A WALKING BIPEDAL ROBOT USING A POSITION CONTROL ALGORITHM BASED ON CENTER OF MASS CRITERION

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ABSTRACT
A position control algorithm based on inverse kinematics and a control strategy utilizing the Center of Mass criterion are implemented to yield a walking bipedal robot. The bipedal robot has no upper body, stands approximately 50 cm and weighs about four kg. Each leg of the robot has five degrees of freedom: two at the hip, two at the ankles, and one at the knee. The closed form solution of each joint angle is derived by using inverse kinematics, following the Denavit-Hartenberg guidelines to determine the structural parameters of the biped. Such closed form equations determine the value of the joint angle to achieve an instance needed to complete the walking activity. Bipedal locomotion is verified by both simulation and experiments. Simulation results provide desired joint angle trajectories and will serve as benchmark for the actual experiments. Experimental results show that actual step length, foot clearance and hip height gait parameters do not exceed one centimeter from the target. Also, the joint angle mean square error has a maximum deviation of 11 degrees.

Keywords: bipedal locomotion, inverse kinematics, center of mass criterion.

1. INTRODUCTION
Mobile robotics has focused on the use of wheels primarily because of ease of control. They have well-developed and understood power and control systems and provide inexpensive solution to every robotics problem [1]. However, the main disadvantage of having wheels is that it is limited to certain types of environments such as flat and smooth surfaces. Because of this, studies regarding legged locomotion have picked up and been undertaken, especially bipedal locomotion. Legged robots are dexterous and have higher mobility that makes them capable of traversing rough and uneven terrains [2].

There are three important things in making a bipedal robot walk: 1) understanding human walking, 2) designing a suitable model for the bipedal robot (mechanical design), and 3) implementing an efficient and stable walking algorithm.

Figure-1. Human Gait Cycle and Parameters.

Bipedal locomotion is a type of movement based on organisms having two legs, particularly humans. Analyzing the human gait is the first step towards the goal of making a bipedal robot walk. This human activity is a simple task yet remains one of the more difficult research problems in multi-body system and robotics [3]. Human walking, shown in Figure-1, is considered as a periodic process, called gait cycle, divided into two phases, namely, 1) swing phase or single support phase (SS) and 2) double support phase (DS). The hip height is the initial height when the robot is in a standing position. A leg is called a stance leg if it is in contact with the ground; otherwise it is called a swing leg. The step length is the horizontal distance of the foot from the hip and the foot clearance is the maximum vertical distance of the foot from the ground. The hip and foot trajectories are the paths undertaken by the hip and foot respectively.

The single support phase, which makes up 60-65% of the walking cycle, takes place when one foot is on the ground (the stance leg) while the other moves (the swing leg). On the other hand, the double support phase, which comprises the remaining 35-40% of the walking cycle, happens when both feet (the stance and swing legs) are on the ground as the body is moving forward. In addition, there are two periods of double support and one period of single-leg stance during the gait cycle [4]. The transitions between the SS and DS stages are also worth noting. The moment of liftoff, immediately at the beginning of the SS, is where the foot is just propelling the body forward so that the leg loses contact with the ground. The other transition between the SS and DS is characterized by a collision of the swing foot with the ground [5].

The main interest and basic requirement for bipedal walking robots have focused on stability in the course of walking [6, 7]. Continuous stable walking can be characterized by some criteria depending on the type of walking. Static walking is characterized by low walking speed and acceleration, and the centrifugal and Coriolis terms are negligible. With increasing speed, the acceleration, centrifugal and Coriolis terms become larger, which illustrates dynamic walking [6, 8]. One criterion considered in static walking or low-speed walking is the Center of Mass (CoM) principle [9]. CoM represents the unique point in a system that describe the system’s response to external forces and torques. This method aims
to maintain the projection of the global CoM of the robot inside the area inscribed by the feet that are in contact with the ground. The closer the vertical projection of the CoM is to the center of the support polygon formed by the feet, the more stable the robot. The lower the CoM, the more stable the system. However, the posture of the biped should imitate that of a human being.

The natural dynamics of a biped is one of the ways of simplifying the control system. The choice of actuators and sensors affects the levels of performance of the biped. According to [10], the natural dynamics can be exploited in the control of a bipedal walking robot.

