



SINGLE-CONTOUR GEODESIC SHELLS WITH QUADRANGLE PLATES

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ABSTRACT

Geodesic domes are a kind of building structures for a breakdown of which are used geodesic lines. In this area, the researches of geometric models of geodesic domes of classical breakdown are conducted. Herewith plates predominantly of triangular and hexagonal shape are used. However, during the formation of shells, you can also use plates of quadrangular and pentagonal shape. The purpose of this research is developing of methods, algorithms and software for geodesic shells with quadrangular plates. To solve these problems were used methods of analytical geometry in space, nonlinear optimization and programming in the language GDL embedded in the ArchiCAD. As a result, a parametric ArchiCAD object was developed for single-contour geodesic shells with quadrangular plates. This parametric object can be used by architectural and construction design of unique buildings.

Keywords: geodesic domes, a breakdown of geodesic shells, parametrical modelling, parametrical objects for modelling of geodesic domes.

INTRODUCTION

Domes are spatial structures which are able to cover large spans without additional intermediate supports. Geodesic domes are a type of domes, which are the result of the breakdown of a sphere into elements using geodesic lines. Geodesic domes are used for unique buildings and structures: stadiums, exhibitions, administrative buildings.

The formations of spherical single-contour plate shells can be based on the following approaches: 1) the breakdown of the spherical surface on the elements (computer breakdown of geodesic shells), 2) using of regular, semi-regular, composite, corrugated polyhedrons (crystal shells), 3) projection of polyhedron faces on a sphere and further breakdown of the resulting triangles (classical breakdown of geodesic shells).

The first approach includes operations to determine the configurations of geodesic domes of computer breakdown using optimization methods [1, 2]. The optimization problem is finding the cross sections of elements, the height for the crown and the number of elements under certain loads for the purpose of the minimum weight of the structure. These problems are solved using heuristic algorithms. You can get a breakdown of the sphere into regular triangles, regular hexagons (with 12 pentagons). An example of such works is the use of computer graphics to generate C-TANGENT domes that approximate the sphere [3].

The second approach includes the work on determining the configuration of crystal shells when using regular polyhedrons with triangular, quadrangular and pentagonal faces [4]. By the augmentation procedure, additional pyramids are placed on the faces of the original polyhedrons. This procedure can be considered as an additional breakdown of the circumscribed sphere. By extending of this method to semi-regular polyhedrons [5] you can get composite and corrugated polyhedrons. The faces of derived polyhedrons can be triangular, quadrangular, pentagonal and hexagonal. Examples of

such polyhedrons are Goldberg polyhedra [6], polyhedra analogues of the Platonic solids [7].

The third approach includes studying of methods for determining the parameters of geodesic domes of the classical breakdown when polyhedron (icosahedrons, tetrahedron, octahedron) is taken as starting point and its faces are projected on the sphere [8, 9]. These faces are regular curvilinear triangles (Moebius triangles) are divided into Schwarz triangles and smaller elements in the same way. This leads to economical breakdowns with a small number of element types. Example of such geodesic domes are Fuller domes [10], Pavlov shells [11, 12] and using the method step-by-step projections [13]. Herewith geodesic domes consist of elements mostly triangular or hexagonal shape, in this case, hexagons are often further divided into triangles.

Thus, the faces of geodesic shells are mainly triangular and hexagonal in shape. While the faces of the crystal shells are more diverse. It is also found that geodesic domes of quadrangular and pentagonal plates are possible [14].

Now a library of parametric objects (ArchiCAD GeoDome Library) of geometric models of geodesic shells in the GDL language is been built. It contains objects of different classes of geodesic shells. In the library of GeoDome Lib. v.1.0, there is no software implementation of the library objects of single-contour geodesic shells with quadrangular plates based on the icosahedron. Therefore, the task was set to develop a parametric object of ArchiCAD geometric models of the corresponding class of geodesic shells.

