the relay element has a rectangular shape. When the number of sources tends to infinity, it will take the form of an impulse created by a point source. Assuming that the action of each source occurs during an infinitesimal time interval, it can be assumed that the control action is created by instantaneous point sources and is represented as a product of delta functions (Chernyshev, 2009, Chernyshev, 2010).

$$u(x,t) = \delta(x-\xi)\delta(t-\tau).$$

The mathematical model takes the form:

$$\frac{\partial T}{\partial t} = a^2 \frac{\ddot{a}^2 T}{\ddot{a}x^2} + \delta(x-\xi)\delta(t-\tau); \; 0 < x < l; \; t > 0. \quad (8)$$

$$T(0,t) = T(l,t) = T(x,0) = 0 \quad (9)$$

The coefficients and the input functions take on the values [3, 4]:

$A = 0; A_1 = 1; C = a^2; B_1 = 0; C_1 = 0$, $f = \delta(x-\xi)\delta(t-\tau)$, $g_0 = 0$, $g_1 = 0$.

Then the general solution of the boundary value problem in integral form takes the form:

$$T(x,t) = \int_0^t \int_0^l G(x,\xi,t,\tau)\, \delta(\xi-\xi_0)\delta(\tau-\tau_0) d\xi d\tau;$$
$$x \in (0,l); \; t > 0 \quad (10)$$

The obtained model describes the process of heat propagation in the rod, under assumptions that allow the system to be considered linear and the control process to be continuous. To stabilize the temperature, it is necessary to consider a closed control system, represented in the form of the following structural scheme (see Figure-2). The regulator of such a system can be realized as a nonlinear discrete algorithm. This algorithm must perform an impact on the deviation of temperature from a given value at certain points at a certain time. Before any negative deviations occur $T(x,t) - T_{зад}$, it is necessary to heat the rod to a temperature exceeding the value $T_{зад}$ along the entire length of the rod. That is, it is necessary to form, so-called, the function of initial heating. This function can be formed as a result of the initial inclusion of all sources (Kolesnikov, 2014).

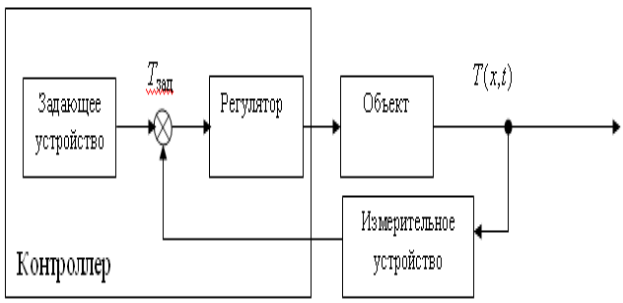

Figure-2. Structural diagram of a closed-loop control system.

Circuit elements: «Задающее устройство» - Master device; «Регулятор» - Regulator; «Контроллер» - Controller; «Объект» - Object; «Измерительное устройство» - Measuring device

In the mathematical model (8) - (9) it is necessary to change the initial conditions. As a result, the mathematical model takes the form:

$$\frac{\partial T}{\partial t} = a^2 \frac{\ddot{a}^2 T}{\ddot{a} x^2} + \delta(x-\xi)\delta(t-\tau);\ 0 < x < l;\ t > 0; \quad (11)$$

$$T(0,t) = T(l,t) = 0; \quad (12)$$

$$T(x,0) = \delta(x-\xi)\ \delta(t) \quad (13)$$

Then, the general solution in integral form will be expressed as:

$$T(x,t) = \int_0^l G(x,\xi,t)\delta(\xi-\xi_0)d\xi + \int_0^t\int_0^l G(x,\xi,t,\tau)$$
$$\delta(\xi-\xi_0)\delta(\tau-\tau_0)d\xi d\tau; \quad (14)$$
$$x \in (0,l);\ t > 0.$$

The mathematical model (11) - (13) obtained and the integral representation of the solution (14) can only be of a basic nature. The real regulatory system under consideration is of a discrete nature. Sources and sensors are located at specific fixed points, their number is limited. The time for switching on the control actions is determined in accordance with the software algorithm, when the output value reaches the set value at the sensor setting point (Kolesnikov, Zarembo, 2014).

