



# FRACTAL DIMENSION ANALYSIS OF ICTAL EEG SIGNAL USING BOX COUNTING METHOD

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## ABSTRACT

Epilepsy is a state that results in the loss of consciousness of patients and is one of the major disorders that affect the brain. It is very dangerous and disturbs the day-to-day life of the patient. Currently, several traditional and advanced methods are used to detect the presence of epilepsy. Even though some of these methods are very accurate, they take more time to administer. Computational complexity increases as the number of channels increases. Thus, the researchers aim for automatic epilepsy detection and prediction. The method that is being presented here uses fractal properties, such as fractal dimension and lacunarity, to analyze the EEG signal and determine the epileptic signal. The proposed method utilizes the box counting algorithm for the calculation of the fractal dimension and the gliding box algorithm to obtain lacunarity. The parametric statistical test is used to test the reliability of the method, and by using this method, the epileptic signal is distinguished from the normal one with high accuracy.

**Keywords:** EEG, fractal dimension, Ictal, lacunarity.

## 1. INTRODUCTION

Electroencephalogram (EEG) signals are biological signals collected from different parts of the brain and are effectively used to understand the functions of the brain and also used in the diagnosis of several disorders associated with malfunctioning of the brain, such as epilepsy [1]. Epilepsy or Apasmara described in the Ayurvedic literature Charaka Samhitha is the state of consciousness loss. The word epilepsy originated from epilambanein, a Greek word meaning attack or seize. A group of brain cells suddenly release some electric discharges due to some stimuli and results in epilepsy [2]. The discharges can originate from the different parts of the brain. The hyper synchronous activity of the brain cell results in the malfunctioning of the brain. It is one of the severe brain disorders that occur among adults and children [3]. EEG monitoring is one of the most efficient ways of epilepsy prediction. However, epileptic signals are low-frequency and high-amplitude signals. Thus, analyzing EEG for the detection of epilepsy will take time. If the number of channels taken is more, then the task becomes a tedious one. Therefore, automatic seizure detection is very useful in the biomedical field.

The researchers have been attracted towards epilepsy monitoring, detecting, and predicting. Dynamical analysis of a time series is used for the prediction of focal epilepsies from the scalp EEGs [4]. Many methods are available for the detection of seizure [5]. Those methods use the features which are extracted from the EEG signal. These extracted features are used for the training and classification of EEG signals [6], [7] such as with seizure or without seizure. However, the selection of feature plays an important role. Energy [8], [9], statespace modeling [10] and entropy [11] are some of the features.

In this study, we consider EEG signals as fractal objects, which also possess features of fractals. Many methods use fractal measures, like fractal intercept and fractal dimension (FD), for the detection of epilepsy [12].

In this paper, we use the fractal properties, such as fractal dimension and lacunarity, for epilepsy detection. FD is a textural feature of an image. It is a non-integer dimension of an object and is widely used in the analysis of biomedical signal to detect the presence of transient signals and also in pattern recognition applications, including segmentation and classification [13]. The FD is calculated by using the box counting method [14]. Box counting is an efficient and easy method for the calculation of FD. The FD thus obtained is compared with the FD obtained using the Higuchi algorithm. Another feature that we are considering is lacunarity. FD cannot provide sufficient data for epilepsy detection in EEG data [15].

This paper consists of the following Sections. Section II provides an overview of fractal properties, such as FD and lacunarity. Section III describes the methodology that we adopted in this study, followed by experimental results in Sections IV. Section V provides the conclusion of this paper.

## 2. FRACTALS AND PROPERTIES

### A. Fractal dimension

Fractals are objects having self-similarity. Self-similarity and FD are the major properties of fractal objects [16]. According to Mandelbrot, if the bounded set A is the collection of non-overlapping scaled copies of itself with a scaling factor r, then the set will be a fractal [17]. The dimension of a fractal object is a non-integer value ranging from 1 to 2. Then, the FD of A is calculated by using the given formula.

$$D = \frac{\log(N_r)}{\log(1/r)} \quad (1)$$

where r is the box size and  $N_r$  is the total number of boxes required to enclosed the whole area of the object.



Many algorithms are available for FD calculation. However, their computational requirements are expensive. The Higuchi, Katz, and Petrosian's algorithm are some of them [18]-[20]. In this paper, we use the box counting method for FD calculation. Even though FD appears to be a useful metric in some situation, FD alone cannot provide sufficient information for this analysis. Thus, we consider two more features of fractals, such as fractal abundance and lacunarity.