METHOD OF BREAKDOWN OF A GEODESIC SHELL, CALCULATION OF COORDINATES

The purpose is to get the shell surface which is mostly composed of flat quadrangular plates when building of library object of single-contour geodesic domes of class I1; 4. The initial library object is of class I1; 6 [11] consisting of flat hexagonal plates.

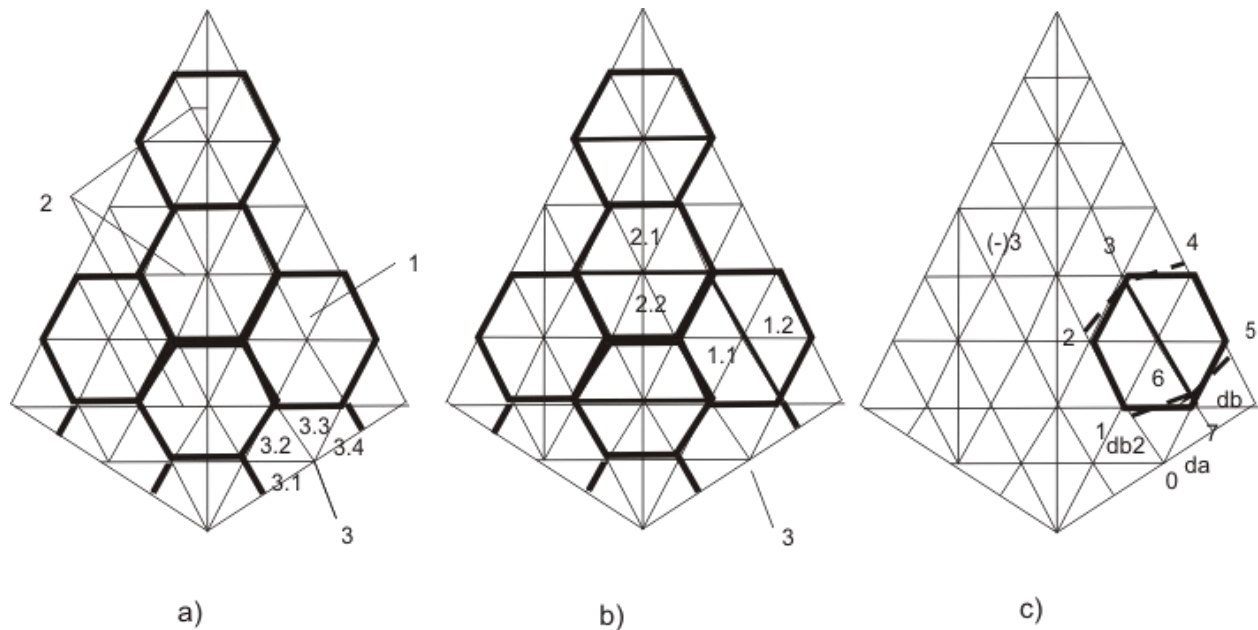


Figure-1. Network breakdown of class I1; 6 (a) and I1; 4 (b), change the lengths of arcs of class I1; 4 (c) (—— – initial, — — – modified).

There are three groups of plates on the surface of the Schwarz triangle: 1) main plates - flat hexagons located along the big leg of the Schwarz triangle, 2) one row of auxiliary plates - hexagons located along the hypotenuse of the Schwarz triangle, 3) one row of auxiliary plates - triangles located along the small leg of the Schwarz triangle (see Figure-1, a).

To form a surface of that quadrangular plates the following changes are proposed (Figure-1, b): 1) the main plates - the hexagons must be divided into two quadrangles 1.1 and 1.2, 2) the auxiliary plates - hexagons must be divided into two quadrangles 2.1 and 2.2, 3) the auxiliary plates - triangles must be united in quadrangles 3.

For the creation of edges forming quadrangular plates (the first group of plates), you must change the length of the arcs $db2 - \delta$, $db + \delta$ ($\delta=0.01$). As the points corresponding to the corners remain on the lines of great circles, the bases of the resulting quadrangular regular trapezoids will rise and the top of the trapezoids will fall. As a result, an edge is formed along the line 3 - 6 (Figure-1, c).