The study of stability and mobility of these mechanisms in a range of environmental conditions have been one of the main topics in advanced robotic research. Genetic algorithm has been used to generate walking motions in an ascending slope [11]. In [12], a fuzzy logic controller is developed to maintain bipedal stability during locomotion while traversing uneven terrains. The study only uses data coming from the IMU sensor monitoring the robot’s posture. The ever-reliable PID controllers are still widely used in implementing bipedal walking [13, 14]. In [13], PID is used to track under actuated systems instead of partial feedback linearization. A predictive PID controller is presented in [14] to track the positions and angular momentum of the right and left legs to reduce modeling errors. These research studies, however, rely heavily on the model that do not focus on achieving closed form solution for each of the joint angle. In most cases, the bipedal locomotion problem becomes an optimization problem, i.e., satisfying an objective function subject to certain constraints. Developing closed form solutions is difficult to derive but is very efficient and fast during implementation.

In this work, a walking bipedal robot employing a position control algorithm satisfying the Center of Mass criterion is implemented. Stable walking is demonstrated along a flat terrain while imitating a human gait. The major contributions of this work are summarized below:

a) Closed form solutions in determining the joint angles are derived using inverse kinematics following the Denavit-Hartenberg (DH) guidelines. The bipedal robot is treated as two decoupled three-link robot manipulators. Online calculations from on-board controllers are implemented.

b) Extensive simulations of bipedal locomotion using combinations of walking gait parameters are tested to provide benchmark for the actual bipedal walking.

c) Experimental results of bipedal locomotion are presented and compared with the simulated biped.

This paper is organized as follows. Section 2 discusses the derivation of the closed form solution of the six joint angles of the biped used in the experiments and the biped and hardware used in the experiments. Section 3 presents the discussion of simulation and experimental results. Finally, Section 4 concludes the work and gives some recommendations for future work.

2. METHODOLOGY

To come up with a suitable position control walking algorithm, motion planning and environmental conditions, and stability criterion and type of walking are noted. Motion planning for the biped dictates which parameters are to be considered, studied, and adjusted for the biped to achieve a certain type of motion. This focuses on the type of surface the biped will be tested. The stability criterion establishes the type of motion that the biped will undergo.

The design and implementation phases are divided into two parts: mechanical (biped) system and control system, as shown in Figure-2. The mechanical phase consists of limiting the biped’s movements and lessening the effects of friction. On the other hand, the control system provides the implementation of the position control algorithm based on the DH guidelines [15].

Before the position control algorithm is implemented in the actual biped, simulations will be performed to verify the walking algorithm. The results of the simulations will then be compared to the data gathered from the actual implementation.

2.1 The biped system

A bipedal robot with no upper limbs will be the testbed in this study [16] [17]. The biped is shown in its front and side views in Figure-3. A rubber is placed at the foot and at the ground to ensure that slippage does not occur. Also, to limit the movement of the bipedal robot along the sagittal plane, the hip and ankle roll joints are physically locked and is attached to a metallic bar overhead by support ropes during experiments to avoid major damage to the bipedal platform.
The biped is made from aluminum joined by bolts and nuts. For its joints, the system uses Tormax DC motors to control the hip movement along the lateral plane while the rest of the joints employ micro-geared DC motors. Each leg has five degrees of freedom, i.e., two at both the hip and ankle and one at the knee. Inexpensive and off-the-shelf potentiometers measure the joint movements. The total mass of the biped is approximately four kilograms (4 kg).

Each leg of the biped has its set of its own motor drivers, signal conditioning circuits and power supply. However, both legs are controlled by a single TMS320LF2407 DSP controller. This is shown in Figure-4 below.

Figure-4. TMS320LF2407 DSP controller (top) and Motor Driver Boxes (bottom) of the Experimental Bipedal Platform.

Figure-5 shows all possible minimum hip heights and step lengths achievable by the biped platform. For a given minimum hip height or step length, a corresponding range of step lengths or minimum hip heights can be computed, respectively. The smaller the given hip height, the wider the range of the step lengths will be and vice versa. Note that the minimum hip height and step length axes are normalized to one. The curve line represents the boundary between reachable and unreachable workspace. Part of the reachable workspace is the unusable part because of two reasons: 1) heights from 0-0.5 do not satisfy human walking and 2) heights ranging from 0.5-0.7 correspond to bipedal platform limitation.