### B. Lacunarity

The properties and characteristics of a fractal object are not completely determined by its FD. Different fractals can have the same dimension but look differently from each other. Thus, lacunarity is an effective tool to differentiate fractals [21]. Lacunarity is correlated to the FD that reflect the characterization of a fractal [22]. It can quantify additional features, such as the heterogeneity of a pattern. It is related to the distribution of the holes size. In other words, if a fractal having large gaps, its lacunarity is also larger [23].

Lacunarity measures the lumpiness of the fractal data [24]. It provides additional information about the computed FD values in an image [25]. Lacunarity (L) can be calculated as the ratio of the variance over the mean value of the function and is given by (2)

$$L = \frac{\frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} I(m,n)^2}{\left( \frac{1}{MN} \sum_{k=0}^{M-1} \sum_{l=0}^{N-1} I(k,l) \right)^2} - 1 \quad (2)$$

where M and N are the sizes of the processed image I.

### 3. PROCEDURE AND METHODOLOGY

Figure-1 shows the flowchart of the system. Here, the EEG data is converted into the image. Then, the FD and L are calculated using the box counting method. Here, the most popular type of box counting, using gliding boxes, is used. Then, all the observed data are validated and compared statistically using parametric statistical test. The same test is conducted, seizure EEG and normal EEG signal, and the irregularity is compared.

#### A. EEG database

The first phase is the image formation from the available EEG signal. The EEG data used here are obtained from the Bonn University database [26]. It is an open-source database. The sampling frequency of the data was 173.61 Hz [27]. This EEG data were segmented and converted into a binary image. These images are used for the calculation of fractal features using the box counting method. This paper uses ictal and normal EEG data. From both dataset, 60 sets were chosen for the analysis.

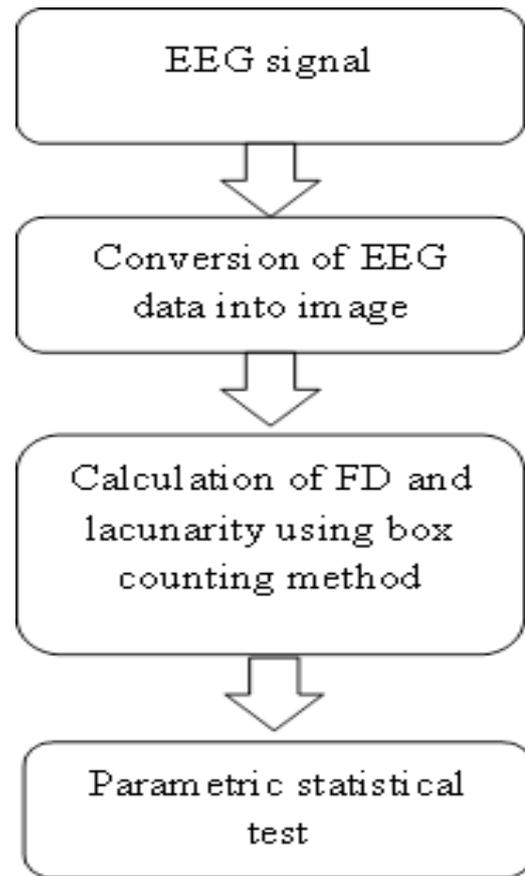


Figure-1. Flowchart of the process.

#### B. Box counting method

The box counting is the one of the most popular methods for FD calculation [28], [29] in this analysis, the complex pattern of a data set or image is partitioned into smaller and smaller pieces. It is one of the important methodologies for fractal analysis. The implementation of box counting algorithm is based on the basic idea of covering an image with a set of measuring box of size "s" and calculating the number of boxes required.

Let the image be  $N \times N$  image and is partitioned in to the grid of size  $s \times s$ . Let  $r = s/N$ . The minimum and maximum gray levels of the image in a particular grid (i, j) are v and u, respectively [30]. The number of boxes between the minimum and maximum gray levels at the (i, j) grid is counted by the equation.

$$n(i, j) = u - v + 1 \quad (3)$$

The proposed work of the box counting algorithm is shown in Figure-2. The size of the box (squares) is set randomly and packed into the fractal image. Then count the number of packed boxes in the fractal area. Then varies the size of the boxes and do the same procedure. The Box dimension is taken as the approximation of FD [31].

