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EFFECT OF SORET NUMBER AND HEAT SOURCE ON UNSTEADY MHD CASSON FLUID FLOW PAST AN INCLINED PLATE EMBEDDED IN POROUS MEDIUM

V. Manjula and K. V. Chandra Sekhar
Department of Mathematics, Koneru Lakshmaiah Education Foundation, Vaddeswaram, Guntur, Andhra Pradesh, India
E-Mail: kasulanati22@rediffmail.com

ABSTRACT

The present paper describes the Soret and heat generation effects on MHD Casson fluid flow past an inclined plate immersed in porous media. The non - dimensional governing equations are solved analytically through Laplace transform technique. Exact solutions for velocity, energy and species concentration are obtained, in terms of exponential and complementary error functions. For better understanding of physical insight of the problem graphical analysis was done under the influence of different critical parameters on fluid flow. Increase in values of Soret number raises the buoyancy force, which in turn results in raise in velocity of the fluid. Increase in thermal radiation results in fall in temperature of the fluid.

Keywords: soret number, heat source, MHD, casson fluid, inclined plate, porous medium.

NOMENCLATURE			
B_0	uniform magnetic field	Kr	chemical reaction parameter
C'	species concentration	Q_0	heat absorption/generation coefficient
D_M	mass diffusion coefficient	T'	fluid temperature
D_T	thermal diffusion coefficient	t	dimensionless time
G_m	mass Grashof number	H	heat generation parameter
k	permeability parameter	a	absorption constant
k'	permeability of porous medium	Greek symbols	
Pr	Prandtl number	β'_T	volumetric coefficient of thermal expansion
M	magnetic parameter	β'_c	volumetric coefficient of species concentration expansion
Sc	Schmidt number	μ_β	plastic dynamic viscosity
u'	fluid velocity in x_0 direction	ν	kinematic viscosity coefficient
t'	Time	ρ	fluid density
q_r	radiative heat flux	σ	electrical conductivity
R	thermal radiation	β	Casson parameter
θ	dimensionless fluid temperature		
u	dimensionless fluid velocity in x irection		
C	dimensionless species concentration		
c_p	specific heat at constant pressure		
g	acceleration due to gravity		
G_r	thermal Grashof number		

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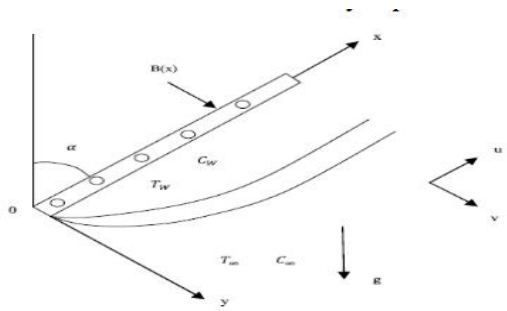
1. INTRODUCTION

Magneto hydrodynamics is the study of the motion of the fluid in the presence of a magnetic field. Significant research was in progress in Magneto hydrodynamics from the past few years in engineering sciences and natural sciences. This was happened after work done by Hartmann [1] in liquid metal duct flows subjecting to strong external magnetic field. Due to vast research in optimization in solidification process of metals, alloys the study of magneto hydrodynamic flow of non - Newtonian fluid in porous medium got good attention by many researchers. Existence of electromagnetic hydrodynamic waves was studied by Alfven [2], whereas Singh *et al.* [3] have studied natural convection in different physical conditions in presence of uniform /radial varying magnetic field. MHD Mixed convection flow of Ferro fluid along vertical channel was studied by Khan [4]. Makinde [5] analized the Magneto - Nanofluid flow past an impulsively started porous flat plate in rotating frame. Nadeem [6, 7] has done intensive study on MHD oblique stagnation point flow of a viscoelastic fluid over a convective surface. Recently Kataria [8] analyzed velocity and temperature distributions of optically thick nano fluid flow past an oscillating vertical plate in presence of Magnetic field and radiation.