2.2 Denavit-hartenberg (DH) guidelines
The biped is treated as a robotic manipulator composed of a set of links connected at various joints. To achieve stable and human-like bipedal locomotion, each link must move to its corresponding desired position. To relate the entire set of joint variables to the position and orientation of the end-effector and vice versa, forward and inverse kinematics are utilized, respectively.
The assignment of coordinate frames to each joint follows the DH convention and is shown in Figure-6. The red arrows represent the x-axis, green arrows correspond to the y-axis and the blue arrows denote the z-axis. Numbers represent the length in cm of each link of the biped.

Since all are revolute joints, the transformation matrix relating the \( i \)th link to the \((i-1)\)th link is given by (1).

\[
\begin{align*}
A_{i-1}^i &= \\
&= \begin{bmatrix}
\cos \theta_i & -\cos \alpha_i \sin \theta_i & \sin \alpha_i \sin \theta_i & a_i \cos \theta_i \\
\sin \theta_i & \cos \alpha_i \cos \theta_i & -\sin \alpha_i \sin \theta_i & a_i \sin \theta_i \\
0 & \sin \alpha_i & \cos \alpha_i & d_i \\
0 & 0 & 0 & 1
\end{bmatrix}
\end{align*}
\]

(1)

For example, the transformation matrix for link \( i=1 \) is given in (2).

\[
\begin{align*}
A_{0}^1 &= \\
&= \begin{bmatrix}
\cos \theta_1 & 0 & a_1 \cos \theta_1 \\
\sin \theta_1 & 0 & a_1 \sin \theta_1 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\end{align*}
\]

(2)

Stance leg:

\[
k_i = s
\]
\[
k_z = h - l_i - l_4
\]
\[
\theta_{knee} = -\cos^{-1}\left(\frac{k_1^2 + k_2^2 - l_2^2 - l_3^2}{2l_1 l_4}\right)
\]

(3)

\[
\theta_{ankle} = \tan^{-1}\left(\frac{k_1 (l_3 \cos \theta_{knee} + l_2) - k_2 l_1 \sin \theta_{knee}}{k_1 (l_3 \cos \theta_{knee} + l_2) + k_2 l_1 \sin \theta_{knee}}\right)
\]

\[
\theta_{hkp} = -\theta_{knee} - \theta_{ankle}
\]

Table-1. Derived structural kinematic parameters of the biped.

<table>
<thead>
<tr>
<th>Link i</th>
<th>( a_i ) (cm)</th>
<th>( \alpha_i ) (in(^\circ))</th>
<th>( \theta_i ) (in(^\circ))</th>
<th>( d_i ) (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.1</td>
<td>90</td>
<td>( \theta_1 )</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>16.9</td>
<td>0</td>
<td>( \theta_2 )</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>17.4</td>
<td>0</td>
<td>( \theta_3 )</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>5.1</td>
<td>-90</td>
<td>( \theta_4 )</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>90</td>
<td>( \theta_5 )</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>5.1</td>
<td>-90</td>
<td>( \theta_6 )</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>17.4</td>
<td>0</td>
<td>( \theta_7 )</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>16.9</td>
<td>0</td>
<td>( \theta_8 )</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>9.1</td>
<td>90</td>
<td>( \theta_9 )</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>0</td>
<td>( \theta_{10} )</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>0</td>
<td>( \theta_{11} )</td>
<td>0</td>
</tr>
</tbody>
</table>

Usable Workspace

Unreachable Workspace

\[
\text{height} = \sqrt{1 - (\text{step length})^2}
\]
Swing leg:

\[ k_1 = -s \]
\[ k_2 = h - l_s - l_{10} \]
\[ \theta_{knee} = -\cos^{-1}\left(\frac{k_1^2 + k_2^2 - h^2 - l_s^2}{2 \cdot k_1 \cdot l_s}\right) \] (4)
\[ \theta_{hip} = \tan^{-1}\left(\frac{k_1 (l_s \cos \theta_{knee} + l_s) - k_2 l_s \sin \theta_{knee}}{k_2 (l_s \cos \theta_{knee} + l_s) + k_1 l_s \sin \theta_{knee}}\right) \]
\[ \theta_{ankle} = -\theta_{knee} - \theta_{hip} \]

where \( s \) = step length, \( h \) = hip height, \( l_s \) = link \( x \) length, \( x=1,2,3,4,7,8,9,10 \) and \( \theta_{hip} \) = hip angle, \( \theta_{knee} \) = knee angle and \( \theta_{ankle} \) = ankle angle.