Let us analyze the influence of the components of the series at $n \to \infty$. At the initial instant of time, when $t = 0$, $\tau = 0$, is obtained:

$$\exp\left[-\left(\frac{\pi n a}{l}\right)^2 (t-\tau)\right] = 1;$$

Then the temperature at the points of the rod will be expressed by the formula:

$$T(x,\xi) = \frac{2}{l}\sum_{n=1}^{\infty} \sin\frac{\pi n}{l}x \sin\frac{\pi n}{l}\xi;$$

Let the instantaneous heat source affect the point $\xi = \frac{l}{4}$, then the formula takes the form:

$$T(x,\xi) = \frac{2}{l}\sum_{n=1}^{\infty} \sin\frac{\pi n}{4} \sin\frac{\pi n}{l}x;$$

The amplitude of each component of the series will look like:

$$A_n = \frac{2}{l}\left|\sin\frac{\pi n}{4}\right|.$$

For the first six components of the series, for $x = \frac{l}{4}$, we get:

$$T_1(x) = \frac{2}{l}\sin\frac{\pi}{4}\sin\frac{\pi}{l}x = \frac{\sqrt{2}}{l}\sin\frac{\pi}{l}x; \quad A_1 = \frac{\sqrt{2}}{l};$$
$$T_1\left(\frac{l}{4}\right) = \frac{1}{l};$$
$$T_2(x) = \frac{2}{l}\sin\frac{\pi}{2}\sin\frac{2\pi}{l}x = \frac{2}{l}\sin\frac{2\pi}{l}x; \quad A_2 = \frac{2}{l};$$
$$T_2\left(\frac{l}{4}\right) = \frac{2}{l};$$
$$T_3(x) = \frac{2}{l}\sin\frac{3\pi}{4}\sin\frac{3\pi}{l}x = \frac{\sqrt{2}}{l}\sin\frac{3\pi}{l}x;$$
$$A_3 = \frac{\sqrt{2}}{l};\ T_3\left(\frac{l}{4}\right) = \frac{1}{l};$$
$$T_4(x) = \frac{2}{l}\sin\pi\sin\frac{4\pi}{l}x = 0;\ A_4 = 0;\ T_4\left(\frac{l}{4}\right) = 0;$$
$$T_5(x) = \frac{2}{l}\sin\frac{5\pi}{4}\sin\frac{5\pi}{l}x = -\frac{\sqrt{2}}{l}\sin\frac{5\pi}{l}x;$$
$$A_5 = \frac{\sqrt{2}}{l};\ T_5\left(\frac{l}{4}\right) = \frac{1}{l};$$
$$T_6(x) = \frac{2}{l}\sin\frac{3\pi}{2}\sin\frac{6\pi}{l}x = -\frac{2}{l}\sin\frac{6\pi}{l}x;\ A_6 = \frac{2}{l};$$
$$T_6\left(\frac{l}{4}\right) = \frac{2}{l};$$

It is known that $G(x,t,\xi,\tau) \geq 0$, at any value x,t,ξ,τ. When $t = 0$ and a large number of terms of the series n negative values of the function will disappear, and the range of values of x for which the function is positive will narrow, approaching from both sides to the point of application of the action, in this case to the point $\xi = \frac{l}{4}$. In this case, the value of the function at the point ξ will tend to infinity (see Figure-3). The expression

$$\frac{2}{l}\sum_{n=1}^{\infty} \sin\frac{\pi n}{l}x \sin\frac{\pi n}{l}\xi$$

is a spatial delta function in the form of a Fourier series, i.e.

$$\delta(x-\xi) = \frac{2}{l}\sum_{n=1}^{\infty} \sin\frac{\pi n}{l}x \sin\frac{\pi n}{l}\xi$$