A good number of applications of MHD non - Newtonian fluids are encountered in biological systems, heat - storage beds, irrigation, petroleum processing, paper, textile industries. Casson fluid is one which never possesses small shear stress. Casson model was first introduced by Casson in 1959 to analyze the flow of pigment - oil suspensions [9]. Dash [10] investigated Casson flow in a pipe with homogeneous porous medium. The analysis of Soret and Dufour effects was done by Hayat [11] on MHD flow of Casson fluid. The MHD flow of casson fluid flow over exponentially stretching sheet was done by Nadeen [12, 13]. Hussnan [14, 15] illustrates that free convective flow past an oscillating plate with Newtonian heating. Rashidi [16, 17] has discussed on steady/ unsteady MHD and slip flow over a rotating porous disk. Kataria [18] *et al* investigated MHD casson fluid flow past an oscillating vertical plate under the influence of Soret and heat generation. In the above discussion it was observed that somehow the presence of inclined plate was neglected. So we included inclined plate for the present analysis.

2. MATHEMATICAL FORMULATION



Geometry of the problem:

Consider a plate inclined with an angle of inclination α immersed in porous media. The flow confined to be along $y' > 0$, where y is the coordinate measured normal to the plate and x' axis is taken along the plate in the upward direction. At time $t' = 0$ both the plate and the fluid are having uniform temperature T'_∞ and the concentration is assumed to be C'_∞ respectively. At time $t' > 0$, the plate is accelerated in the upward direction opposite to the gravitational forces, with velocity $U_0 e^{at'}$ with constant heat flux $T_\infty' + (T_w' + T_\infty')t'/t_0$ when $t' \le t_0$ and T_w' when $t' > t_0$ respectively. Thereafter it is maintained constant. Similarly mass transfer at the surface is raised to $C_\infty' + (C_w' + C_\infty')t'/t_0$ when $t' \le t_0$, lowered to C_w' when $t' > t_0$ respectively, there after concentration level maintained constant C_w'. A magnetic field of strength B_0 is applied uniformly along y' axis direction. The induced magnetic field is considered negligible. Neglecting rigidness of the plate, incompressibility, on-Newtonian nature, one dimensional flow, free convection, and viscous dissipation term in the energy equation, we considered the following partial differential equations governing the flow with initial and boundary conditions.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\rho \frac{\partial u'}{\partial t'} = \mu_B \left(1 + \frac{1}{\beta}\right) \frac{\partial^2 u'}{\partial y'^2} - \sigma B_0^2 u' - \frac{\mu\varphi}{k_1'} u' + \rho g \beta_T' \left(T' - T_\infty\right)\cos\alpha + \rho g \beta_c' \left(C' - C_\infty\right)\cos\alpha \tag{2}$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y'} + \frac{Q_0}{\rho c_p}\left(T' - T_\infty\right) \tag{3}$$

$$\frac{\partial C'}{\partial t'} = D_M \frac{\partial^2 C'}{\partial y'^2} + D_T \frac{\partial^2 T'}{\partial y'^2} - kr^2 \left(C - C_\infty\right) \tag{4}$$

$$\begin{aligned}
&u' = U_0, T' = T_\infty', C' = C_\infty' \text{ as } y' \ge 0 \text{ and } t' \le 0 \\
&u' = \cos wt / \sin wt \text{ as } t' > 0 \text{ and } y' = 0 \\
&T' = T_\infty' + \left(T_w' - T_\infty'\right) t'/t_0 \; if \; 0 < t < t_0 \\
&\quad = T_w' \; if \; t' \ge t_0 \; for \; y' = 0 \\
&C' = C_\infty' + \left(C_w' - C_\infty'\right) t'/t_0 \; if \; 0 < t' < t_0 \\
&\quad = C_w' \qquad if \; t' \ge t_0 \qquad for \; y' = 0
\end{aligned} \tag{5}$$

$$u' \to 0, T' \to T_\infty', C' \to C_\infty' \; as \; y' \to \infty \; and \; t' \geq 0$$

Using Roseland approximation

$$\frac{\partial q_r}{\partial y'} = -4a\sigma\left(T_\infty^{\,4} - T'\right) \tag{6}$$