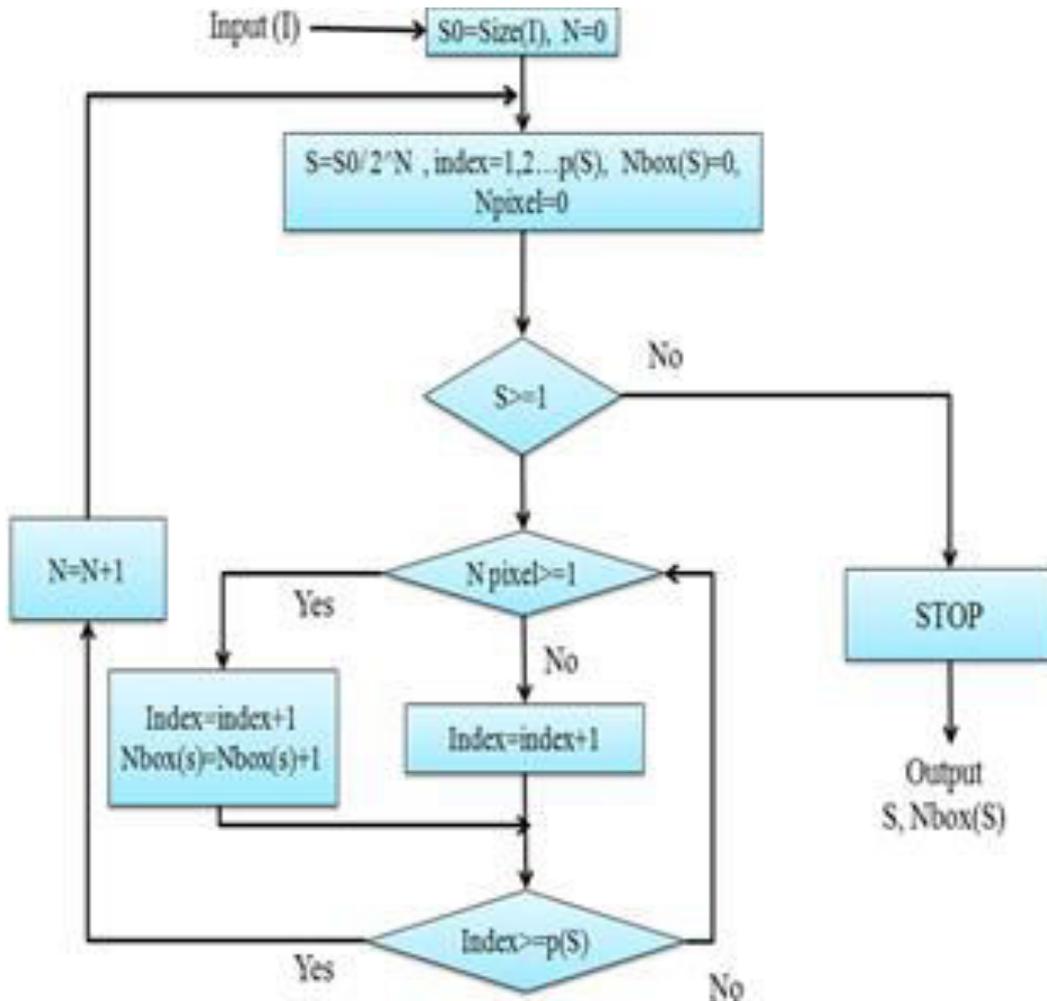


Figure-2. Box counting algorithm.

The total number of boxes required to cover that image can be calculated by using the summation equation

$$N_r = \sum_{i,j} n(i,j) \tag{4}$$

$N_r$  is the number of boxes dependent on the box size  $r$ , and the FD of an image can be obtained from the least square linear fit of  $\log(N_r)$  and  $\log(1/r)$ , and the slope of the fit line will be the FD. Equation (1) will give the FD of the image. However, the FD calculated by this method has some disadvantage. The pixels in the common border may be counted twice. Thus, another method, called the modified box counting method, is used. In this method, both the number of pixels and the gray levels of the picture are also considered [32]. In that method, the number of boxes in the  $(i, j)$  grid having size “ $s \times s$ ” is given by.

$$n(i,j) = 1 + \sum_{m=1}^{s^2} w(m) \bar{g}_r(m) \tag{5}$$

where  $w(m)$  is the weight of the gray levels and  $\bar{g}_r(m)$  is the set of gray levels of images. The total number of boxes can be counted by using (4). From (1), we will obtain the FD.