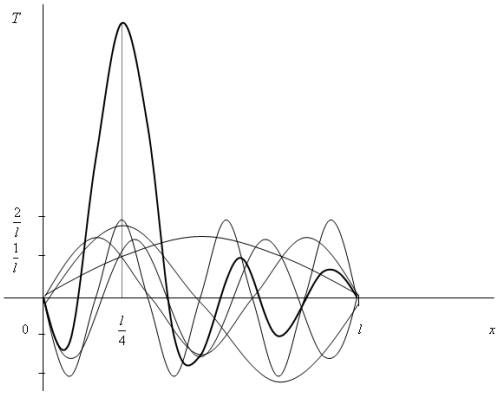

Figure-3. Graphs $\delta-$ functions with $n=6$ and with $n\to\infty$.

$\delta-$function belongs to the class of generalized functions. Apparatus $\delta-$functions are widely used in the study of models of control objects with distributed parameters. The argument of $\delta-$functions $\delta(x-\xi)$ is the spatial coordinate, considered as a scalar quantity in a spatially one-dimensional problem or as a vector coordinate for multidimensional spatial domains. The function $\delta(x-\xi)$ is zero within the entire spatial domain occupied by the object, except for the point ξ, at which $\delta(x-\xi)$ it assumes a value equal to infinity (Pleshivtseva, Rapoport, 2014).

$$\delta(x-\xi)=\begin{cases}\infty, & \text{ïðè} \quad x=\xi\\ 0, & \text{ïðè} \quad x\neq\xi\end{cases}.$$

For functions $f(x)$, distributed over a segment $[0,l]$, we have the equality:

$$\int_0^l f(x)\delta(x-\xi)dx=f(\xi);$$

Similarly, we introduce the concept of a time $\delta-$function $\delta(t-\tau)$, with the purpose of describing the impulse actions that are limited in energy consumption of a sufficiently short duration, but of very great magnitude, on objects of a different physical nature. In the limit, infinitely large impacts, concentrated at fixed instants of time. Delta function $\delta(t-\tau)$ is equal to zero for all t, except for the value of $t=\tau$, at which it turns to infinity, and

$$\int_{-\infty}^{+\infty}\delta(t-\tau)dt=1.$$

If $f(t)$ – any bounded and continuous function on the interval $[t_0,t]$, so

$$\int_{t_0}^{t} f(t)\delta(t-\tau)dt=\begin{cases}f(\tau), \text{åñëè} & \tau\in[t_0,t];\\ 0, \text{åñëè} & \tau\notin[t_0,t],\end{cases}$$

As applied to the function $f(x,t)$ two variables - a scalar spatial coordinate x and time t, vector $\delta-$function, simulating the impulse action applied at the time $\tau\in[t_0,t]$ at the point $\xi\in[0,l]$, described by $\delta(x-\xi)\delta(t-\tau)$. The integral representation takes the form:

$$\int_0^l\int_{t_0}^{t} f(x,t)\delta(x-\xi)\delta(t-\tau)dxdt=f(\xi,\tau).$$

Thus, assuming that the input of the object under consideration received a single impulse disturbance applied at the point ξ_0 at the time τ_0, at the output we get:

$$T(x,t)=\int_0^t\int_0^l G(x,t,\xi,\tau)\delta(\xi-\xi_0)\delta(\tau-\tau_0)d\xi d\tau=G(x,t,\xi_0,\tau_0)$$

For a fixed time $t\neq 0$, the amplitude of the components of the Fourier series will have the form:

$$A_n=\left|\frac{2}{l}\sin\frac{\pi n}{l}\xi\exp\left[-\left(\frac{\pi na}{l}\right)^2 t\right]\right|;$$