Using Taylor series expansion expanding T'^4 linearly about T'_∞ by neglecting higher order terms,

$$T'^4 \cong 4T'_\infty T' - 3T'^{\,4}_\infty \tag{7}$$

$$u = \frac{u'}{U_0}\; y = \frac{y'}{U_0 t_0}, t = \frac{t'}{t_0}, \theta = \frac{T' - T_\infty'}{T_w' - T_\infty'}$$

$$C = \frac{C' - C_\infty'}{C_w' - C_\infty'}, Gr = \frac{v g \beta_T' T_w' - T_\infty'}{U_0^{\,3}}, Gm = \frac{v g \beta_C' \left(C_w' - C_\infty'\right)}{U_0^{\,3}}$$

$$M = \frac{\sigma B_0^{\,2} v}{\rho U_0^{\,2}}, \Pr = \frac{\rho v c_p}{k}, R = \frac{16 a \sigma v^2 T_\infty'^{\,3}}{k U_0^{\,2}},$$

$$H = \frac{Q_0 v^2}{k U_0^{\,2}}, Sc = \frac{v}{D_M}, Kr = \frac{v k r'^2}{U_0^{\,2}} \tag{8}$$

$$Sr = \frac{D_T\left(T_w' - T_\infty'\right)}{v\left(C_w' - C_\infty'\right)}, \beta = \frac{\mu_B\sqrt{2\pi_c}}{P_y}, \tau = \frac{\tau'}{\rho u^2}$$

Using the above non-dimensional scheme,

$$\frac{\partial u}{\partial t} = \left(1 + \frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} - \left(M^2 + \frac{1}{k}\right)u + Gr\theta\cos\alpha + GmC\cos\alpha - \frac{u}{K} \tag{9}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} + \frac{(H - R)}{\Pr}\theta \tag{10}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} + Sr\frac{\partial^2 \theta}{\partial y^2} - krC \tag{11}$$

With initial and boundary conditions

$$u = \theta = C = 0, y \geq 0, t \leq 0, u = 1 \; at \; y = 0, t > 0$$
$$\theta = \begin{cases} t, 0 < t \leq 1 \\ 1, t > 1 \end{cases} \quad C = \begin{cases} t, 0 < t \leq 1 \\ 1, t > 1 \end{cases} \tag{12}$$

$$\text{at} \begin{array}{l} y = 0, t > 0, \\ u \to 0, \theta \to 0, C \to 0, as \; y \to \infty, t > 0 \end{array} \tag{13}$$

3 SOLUTION OF THE PROBLEM

Applying Laplace transform technique exact solutions were obtained from equations (9), (10) and (11) with initial and boundary conditions (12) and (13).