**C. Gliding box counting method**

Box counting is also used in the calculation of  $L$ . As already mentioned,  $L$  is the measure of irregularity directly related to scale, density, emptiness, and variance. It gives the idea of deviation of a geometric structure from its translational invariance [33]. The gap distribution in different scales helps distinguish patterns from one another. The higher the  $L$  of a spatial pattern, the higher will be the variability of its gaps in an image, and the more heterogeneous will be its texture [34]. Many methods are available for the calculation of  $L$  and we use the gliding box counting method here [35].

According to the gliding box algorithm proposed by Allain and Cloitre, a box of size  $r$  is used to slide over an image. Then, the probability distribution  $Q(M, r)$  is obtained by dividing the number of gliding boxes having radius  $r$  and mass  $M$  with the total number of boxes [36]. Let  $n(M, r)$  be the number of gliding boxes, and the  $L$  can be obtained by dividing the mean square deviation of the variation of mass distribution probability  $Q(M, r)$  by its mean square, as follows:

$$L(r) = \frac{\sum_M M^2 Q(M,r)}{[\sum_M M^2 Q(M,r)]^2} \tag{6}$$



where  $L(r)$  is the lacunarity and  $M$  is the mass of box having size “ $r$ ”. In this research, the input image is the binary image; hence the gliding box algorithm counts only the foreground pixels. This is because the binary images have only two values, and the gray scale and color images have many values.

#### 4. EXPERIMENTAL RESULTS

The FD of EEG signal has different values ranging from 1 to 2. The FD result obtained from the experimental analysis of EEG data reveals that the FD obtained from the box counting method is easy, and using the box counting method will obtain different values of FD for the different box sizes.

The EEG data collected are in another format. Thus, the first step is the conversion of EEG data into images. Then, the box counting method is used to calculate the FD of the binary image. The normal and ictal inputs are given to the box counting algorithm. The box counting algorithm is implemented for different boxes with different sizes to cover the image.

Corresponding to each image, the FD is calculated for the different values of box sizes. The log plot is shown in Figures 3 and 4 give the idea of FD for different box size. The FD obtained from the box counting algorithm purely depends on the number of boxes and the sizes of the box. In this fractal analysis, we use 15 normal EEG signals and 15 ictal EEG signals. The normal input and its box counting dimensions are shown below. These dimensions are known as local dimensions.

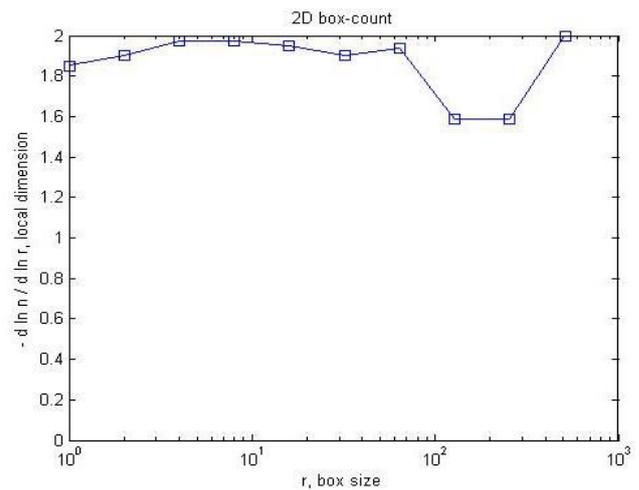


Figure-3. Fractal dimension plot of normal EEG.