For the second group of plates is the same effect: points 2 will fall and point 3 will rise, along with the line $(-3) - 3$ will appear an edge forming quadrangular plates in the form of regular trapezoids.

For the third group of plates in the form of quadrangles (for example, points 0-1-6-7) the automatic flat form is not possible. It is necessary to make calculating of the coordinates of the fourth point (point 0) so that it is in the plane 1-6-7 (Figure-1, c).

Point p_4 in the original version lies on the line of the great circle and at distance $r=1$ from the centre of the

sphere (Figure-2, a). However, it doesn't lie in the plane $p_1-p_2-p_3$, so the quadrangle is not flat.

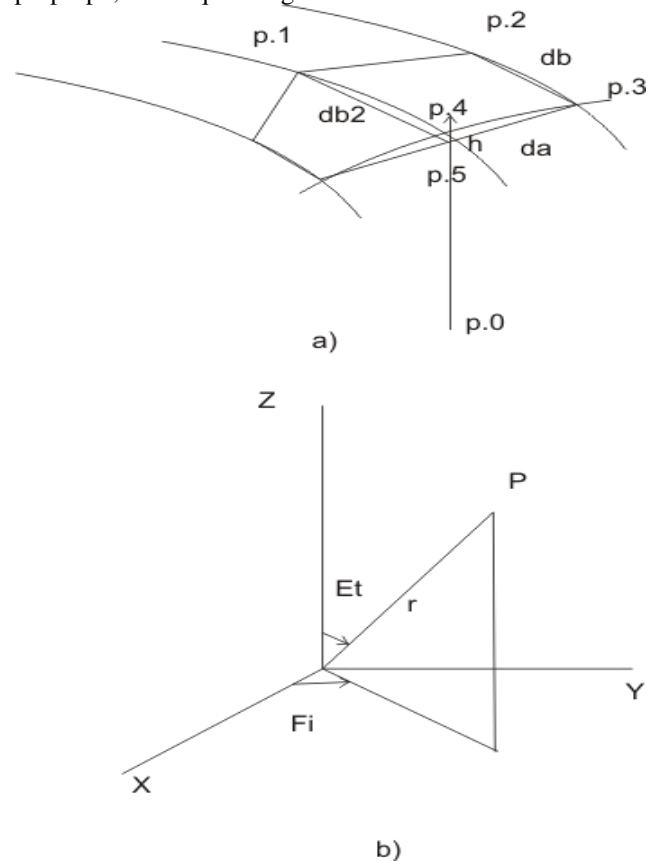


Figure-2. Calculating the coordinates of a point of the quadrangle a), a relationship of spherical and Cartesian coordinates b).



To form a flat quadrangle p1-p2-p3-p5 it is necessary to move the point p5 along the vector p0-p4 at the required distance h. The movement of the point p5 be performed in the spherical coordinate system (Figure-1, b) and check the belonging of the point p5 to the plane p1-p2-p3 in the Cartesian coordinate system. For this purpose, the formulas of transition from the spherical coordinate system to the Cartesian coordinate system and back are used.

$$X = r \cdot \sin(Et) \cdot \cos(Fi) \quad (1)$$

$$Y = r \cdot \sin(Et) \cdot \sin(Fi) \quad (2)$$

$$Z = r \cdot \cos(Et) \quad (3)$$

$$r = \sqrt{X^2 + Y^2 + Z^2} \quad (4)$$

$$Fi = \arctg \frac{Y}{X} \quad (5)$$

$$Et = \arctg \left(\frac{\sqrt{X^2 + Y^2}}{Z} \right) \quad (6)$$

where, X, Y, Z are Cartesian coordinates, r, Fi, Et are spherical coordinates (r is the radius of the sphere, Fi is longitude, Et is polar distance).