$$u(y,t) = \frac{1}{4i}[\{e^{A1}erfc(B1) + \{e^{A2}erfc(B2) - \{e^{A3}erfc(B3) +$$
$$\{e^{A4}erfc(B4)\}]-\{e^{A7}erfc(B7) + \{e^{A8}erfc(B8)\}$$
$$-\{e^{A17}erfc(B17) + \{e^{A18}erfc(B18)\} + \frac{-1}{2a_7^{\,2}}(\{e^{D7}erfc(E7)$$
$$+\{e^{D8}erfc(E8)\} - \{e^{D17}erfc(E17) + \{e^{D18}erfc(E18)\}]-$$
$$\frac{a_5}{a_6}G_r\cos\alpha[\frac{-1}{2a_7}(\{c9\{e^{A9}erfc(B7) + c10\{e^{A10}erfc(B8)\}]-$$
$$\{c11\{e^{A19}erfc(B17) + c12\{e^{A20}erfc(B18)\} + \frac{1}{2a_7}(\{F9$$
$$\{e^{A9}erfc(E7) + F10\{e^{A10}erfc(E8)\} - \{F11\{e^{A19}erfc(E17)$$
$$+F12\{e^{A20}erfc(E18)\} + \frac{a_5}{a_6}G_r\cos\alpha[\frac{-1}{2a_7^{\,2}}(\{e^{A5}erfc(B5)$$
$$+\{e^{A6}erfc(B6)\} - \{e^{A21}erfc(B21) + \{e^{A22}erfc(B22)\} -$$
$$\frac{1}{2a_7^{\,2}}(\{e^{D5}erfc(E5) + \{e^{D6}erfc(E6)\}\mathrm{H(t-1)} - \{e^{D21}erfc(E21) +$$
$$\{e^{D22}erfc(E22)\}\mathrm{H(t-1)} + \frac{a_5}{a_8}G_m\cos\alpha[\frac{-1}{2a_9^{\,2}}\{e^{A11}erfc(B11) +$$
$$\{e^{A12}erfc(B12)\} - \{e^{A23}erfc(B23) + \{e^{A24}erfc(B24)\} + \frac{1}{2a_9^{\,2}}$$
$$(\{\{e^{D11}erfc(E11) + \{e^{D12}erfc(E12)\}\mathrm{H(t-1)} - \{e^{D23}erfc(E23) +$$
$$\{e^{D24}erfc(E24)\}\mathrm{H(t-1)} + \frac{a_5}{a_8}G_m\cos\alpha$$
$$[\frac{-1}{2a_9}\{c9e^{A9}erfc(B7) + c10e^{A10}erfc(B8)\} -$$
$$\{c13e^{A25}erfc(B23) + c14e^{A24}erfc(B24)\} + \frac{1}{2a_9}[\{F9e^{A9}erfc(E7) + F10e^{A10}erfc(E8)\} -$$
$$\{F13e^{A25}erfc(E23) + F14e^{A26}erfc(E24)\}]\mathrm{H(t-1)} +$$
$$\{e^{A11}erfc(B11) + e^{D12}erfc(E12)\}$$
$$-\{e^{D27}erfc(E27) + e^{D28}erfc(E28)\}\mathrm{H(t-1)}$$
$$+\frac{a_5}{aa_2\,a_8}S_c\,S_r\,P_r\,G_m Cos\alpha[\frac{1}{2a_{12}}\{e^{A11}erfc(B11) +$$
$$e^{A12}erfc(B12)\} - \{e^{A27}erfc(B27) + e^{A28}erfc(B28)\} - \frac{1}{2a_{12}}$$
$$\{e^{D11}erfc(E11) + e^{D12}erfc(E12)\}H(t-1) -$$
$$\frac{a_5}{aa_2\,a_8}S_c\,S_r\,P_r\,G_m Cos\alpha[-\frac{1}{2a_{12}}\{e^{A13}erfc(B13) + \{e^{A14}erfc(B14) +$$
$$e^{A29}erfc(B29)\} + \{e^{A30}erfc(B30)\} + [\frac{1}{2a_{12}}\{e^{D13}erfc(E13) +$$
$$\{e^{D14}erfc(E14)\} + \{e^{D9}erfc(E29)\} + \{e^{D30}erfc(E30)\} +$$
$$\frac{a_5}{aa_2\,a_8}S_c\,S_r\,P_r\,G_m Cos\alpha$$

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$[-\frac{1}{2a_9}\{e^{A11}erfc(B11)+e^{A12}erfc(B12)\}-\{e^{A27}erfc(B27)+$
$e^{A28}erfc(B28)\}+[\frac{1}{2a_9}\{e^{D11}erfc(E11)+\{e^{D12}erfc(E12\}]H(t-1)$

$+\frac{a_5}{aa_2\,a_8}S_c\,S_r\,P_r\,G_m Cos\alpha[\frac{1}{2a_9}\{e^{A7}erfc(B7)+$
$\{e^{A8}erfc(B8)+\{e^{A23}erfc(EB3)-\{e^{A23}erfc(EB3)+$
$\{e^{A24}erfc(B24)\}-\frac{1}{2a_9}\{e^{D7}erfc(E7)+e^{D8}erfc(E8)\}H(t-1)-$