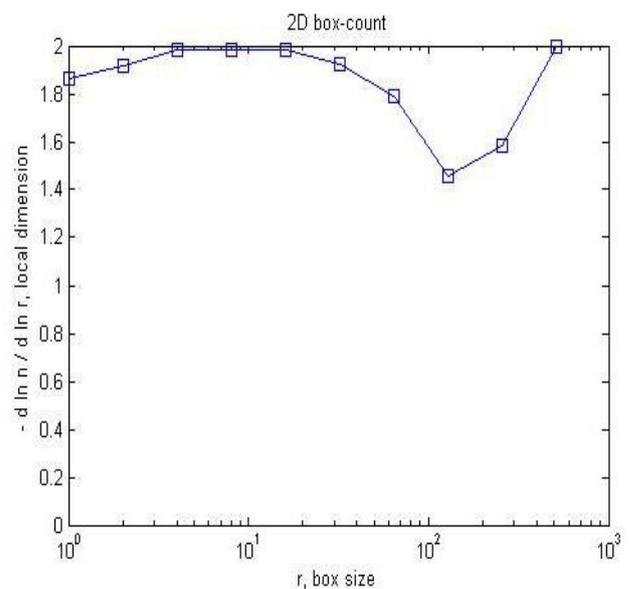


Figure-4. Fractal dimension plot of ictal EEG.

Using gliding box algorithm, the  $L$  of each input is calculated. Table-1 gives the value of FD and  $L$  for different set of images of the normal EEG data and ictal EEG data

**Table-1.** Fractal dimension and lacunarity of EEG data.

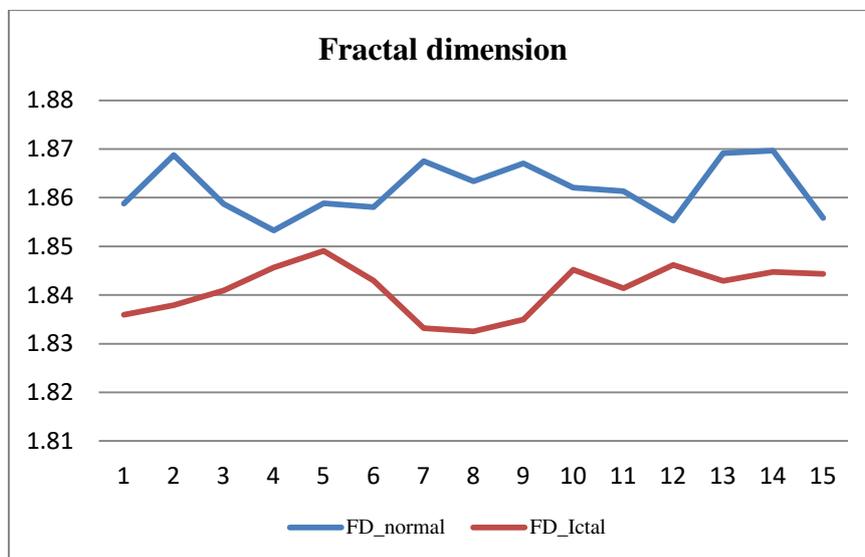
data	Normal EEG		Ictal EEG	
	FD	Lacunarity	FD	Lacunarity
1	1.8587938	1.0712	1.83593773	1.0583
2	1.8687584	1.0667	1.83790734	1.0604
3	1.8587412	1.0667	1.84092056	1.0612
4	1.8532658	1.0758	1.84562964	1.0673
5	1.85885	1.103	1.84906309	1.0867
6	1.8580707	1.0795	1.8429844	1.0789
7	1.8674987	1.0784	1.83321027	1.0757
8	1.8633969	1.0813	1.83254308	1.0763
9	1.8670611	1.0828	1.83497991	1.0771
10	1.8620664	1.1094	1.84517828	1.0779
11	1.8613593	1.1067	1.84138684	1.0777
12	1.8552895	1.0887	1.84618899	1.0799
13	1.8691467	1.0681	1.84292262	1.0584
14	1.869693	1.0669	1.84473911	1.065
15	1.8558071	1.0618	1.84432064	1.0637

Table-1 shows that the value of FD is not changing much as they approximately have the same values for FD. L also helps to differentiate the data from each other.

Similarly, different values of FD can be obtained by applying different seizure signal to the box counting algorithm. To analyze the FD of pre-ictal and ictal stages of EEG data can finalize the scope of arriving at the information in the EEG data having seizure attacks. From the result, it concludes that the value of FD of the seizure

signal decreases sharply; its FD is less compared to the FD of the normal signal.

Figure-5 shows the FD plot of the normal and ictal EEG data. The difference in the FD value in both classes of EEG data is clearly observed in the plot. The FD of ictal EEG has less value compared with FD of the normal data. The descriptive statistics shows the maximum and minimum values are (1.8696, 1.8490) and (1.85580, 1.83593), respectively.