The equation of the plane in three-dimensional space, passing through three points (p1-p2-p3) has the form:

$$A \cdot X + B \cdot Y + C \cdot Z = D \quad (7)$$

where $A = (Y_2 - Y_1) \cdot (Z_3 - Z_1) - (Z_2 - Z_1) \cdot (Y_3 - Y_1)$, $B = (X_2 - X_1) \cdot (Z_3 - Z_1) - (Z_2 - Z_1) \cdot (X_3 - X_1)$, $C = (X_2 - X_1) \cdot (Y_3 - Y_1) - (Y_2 - Y_1) \cdot (X_3 - X_1)$, $D = X_1 \cdot A + Y_1 \cdot B + Z_1 \cdot C$.

We can calculate the discrepancy to check the belonging of the point p5 of this plane

$$D5 = A \cdot X5 + B \cdot Y5 + C \cdot Z5 - D \quad (8)$$

Thus, we can formulate the problem of nonlinear one-dimensional optimization

$$D5 = F1(A, B, C, D, X5, Y5, Z5) \rightarrow \min \quad (9)$$

where, A, B, C, D are the parameters of the plane, $X5, Y5, Z5$ are coordinates of p5, $\text{Coord}(p5) = F2(Et, Fi, r)$, $r = 1 - h$, h is the displacement of the point p5.

This problem can be solved by one-way descent method. The program flowchart is shown in Figure-3.

In Block 2 the cycle by breakdown variants of the geodesic shell of class I1; 4 (from 3 to 8 variants) is organized. Block 3 specifies the arc length of the original

quadrangle taking into account the change in arc length by $\delta = 0.01$ ($da3 = da$, $db3 = db + \delta$, $bd23 = db2 - \delta$). In Block 4 the coordinates of point's p1, p2, p3 and parameters of the plane passing through these points are determined. Block 5 sets the initial value of the optimization parameter $h = 0$. Next, the coordinates of the point p5 and the value of the discrepancy $D5$ are calculated (Block 6). Then you must check: is the value of discrepancy less than the specified calculation error or not. If the value of the discrepancy is less, the output of the parameter value h is performed in Block 9. After that, the transition to Block 2 is performed. If the value of the discrepancy is more then an error, the sign of the discrepancy is tested in Block 8. If $D5 > 0$, it means that the point p5 fell below the plane p1-p2-p3, perform the transition to the Block 10, where to perform a step back and the value of the step is reduced by 10 times. After that, the transition to Block 6 is performed. If $D5 < 0$ it means that the point p5 is located above the plane p1-p2-p3, then move to the Block 11, in which perform a step forward. After that return to Block 6.

Using the algorithm for calculating the coordinates of the point of the quadrangle of the third type, which is presented in the Flowchart (Figure-3), the program is developed in the programming language Visual Basic.

Table-1. The results of calculation using this program are presented in Table-1.

No breakdown variants	R5	H
3	0.9967410946469	0.003258905353058
4	0.9984481618147	0.001551838185202
5	0.9991802918548	0.0008197081451015
6	0.9995460514618	0.0004539485381978
7	0.9997472410740	0.0002527589259986
8	0.9998652362193	0.0001347637806993