$\{e^{D23}erfc(E23)+e^{D24}erfc(E24)\}H(t-1)\ -\frac{a_5}{a_8a_2^2}S_c\,S_r\,(R-$
$H)\,G_m Cos\alpha[\frac{1}{2a_9}\{e^{A11}erfc(B11)\}+\{e^{A12}erfc(B12)-$
$e^{A23}erfc(B23)+e^{A24}erfc(B24)\}[-\frac{1}{2a_9}\{e^{D11}erfc(E11)+$
$e^{D12}erfc(E12)\}H(t-1)-\{e^{D23}erfc(E23)+$
$e^{D24}erfc(E24)\}H(t-1)\ +\frac{a_5}{a_8a_2^2}S_c\,S_r\,(R-H)\,G_m Cos\alpha[-$
$\frac{1}{2a_9}\{e^{A7}erfc(B7)+e^{A8}erfc(B8)\}-\{e^{A23}erfc(B23)+$
$\{e^{A24}erfc(B24)\}-\frac{1}{2a_9}\{e^{D7}erfc(E7)+$
$e^{D8}erfc(E8)\}H(t-1)-e^{D23}erfc(E23)$
$+e^{D24}erfc(E24)\}H(t-1)\ +\frac{a_5}{a_8a_2}S_c\,S_r\,(R-$
$H)\,G_m Cos\alpha[-\frac{1}{2a_9^2}\{e^{A7}erfc(B7)\}-\frac{1}{2a_9^2}$
$\{e^{D7}erfc(E7)+\{e^{D8}erfc(E8)\}H(t-1)-$
$\{e^{D23}erfc(E23)+\{e^{D24}erfc(E24)H(t-1)$

$-\frac{a_5}{a_8a_2}S_c\,S_r\,(R-H)G_m Cos\alpha\frac{1}{2a_9}[\{c9e^{A9}erfc(B7)$
$+c10e^{A10}erfc(B8)\}-c13e^{A25}erfc(B23)$
$+c14e^{A25}erfc(B24)\}+\frac{1}{2a_9}[\{F13e^{A25}erfc(E23)$
$+F14e^{A26}erfc(E24)\}H(t-1)][-\frac{1}{2a_9}\{e^{A11}erfc(B11)$

$+\{e^{A12}erfc(B12)\}-\{e^{A27}erfc(B27)\}+$
$\{e^{A28}erfc(B28)\}-[\frac{1}{2a_9^2}\{e^{D11}erfc(E11)+$
$\{e^{D12}erfc(E12\}]H(t-1)-\{e^{D27}erfc(E27)+$
$e^{D28}erfc(E28)\}H(t-1)\ -[\frac{1}{2a_{12}}\{e^{D11}erfc(E11)+$
$\{e^{D12}erfc(E12)\}H(t-1)-\{e^{D27}erfc(E27)+$
$\{e^{D28}erfc(E28)\}H(t-1)-\frac{1}{2a_{12}}\{e^{A13}erfc(B13)+$
$\{e^{A14}erfc(B14)\}\ -\{e^{A29}erfc(B29)+\{e^{A30}erfc(B30)\}+$

$\frac{1}{2a_{12}}\{e^{D13}erfc(E13)+e^{D14}erfc(E14)H(t-1)\}]\ -\{e^{D29}erfc(E29)+$
$\{e^{D30}erfc(E30)\}H(t-1)+\{e^{D15}erfc(E15)+e^{D16}erfc(E16)H(t-1)$
$[\{e^{D31}erfc(E31)+\{e^{D32}erfc(E32)\}H(t-1)-$
$\frac{1}{2a_{13}}[\{e^{A15}erfc(B15)+\{e^{A16}erfc(B16)\}-$
$\{e^{A31}erfc(B31)+\{e^{A32}erfc(B32)\}H(t-1)+$
$\frac{a_5}{aa_2\,a_{10}}S_c\,S_r\,P_r\,G_m Cos\alpha[-\frac{1}{2a_{13}}\{e^{D13}erfc(E13)+e^{D14}erfc(E14)H(t-1)\}-$
$\{e^{D14}erfc(E14)H(t-1)\}-\{e^{D33}erfc(E33)+$
$\{e^{D34}erfc(E34)H(t-1)+\frac{1}{2a_{13}}\{e^{A13}erfc(B13)\}+$
$\{e^{A14}erfc(B14)\}-\{e^{A33}erfc(B33)\}+\{e^{A34}erfc(B34)\}-$
$\frac{1}{2a_{11}}\{e^{D15}erfc(E15)+\{e^{D16}erfc(E16)H(t-1)\ -\{e^{D31}erfc(E31)$
$+\{e^{D32}erfc(E32)\}H(t-1)-\frac{1}{2a_{11}}\{e^{A15}erfc(B15)+$
$e^{A16}erfc(B16)\}-\{e^{A31}erfc(B31)+e^{A32}erfc(B32)\}-[$
$\frac{1}{2a_{11}}\{e^{D7}erfc(E7)+\{e^{D8}erfc(E8)\}H(t-1)-$
$\{e^{D17}erfc(E17)+e^{D18}erfc(E18)\}H(t-1)+\frac{1}{2a_{11}}$
$\{e^{A7}erfc(B7)+e^{A8}erfc(B8)\}-\{e^{A17}erfc(B17)+e^{A18}erfc(B18)\}-$
$\frac{a_5}{aa_{10}a_2^2}s_c s_r(R-H)G_m\cos\alpha[\frac{1}{2a_{11}}\{e^{D15}erfc(E15)+$
$\{e^{D16}erfc(E16)H(t-1)-\{e^{D31}erfc(E31)+e^{D32}erfc(E32)\}]$
$H(t-1)-\frac{1}{2a_{11}}[\{e^{A15}erfc(B15)+\{e^{A16}erfc(B16)-$
$\{e^{A31}erfc(B31)+\{e^{A32}erfc(B32)\}-\frac{a_5}{aa_{10}a_2^2}s_c s_r(R-H)G_m\cos\alpha$