**Figure-5.** Fractal dimension of ictal EEG and normal EEG signals.

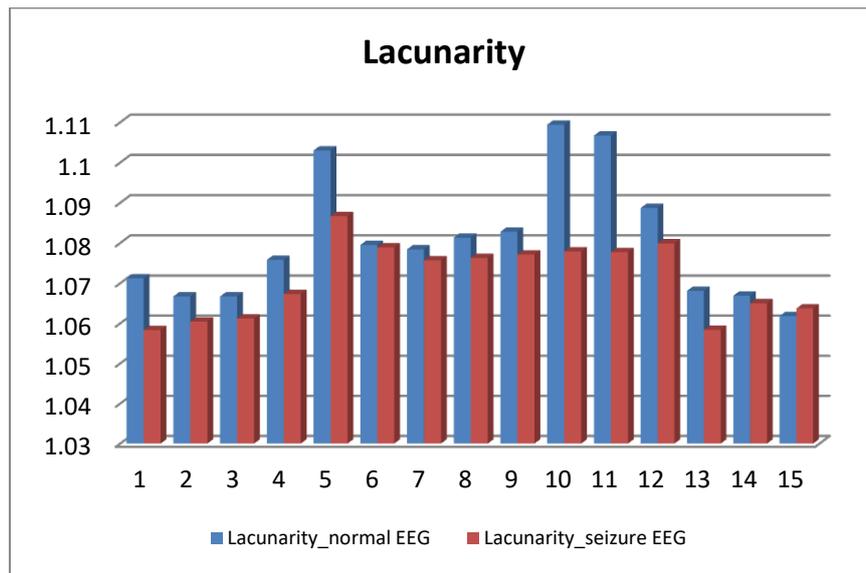


Figure-6. Lacunarity of ictal EEG and normal EEG signals.

The L plot of the normal EEG and seizure EEG data are shown in Figure-6. The graph shows that the values obtained for the normal data set is lower than the values of the ictal EEG data. In other words, the seizure data is more heterogeneous than the normal one. As the heterogeneity increases, the signal becomes more complex, and a corresponding change in the L measure occurs.

**A. Parametric statistical test**

To analyze the data, a parametric statistical test of the FD and L of the normal and ictal data set was conducted with a significance level of 5%. The result

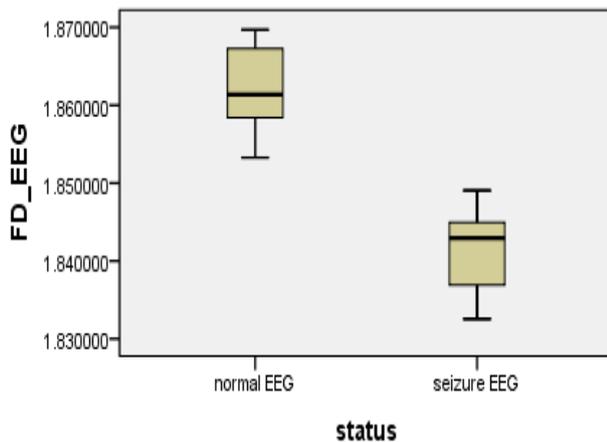
shows that its p value is less than the significance level ( $p < 0.001$ ). Therefore, our null hypothesis is rejected and an alternate hypothesis is that a significant difference in two classes of EEG data sets exists. Table-2 shows the descriptive statistics of FD and L in both classes. Table-3 shows a statistically significant difference between the normal EEG and ictal EEG groups as determined by t test ( $t = 864.853, p < 0.001$ ;  $t = 441.679, p < 0.001$ , respectively). Additionally, a difference exists in the FD of ictal EEG and normal EEG signals. The box plot of FD is shown in Figure-7.

Table-2. Descriptive statistics.

	N	Min.	Max.	Mean	SD
FD_EEG	30	1.83254	1.86969	1.85152	0.01173
Lacunarity_EEG	30	1.0583	1.1094	1.07572	0.01334
Valid N (list wise)	30				

Table-3. Result of one-sample t test.

	Test Value = 0					
	t	df	Sig. (2-tailed)	Mean difference	95% confidence interval of the difference	
					Lower	Upper
FD_EEG	864.853	29	.000	1.851523703	1.84714517	1.85590224
Lacunarity_EEG	441.679	29	.000	1.0757167	1.070735	1.080698



**Figure-7.** Box plot of fractal dimension.

## 5. CONCLUSIONS

The EEG analysis for epilepsy detection is one of the important studies in the biomedical field. Automatic epilepsy detection is very important because of the random and complex nature of EEG. Thus, EEG analysis is time-consuming and tedious for physical analysis and verification. Thus, in this paper, a new method is introduced. It involves the FD calculation, and we predict the presence of ictal from that. According to the EEG image, the box counting method is used for the calculation of FD.