When deriving the inverse kinematic equations, the biped can be decoupled and treated as two three-link planar manipulators, instead of a single six degree of freedom manipulator. By decoupling the system, we can identify which of the two is the stance or swing leg. The world coordinate frame is assigned at the stance leg; thus, it is necessary to obtain the unknown joint angles for this leg before obtaining the joint angles for the swing leg. We also assume that the orientations of the hip and foot for both the stance and swing legs are set to be perpendicular to the ground. This assumption provides a relationship between the hip, knee and joint angles, i.e., \( \theta_{hip} + \theta_{knee} + \theta_{ankle} = 0 \). To realize the movement of the biped along the sagittal plane only, the hip and ankle roll joint angles of the two legs are equated to zero.

Given these, the solutions to the inverse kinematics problem for both the stance and swing legs are given in (3) and (4), respectively. For a complete derivation of the equations, the reader is referred to [18]. Since the derived closed form solutions of each angle are already determined, an online approach can be implemented efficiently and fast by a digital computer.

### 2.3 Control system

The walking pattern of the biped is dependent on the gait parameters: step length, maximum foot clearance, and minimum and maximum hip heights specified by the researcher. Simulations are done to check on the walking gait parameters through stick diagrams. The simulation also outputs the trajectories taken by the hip, knee and ankle joints, the location of the center of mass of the biped while walking and the locations of the foot and hip.

Since the hip is unactuated in the lateral plane, the center of mass of the biped is always maintained at the middle of the two legs. This instability allows the biped to move forward, thus, allowing the robot to walk. The ropes tied to the platform maintain the hip’s orientation to be always perpendicular to the ground. More importantly, it prevents the biped from falling to its swing leg side during the SS stage, if the swing leg is swinging too much.

The control system implements position control of the biped by monitoring the movements of each link by using the TMS320LFP2407A-40MHz DSP board. The DSP monitors the current position of the biped, computes the error for each joint and produces the appropriate PWM duty cycle to achieve desired joint position by using a digital PID controller. The discrete PID controller has the form given in (5).

\[ u(n) = u(n-1) + K_p e(n) + K_i e(n-1) + K_d e(n-2) \]
\[ K_1 = K_p + \frac{K_i T}{2} + \frac{K_d}{T} \]
\[ K_2 = -K_p + \frac{K_i T}{2} - \frac{2K_d}{T} \]
\[ K_3 = \frac{K_d}{T} \]

where \( u(n) \) and \( e(n) \) denote the PWM control value and joint angle error, respectively. \( K_1, K_2, K_3 \) are the digital PID constants while \( K_p, K_i \) and \( K_d \) are the analog proportional, integral and derivative constants. \( T \) is the sampling time of the DSP equal to 20 ms.

### 3. RESULTS AND DISCUSSIONS

Using the biped workspace, the gait parameters, step length, hip heights (minimum and maximum) and maximum foot clearance are chosen. Also, the number of intervals, \( N \), that will dictate in how many positions will there be for the biped to attain the walking process is given. Table-2 shows a sample of a set of gait parameters being investigated during locomotion. The sampling time of the trajectory is chosen to be at 200 ms, thus, a step rate of 1 step/sec.

<table>
<thead>
<tr>
<th>Gait Parameters</th>
<th>Trial 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step Length [cm]</td>
<td>2.675</td>
</tr>
<tr>
<td>Minimum hip height [cm]</td>
<td>46.075</td>
</tr>
<tr>
<td>Maximum hip height [cm]</td>
<td>48</td>
</tr>
<tr>
<td>Maximum foot clearance [cm]</td>
<td>5</td>
</tr>
<tr>
<td>N</td>
<td>5</td>
</tr>
<tr>
<td>Joint angle sample time interval [ms]</td>
<td>200</td>
</tr>
</tbody>
</table>

### 3.1 Simulation results

The simulation results for Trial 1 are shown below. Figure-7 shows the bipedal locomotion represented by a stick diagram. The hip and ankle links are always perpendicular to the ground, while the foot is at all the times maintained parallel to the ground. There are exactly five different positions when the biped attempts a single step movement, as emphasized by the numbers 1, 2, 3, 4, and 5 in the figure.
Figure-7. Bipedal Locomotion presented as Stick Diagram.