$[-\frac{1}{2a_{11}}\{e^{D7}erfc(E7)+e^{D8}erfc(E8)\}H(t-1)-\{e^{D17}erfc(E17)\}+$
$\{e^{D18}erfc(E18)\}H(t-1)+\frac{1}{2a_{11}}[\{e^{A7}erfc(B7)+e^{A8}erfc(B8)\}-$
$\{e^{A17}erfc(B17)\}+\{e^{A18}erfc(B18)\}$
$+\frac{a_5}{aa_2\,a_{10}}S_c\,S_r\,P_r\,G_m Cos\alpha(R-H)[\frac{-1}{2a_{11}^2}\{e^{D7}erfc(E7)+$
$\{e^{D8}erfc(E8)\}-\{e^{D17}erfc(E17)+e^{D18}erfc(E18)\}]H(t-1)+$

$[\frac{1}{2a_{11}^2}\{e^{A7}erfc(B7)+e^{A8}erfc(B8)\}-\{e^{A17}erfc(B17)+$
$e^{A18}erfc(B18)\}-\frac{a_5}{aa_2\,a_{10}}S_c\,S_r\,P_r\,G_m Cos\alpha(R-H)[-$
$\frac{1}{2a_{11}}\{F9e^{A9}erfc(E7)+F10e^{A10}erfc(E8)\}-\{F11\{e^{A19}erfc(E17)$
$+F12\{e^{A20}erfc(E18)\}H(t-1)+[\frac{1}{2a_{11}}\{c11e^{A19}erfc(B17)$
$+c12e^{A20}erfc(B18)\}-\frac{a_5}{aa_2\,a_{10}}S_c\,S_r\,P_r\,G_m Cos\alpha(R-H)$

$$[\frac{1}{2a_{11}^2}\{c9e^{A9}erfc(B7)+c10e^{A10}erfc(B8)\}-\{e^{D15}erfc(E15)+$$
$$\{e^{D16}erfc(E16)H(t-1)-\{e^{D31}erfc(E31)+$$
$$\{e^{D32}erfc(E32)\}H(t-1)-[\frac{-1}{2a_{11}^2}\{e^{A15}erfc(B15)+$$
$$\{e^{A16}erfc(B16)-\{e^{A31}erfc(B31)+\{e^{A32}erfc(B32)\}+$$
$$\frac{a_5}{aa_{10}a_2^2}s_cs_r(R-H)G_m\cos\alpha[\frac{1}{2a_{12}}\{e^{D15}erfc(E15)+$$
$$\{e^{D16}erfc(E16)H(t-1)-\{e^{D31}erfc(E31)+$$
$$\{e^{D32}erfc(E32)\}]H(t-1)-\frac{1}{2a_{12}}\{e^{A15}erfc(B15)+\{e^{A16}erfc(B16)-$$
$$\{e^{A31}erfc(B31)+\{e^{A32}erfc(B32)\}+\frac{a_5}{aa_{10}a_2^2}s_cs_r(R-H)G_m\cos\alpha[$$
$$-\frac{1}{2a_{12}}\{e^{D13}erfc(E13)+\{e^{D34}erfc(E34)H(t-1)+$$
$$\{e^{D13}erfc(E13)+\{e^{D34}erfc(E34)H(t-1)+$$
$$\frac{1}{2a_{13}}[\{e^{A13}erfc(B13)\}+\{e^{A14}erfc(B14)\}-\{e^{A33}erfc(B33)\}+$$
$$\{e^{A33}erfc(B33)\}+\{e^{A34}erfc(B34)\} \quad (14)$$