According to the fractal analysis of EEG signal using box counting algorithm, an effective method to detect the presence of ictal signal from normal signal is shown. The FD of the ictal image decreases sharply from a higher value. It is low compared with the FD of the normal EEG. The heterogeneity is also calculated by measuring the L of the ictal and normal EEG data. The ictal EEG signal is more heterogeneous than the normal EEG. Based on these assumptions, the fractal measures, such as the calculated FD and L are statistically tested using parametric test. In this analysis, the p value is low ( $p < 0.001$ ), and this helps differentiate the normal data set from the ictal data set with high accuracy.

## REFERENCES

- [1] Akareddy, Sharanreddy Mallikarjun, P. K. Kulkarni. 2013. EEG signal classification for epilepsy seizure detection using improved approximate entropy. *Int J public health science (IJPHS)*. 2(1): 23-32.
- [2] Sanei Saeid, J. A. 2013. *Chambers. EEG signal processing*. John Wiley & Sons.
- [3] Iasemidis Leon D. 2003. Epileptic seizure prediction and control. *IEEE T BIO-MED ENG*. 50(5): 549-558.
- [4] Chaudhuri, Bidyut Baran, Nirupam Sarkar. 1995. Texture segmentation using fractal dimension. *IEEE T pattern anal*. 17(1): 72-77.
- [5] Acharya, U. Rajendra, S. Vinitha Sree, G. Swapna, Roshan Joy Martis, Jasjit S. Suri. 2013. Automated EEG analysis of epilepsy: a review. *Knowl-based Syst*. 45: 147-165.
- [6] Li, Mingyang, Wanzhong Chen, Tao Zhang. 2016. Automatic epilepsy detection using wavelet-based nonlinear analysis and optimized SVM. *Biocybern biomed Eng*. 36(4): 708-718.
- [7] Fu, Kai, Jianfeng Qu, Yi Chai, and Yong Dong. 2014. Classification of seizure based on the time-frequency image of EEG signals using HHT and SVM. *Biomed signal proces*. 13: 15-22.
- [8] Hao Qu, Jean Gotman. 1997. A patient-specific algorithm for the detection of seizure onset in long-term EEG monitoring: Possible use as a warning device. *IEEE T bio-med Eng*. 44 (2): 115-122.
- [9] Khan, Yusuf Uzzaman, Omar Farooq. 2015. Wavelet-based multi-class discrimination of EEG for seizure detection. *International Journal of Biomedical Engineering and Technology*. 19(3): 266-278.
- [10] Biju K. S., Jibukumar M. G. 2018. Ictal EEG Classification based on State Space Modeling of Intrinsic Mode Function. *Procedia Computer Science*. 125: 468-475.
- [11] Acharya, U. Rajendra, Hamido Fujita, Vidya K. Sudarshan, Shreya Bhat, Joel EW Koh. 2015. Application of entropies for automated diagnosis of epilepsy using EEG signals: a review. *Knowl-based Syst*. 88: 85-96.
- [12] Yuan Q, Zhou W, Liu Y, Wang J. 2012. Epileptic seizure detection with linear and nonlinear features. *Epilepsy behav*. 24(4): 415-421.
- [13] Beheshti S. M. A., H. Ahmadi Noubari, E. Fatemizadeh, M. Khalili. 2016. Classification of abnormalities in mammograms by new asymmetric fractal features *Biocybern Biomed Eng*. 36(1): 56-65.
- [14] Long, Min, Fei Peng. 2013. A box-counting method with adaptable box height for measuring the fractal feature of images. *Radio engineering*. 22(1): 208-213.
- [15] Zhang, Yanli, Weidong Zhou, Shasha Yuan, Qi Yuan. 2015. Seizure detection method based on fractal dimension and gradient boosting. *Epilepsy behav*. 43: 30-38.