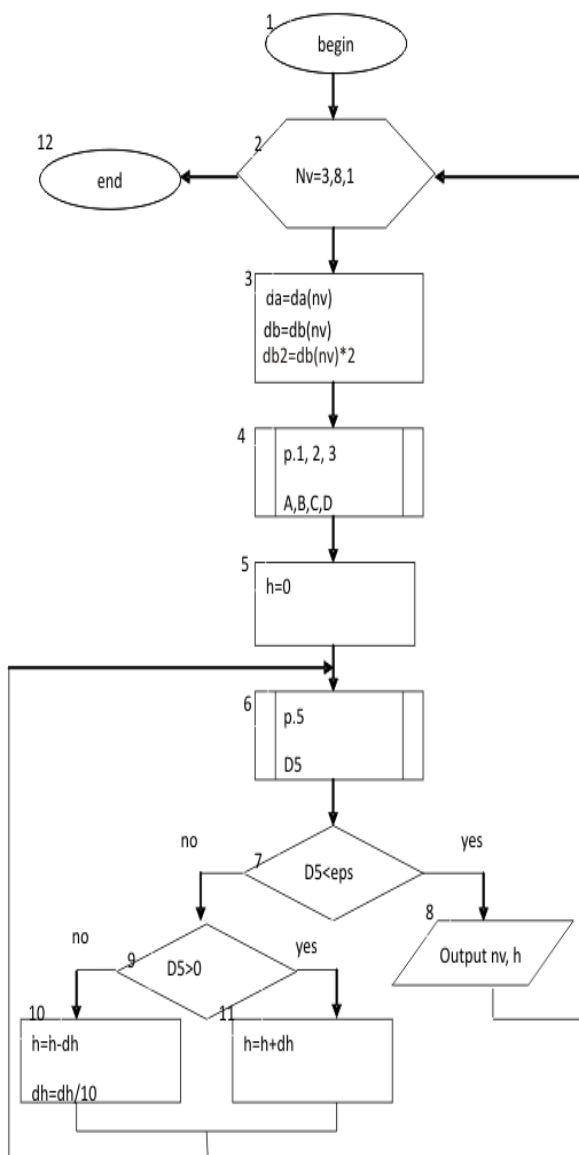


Figure-3. Flowchart of the program for calculating the coordinates of a point of the quadrangle.

IMPLEMENTATION IN GDL LANGUAGE

Using the results of calculations we perform modification of the library object of class I1; 6 for the implementation of the library object of class I1; 4. In this case, we can change the length of the arcs along the line of great circles and the method of calculating the coordinates of the points of quadrangles of the third type.

Listing-1. Changing the lengths of the arcs of quadrangles along the lines of great circles

```
6000:
! Arches new v0.8
if nv=3 then 6003
if nv=4 then 6004
if nv=5 then 6005
if nv=6 then 6006
if nv=7 then 6007
if nv=8 then 6008
6003:
```

```
d[1]=0.079930876325363600
d[6]=0.079930876325363600
d[3]=0.129861752650727000
d[5]=0.129861752650727000
d[1]=d[1]*180.0/ppir
d[6]=d[6]*180.0/ppir
d[3]=d[3]*180.0/ppir
d[5]=d[5]*180.0/ppir
RDEL=0.00325890535305839
goto 6009
6004:
d[1]=0.062542152336684200
d[6]=0.062542152336684200
d[19]=0.062542152336684200
d[3]=0.095084304673368400
d[5]=0.095084304673368400
d[13]=0.095084304673368400
d[1]=d[1]*180.0/ppir
d[6]=d[6]*180.0/ppir
d[19]=d[19]*180.0/ppir
d[3]=d[3]*180.0/ppir
d[5]=d[5]*180.0/ppir
d[13]=d[13]*180.0/ppir
RDEL=0.00155183818520233
goto 6009
```

! for breakdown variants 5, 6, 7, 8 similarly

Listing-2. Subroutine for recalculating the coordinates of a quadrangle point

```
! Quadrangle visualization example
XTMP=x[5] : YTMP=y[5] : ZTMP=z[5]
GOSUB 8000
x[5]=XTMP : y[5]=YTMP : z[5]=ZTMP
!squad 9-10-6-5
slab 4,c,
r*x[9],r*y[9],r*z[9],
r*x[10],r*y[10],r*z[10],
r*x[6],r*y[6],r*z[6],
r*x[5],r*y[5],r*z[5]
!SUB PLATE
```

8000:

```
! Spherical coordinates
FITMP=atn(YTMP/XTMP)
ETTMP=atn(sqrt(XTMP^2+YTMP^2)/ZTMP)
RTMP=1-RDEL
! Radial shift
XTMP=RTMP*sin(ETTMP)*cos(FITMP)
YTMP=RTMP*sin(ETTMP)*sin(FITMP)
ZTMP=RTMP*cos(ETTMP)
return
```

After calculating the coordinates of the plate angle points the shell is rendered. To do this, use the GDL slab function, which as input parameters receives the number of the polygon points, the thickness of the plate and the coordinates of the angle points. It renders the specified plate. By calling slab for all plates, we obtain a geometric model of all shell.