When $n\to\infty$, value A_n will tend to zero, i.e. the effect of each subsequent harmonic of the series decreases. We define with a given accuracy ε, starting with which number n all subsequent components of the series can be neglected. Let is $|A_n|\le\varepsilon$, it is known that

$$\left|\sin\frac{\pi n}{l}\xi\right|\le 1,$$

then

$$\left|\frac{2}{l}\sin\frac{\pi n}{l}\xi\right|\le\frac{2}{l},\quad \left|\exp\left[-\left(\frac{\pi na}{l}\right)^2 t\right]\right|\le\frac{\varepsilon l}{2},$$

$$\exp\left[\left(\frac{\pi na}{l}\right)^2 t\right]\ge\frac{2}{\varepsilon l},\qquad \left(\frac{\pi na}{l}\right)^2 t\ge\ln\frac{2}{\varepsilon l},$$

$$n^2 t\ge\left(\frac{l}{\pi a}\right)^2\ln\frac{2}{\varepsilon l},$$

eventually:

$$n \geq \frac{l}{\pi a \sqrt{t}} \sqrt{\ln \frac{2}{\varepsilon l}}.$$

For example, for values $l = 10$, $a = 0{,}01$, $t = 10000$, $\varepsilon = 0{,}0001$; is obtained: $n \geq 8{,}7757$, that is, to determine the value of the function with accuracy $0{,}0001$ it suffices to take 8 components of the series. The amplitude decreases also when $t \to \infty$. It is possible to determine from which time value t components of the series with $n \geq n_0$, can be neglected.

$$t \geq \left(\frac{l}{\pi n a} \right)^2 \ln \frac{2}{\varepsilon l}.$$

Let's define as an example, starting from which moment of time, with an accuracy of 0.0001, the value of the function will be determined by only one first harmonic.

$$t \geq \left(\frac{10}{0{,}02\pi} \right)^2 \ln \frac{2}{0{,}001}; \; t \geq 192728.$$

Dependence of temperature distribution in the rod on time, as a result of the action of the source at the point $\xi = \frac{l}{4}$, can be represented in the form of a graph (see Figure-4). ($t_1 < t_2 < \ldots < t_5$).

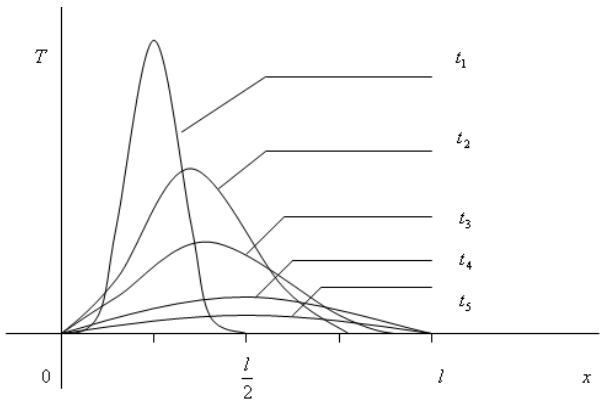

Figure-4. Spatio-temporal dependence of temperature distribution.

CONCLUSIONS

Thus, it can be observed that the task of implementing control systems for objects with distributed parameters is considerably more complicated than for systems with lumped parameters. The number of control actions of such systems can include space-time controls, described by functions of several arguments - time and spatial coordinates. For the analysis and synthesis of systems with distributed parameters, it is required to create a new apparatus on the basis of non-traditional for the classical theory of control of mathematical means.

Based on the theory of impulse response functions, research methods and methods for parametric optimization of discrete distributed systems have been developed, which allowed obtaining analytical dependencies between a given error and sampling parameters that make it possible to justify the choice of the discretization step in practical implementation.

ACKNOWLEDGEMENTS

We should note that persons without whom these studies could not have been carried out. The first is Pershin Ivan Mitrofanovich (Doctor of Technical Dciences, Professor, actual member of the Academy of Natural Sciences, Honored Worker of Higher Professional Education of the Russian Federation), Kolesnikov Anatoly Arkadievich (Honored Worker of Science and Technology of the Russian Federation, Doctor of Technical Sciences, Professor, member of the Academy of Sciences and the Academy of electrical-technical sciences and Motion Control, Corresponding Member of the Russian Academy of Natural Sciences, Soros Professor (four) in the exact sciences field), and Chernyshev Alexander Borisovich (Doctor of Technical Sciences, Professor) who laid the foundation for the synthesis of distributed controllers on the basis of the Green's function.

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