- [16] Mandelbrot Benoit B. 1983. The fractal geometry of nature, Revised and enlarged edition. New York, WH Freeman and Co. 495, p. 1.
- [17] Chaudhuri, Bidyut Baran, Nirupam Sarkar. 1995. Texture segmentation using fractal dimension. *IEEE T pattern anal.* 17(1): 72-77.
- [18] Beheshti, S. M. A., H. Ahmadi Noubari, E. Fatemizadeh, M. Khalili. 2016. Classification of abnormalities in mammograms by new asymmetric fractal features. *Biocybern biomed eng.* 36(1): 56-65.
- [19] Esteller R, Vachtsevanos G, Echauz J, Litt B. 2001. A comparison of waveform fractal dimension algorithms. *IEEE T circuits-I.* 48(2): 177-83.
- [20] Kesić S, Spasić SZ. 2016. Application of Higuchi's fractal dimension from basic to clinical neurophysiology: A review. *Computer Methods and Programs in Biomedicine.* 30(133): 55-70.
- [21] Dougherty, Geoffrey, Geoffrey M. Henebry. 2001. Fractal signature and lacunarity in the measurement of the texture of trabecular bone in clinical CT images. *Med Eng Phys.* 23(6): 369-380.
- [22] Lee, Bum Han and Sung Keun Lee. 2013. Effects of specific surface area and porosity on cube counting fractal dimension, lacunarity, configurational entropy, and permeability of model porous networks: Random packing simulations and NMR micro-imaging study. *J HYDROL.* 496: 122-141.
- [23] Vernon-Carter J, Lobato-Calleros C, Escarela-Perez R, Rodriguez E, Alvarez-Ramirez J. 2009. A suggested generalization for the lacunarity index. *Physica A.* 388(20): 4305-14.
- [24] Al-Kadi, Omar S., D. Watson. 2008. Texture analysis of aggressive and nonaggressive lung tumor CE CT images. *IEEE T Bio-med Eng.* 55(7): 1822-1830.
- [25] Petrou, Maria, Pedro García Sevilla. 2006. Image processing: dealing with texture.
- [26] [http://epileptologie-bonn.de/cms/front\\_content.php?idcat=495&idart=855](http://epileptologie-bonn.de/cms/front_content.php?idcat=495&idart=855)
- [27] Andrzejak RG, Lehnertz K, Mormann F, Rieke C, David P, Elger CE. 2001. Indications of nonlinear deterministic and finite-dimensional structures in time series of brain electrical activity: Dependence on recording region and brain state. *Phys Rev E.* 64.6: 061907.
- [28] Li Jian, Qian Du, and Caixin Sun. 2009. An improved box-counting method for image fractal dimension estimation. *Pattern Recogn.* 42(11): 2460-2469.
- [29] Liu, Yu, Lingyu Chen, Heming Wang, Lanlan Jiang, Yi Zhang, Jiafei Zhao, Dayong Wang, Yuechao Zhao, Yongchen Song. 2014. An improved differential box-counting method to estimate fractal dimensions of gray-level images. *J VIS Commun image R.* 25(5): 1102-1111.
- [30] Theera-Umpon, N. 2002. Fractal dimension estimation using modified differential box-counting and its application to MSTAR target classification. In. *IEEE 2002 on Systems, Man and Cybernetics conference; 6-9 Oct.* pp. 537-541.
- [31] Foroutan-pour, Kayhan, Pierre Dutilleul, Donald L. Smith. 1999. Advances in the implementation of the box-counting method of fractal dimension estimation. *Appl Math Comput.* 105(2): 195-210.
- [32] Sarkar N, Chaudhuri BB. 1994. An efficient differential box-counting approach to compute fractal dimension of image. *IEEE T Syst Man Cyb.* 24(1): 115-20.
- [33] Smith TG, Lange GD, Marks WB. 1996. Fractal methods and results in cellular morphology-dimensions, lacunarity and multifractals. *J Neurosci Meth.* 69(2): 123-36.
- [34] Do Eirado Amorim LM, Barros Filho MN, Cruz D. 2014. Urban texture and space configuration: an essay on integrating socio-spatial analytical techniques. *CITIES.* 39: 58-67.
- [35] Klonowski W. 2000. Signal and image analysis using chaos theory and fractal geometry. *Machine Graphics and Vision.* 9(1/2): 403-32.
- [36] Barros Filho MN, Sobreira FJ. 2008. Accuracy of lacunarity algorithms in texture classification of high spatial resolution images from urban areas. *ISPRS Archives.* 37: 417-22.