As a result, a library object of single-contour geodesic shells of class I1; 4 (program in the program



language GDL SystemOv.08.gsm) is developed. It can be used to form geometric models of geodesic shells with quadrangular plates. You can get a geometric model corresponding to one Moebius triangle (Figure-4), hemispherical dome (Figure-5), the shell in the form of a sphere (Figure-6).

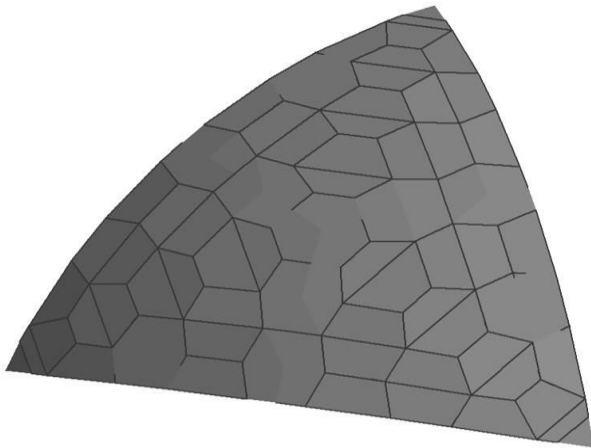


Figure-4. Geometric model of one Moebius triangle of a class I1; 4 geodesic shell.

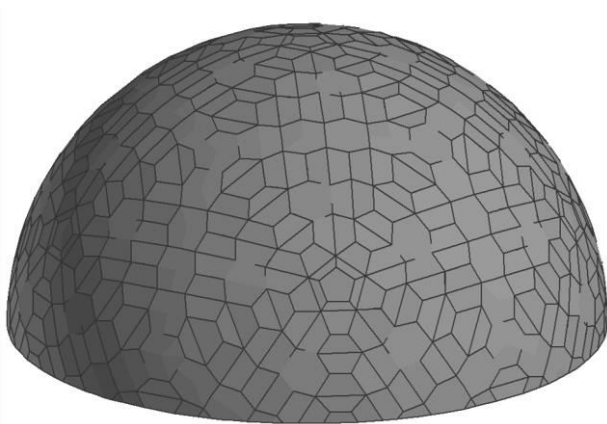


Figure-5. Geometric model of a hemispherical dome of a class I1; 4 geodesic shell.

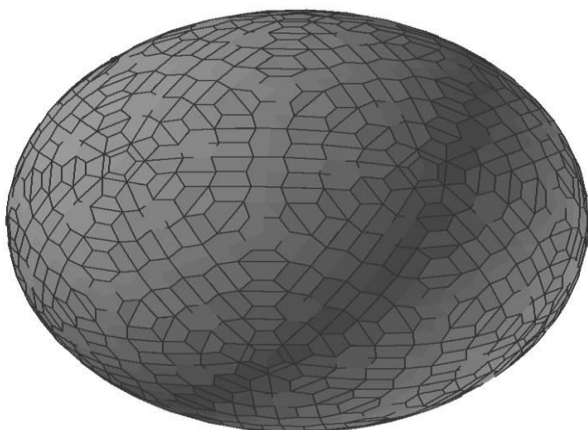


Figure-6. Geometric model of a class I1; 4 geodesic shell in the form of a sphere.

CONCLUSIONS

It was reviewed existing software for geometric modelling of single-contour geodesic shells. It is found that there is no software implementation of single-contour geodesic shells with quadrangular plates. A parametric object ArchiCAD for class I1; 4 of the geodesic shell is developed. This parametric object can be used in the architectural and structural design of unique buildings.

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