The first and second positions shift the biped from the double support to the single support phase while position 3 depicts that the biped has reached its maximum allowable foot clearance. From the fourth to the fifth position, the biped enters the double support phase again and reaches the given step length. At this time, the hip is located at the middle of the two legs. The distance of either foot from the hip before and after a step is taken is equal to $\pm$ the given step length.

Figure-8 presents the trajectories and positions undergone by the hip and foot during the walking process. The hip trajectory has twice the frequency of the right or left foot trajectory. The movements of the hip and foot follow a triangular trajectory so that computations are also simplified because they follow a linear function.

Figure-8. Desired Hip (Top) and Foot (Bottom) Trajectories.

In Figure-8, note that as the swing foot and hip traverse the numbered positions, the stance foot stays in only one position, as indicated by the boxed portion of the diagram.

Figure-9 illustrates the trajectories of the different joint angles. The knee joint angle trajectories follow an almost linear function since it has a wider range of movement compared to the other two joints. This is one of the reasons why the knees consume more energy and is a factor considered why each leg employs its own power supply. On the other hand, the hip and ankle trajectories display an even symmetry with respect to each other.

Figure-9. Desired Joint Angle Trajectories: Ankle (Top), Knee (Center) and Hip (Bottom).

The encircled regions of Figure-8 and Figure-9 depict the moment when the biped proceeds from a standing position to its starting position. Shortly after, the biped begins to walk.

Figure-11 shows the location of the center of mass of the biped during locomotion at each $N$. The CoM of each link is placed at its respective geometric center. The global CoM of the biped is then computed by utilizing (6).
Figure-10. Actual and Desired Joint Angle Trajectories: hip (left), knee (center) and ankle (right).

\[ X_{\text{CoM}} = \frac{1}{M} \sum_{i=1}^{L} m_i x_i \]
\[ Y_{\text{CoM}} = \frac{1}{M} \sum_{i=1}^{L} m_i y_i \]
\[ Z_{\text{CoM}} = \frac{1}{M} \sum_{i=1}^{L} m_i z_i \]

where \( M \) is the total mass of the biped, \( m_i \) is the mass of link \( i \), \( x_i, y_i \) and \( z_i \) are the coordinates of the geometric center of each link and \( i = 1, 2, 3, \ldots, L \) denotes the links of the biped.

From the first and third plots, the center of mass changes accordingly as the biped is in motion. The circles seen at the center of the first plot is the position of the stance leg. Circles above and below these depict the movement of the swing leg, as also manifested by the up and down movement of the \( Z_{\text{CoM}} \) in the third graph. The second plot reveals that the center of mass is always at the center of the two feet. This instability allows the biped to fall on its front (swing) leg, therefore, achieving the walking motion.

3.2 Experimental results

The analog PID controller constants for each of the joint are given in Table-3 (5) is used to convert these analog PID constants to its discrete counterparts. These constants are obtained experimentally even though a single brand of DC motors is used. The last four joints only use the discrete PI controller to achieve its desired positions. Before doing the walking experiments, calibration of the potentiometers is done to ensure that the zero-angle value for each joint is approximately equal to 512.

The experimental results are shown in Figure-10. At some instances, the sampled data contain glitches and spikes. These are due to the non-linearities of the potentiometers at the value limits. At some joint angle sampling intervals, large deviations are evident. A difference of 90 (unsigned int representation) is equal to a 20-degree angle swing. An error approximately equal to 13 degrees is seen. One reason is attributed to the positions of the potentiometers during locomotion. As the biped moves, the respective joint angle potentiometer displaces at a distance.

Table-3. Analog PID Controller Constants.

<table>
<thead>
<tr>
<th>Joint Angle</th>
<th>( K_P )</th>
<th>( K_I )</th>
<th>( K_D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Right hip</td>
<td>25</td>
<td>2</td>
<td>0.5</td>
</tr>
<tr>
<td>Right knee</td>
<td>25</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>Right ankle</td>
<td>11</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>Left hip</td>
<td>27</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Left knee</td>
<td>20</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Left ankle</td>
<td>25</td>
<td>10</td>
<td>0</td>
</tr>
</tbody>
</table>

Out of the 250 samples that have been obtained, selected intervals are used for the evaluation of the gait parameters. In computing for the errors for the step length, the sample intervals necessary are the 5th, 9th, 13th, 17th, 21st and 25th intervals. This is because during this interval the biped has already made a step. For the hip height, the odd intervals from 1st to the 25th intervals are considered while the 3rd, 7th, 11th, 15th, 19th, and the 21st interval are noted in evaluating foot clearance.

Figure-11. Location of Center of Mass.
From these intervals, forward kinematics is used to compute the actual gait parameters and then compare it with the predefined set of gait parameters for each experiment. The error plot for the step length is shown in Figure-12. The maximum absolute error is 0.53 cm. Though there are angle errors ranging from 10-20 degrees, the effect on the step length is minimal; this is because the errors are present in the intervals not affecting the step length. Figure-13 and Figure-14 show the hip height and foot clearance errors, respectively. The hip height and foot clearance experience small errors with an absolute maximum deviation of 0.81 and 0.59 cm, respectively. Though with some deviations, the biped still manages to achieve the desired walking process as seen from the snap shots below. (Figure-15)

Table-4 shows the mean square (in cm) and percent errors (% E) of the gait parameters step length (SL), foot clearance (FC) and hip height (HH) observed in the experiments conducted. Trials 1-3 focus on step length while Trials 4–5 deal with foot clearance. As the step length and foot clearance increase, the errors also increase. One factor contributing to this observation is that the higher these gait parameters are, the more energy is required to power up the motor so that the desired position is attained.

<table>
<thead>
<tr>
<th>Trial</th>
<th>SL % E</th>
<th>FC % E</th>
<th>HH % E</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.43</td>
<td>0.89</td>
<td>0.14</td>
</tr>
<tr>
<td>2</td>
<td>0.56</td>
<td>1.15</td>
<td>0.14</td>
</tr>
<tr>
<td>3</td>
<td>0.94</td>
<td>1.94</td>
<td>0.15</td>
</tr>
<tr>
<td>4</td>
<td>0.47</td>
<td>0.97</td>
<td>0.14</td>
</tr>
<tr>
<td>5</td>
<td>0.21</td>
<td>0.43</td>
<td>0.26</td>
</tr>
</tbody>
</table>

Table-5 illustrates the mean square error of the joint angles (in degrees) in each experiment. R and L denote Right and Left, respectively, while H, K and A pertain to Hip, Knee and Ankle, respectively. The maximum mean square error does not exceed 12 degrees. This error does not affect much of the observed gait parameters because this happens at the intervals where the gait parameters are not recorded, i.e., during the transition stages of bipedal locomotion.

Table-5. Joint angle mean square errors.

<table>
<thead>
<tr>
<th>Trial</th>
<th>RH</th>
<th>RK</th>
<th>RA</th>
<th>LH</th>
<th>LK</th>
<th>LA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.50</td>
<td>6.78</td>
<td>8.86</td>
<td>3.77</td>
<td>7.24</td>
<td>9.84</td>
</tr>
<tr>
<td>2</td>
<td>4.40</td>
<td>7.23</td>
<td>11.30</td>
<td>6.12</td>
<td>8.16</td>
<td>6.85</td>
</tr>
<tr>
<td>3</td>
<td>7.49</td>
<td>4.88</td>
<td>9.31</td>
<td>5.40</td>
<td>5.51</td>
<td>5.89</td>
</tr>
<tr>
<td>4</td>
<td>5.94</td>
<td>8.56</td>
<td>9.82</td>
<td>6.35</td>
<td>10.3</td>
<td>7.77</td>
</tr>
<tr>
<td>5</td>
<td>3.11</td>
<td>11.3</td>
<td>10.12</td>
<td>7.02</td>
<td>8.10</td>
<td>4.19</td>
</tr>
</tbody>
</table>
4. CONCLUSIONS

In this work, the closed form solutions for determining the necessary joint angles to achieve a stable bipedal locomotion in the sagittal plane have been derived. Closed form solution allows efficient and fast calculations such that an online walking is possible. Sagittal movements of the robot are achieved without further control of the lateral joints. Precise position control of each joint motor allows the biped to achieve a human gait-type of walking even though the actual joint angle plots exhibit a relatively small error from the simulated trajectories of each angle. In this study, the biped can walk as fast as 1 step/sec with different combinations of gait parameters, as provided by the user, in a smooth and flat environment. Static walking is implemented by using the center of mass criterion with the aid of support ropes. The support ropes restrict the center of mass to stay in between the two legs and prevent the biped from falling to its side when walking. Future work will rely on implementing the position control algorithm to a bipedal robot having an upper limb and experimenting on faster walking speeds.

REFERENCES