$$\theta(y,t)=\frac{1}{2}[\{L1e^{M1}erfc(N1)\}+\{L2e^{M2}erfc(N2)\}-$$
$$\{L3e^{M1}erfc(N3)\}+\{L4e^{M2}erfc(N4)\}\frac{H(t-1)}{2}$$
(15)

$$c(y,t)=\frac{1}{2}[\{P1e^{Q1}erfc(R1)+\{S1e^{T1}erfc(U1)\}-$$
$$\{P2e^{Q2}erfc(R2)\}+\{S1e^{T1}erfc(U1)\}+\frac{s_cs_rp_r}{2aa_2}$$
$$\{e^{P3}erfc(Q3)+\{e^{R3}erfc(S3)\}-\{e^{P4}erfc(Q4)+$$
$$\{e^{R4}erfc(S4)\}-\frac{s_cs_rp_r}{2aa_2}\{e^{P5}erfc(Q5)+\{e^{R5}erfc(S5)\}-$$
$$\{e^{P6}erfc(Q6)+\{e^{R6}erfc(S6)\}\text{H(t-1)}+\frac{s_cs_r(R-H)}{2a}[-\frac{1}{a_2^2}$$
$$\{e^{P7}erfc(Q7)+\{e^{R7}erfc(S7)\}-\frac{1}{a_2}\{P8e^{Q8}erfc(R8)\}+$$
$$\{S8e^{T8}erfc(U8)\}+\frac{1}{a_2^2}\{e^{P9}erfc(Q9)+\{e^{R9}erfc(S9)\}-$$
$$\frac{s_cs_r(R-H)}{2a}-\frac{1}{a_2^2}\{e^{P10}erfc(Q10)+\{e^{R10}erfc(S10)\}-\frac{1}{a_2}$$
$$[\{P11e^{Q11}erfc(R11)\}+\{S11e^{T11}erfc(U11)\}+\frac{1}{a_2^2}[\{e^{R12}erfc(S12)\}+$$
$$\frac{s_cs_rp_r}{2aa_2}\{e^{P13}erfc(Q13)+\{e^{R13}erfc(S13)\}-\{e^{P14}erfc(Q14)+$$
$$\{e^{P12}erfc(Q12)\{e^{R14}erfc(S14)\}\text{H(t-1)}-\frac{s_cs_rp_r}{2aa_2}\{e^{P15}erfc(Q15)+$$
$$\{e^{R15}erfc(S15)\}-\{e^{P16}erfc(Q16)+\{e^{R16}erfc(S16)\}+$$
$$\frac{s_cs_r(R-H)}{2a}[-\frac{1}{a_2^2}\{e^{P17}erfc(Q17)+\{e^{R17}erfc(S17)\}$$
$$-\frac{1}{a_2}\{P18e^{Q18}erfc(R18)\}+\{S11e^{T18}erfc(U18)\}+$$
$$\frac{1}{a_2^2}\{e^{P19}erfc(Q19)+\{e^{R19}erfc(S19)\}\text{H(t-1)}-$$
$$\frac{s_cs_r(R-H)}{2a}[-\frac{1}{a_2^2}\{e^{P20}erfc(Q20)+\{e^{R20}erfc(S20)\}$$
$$-\frac{1}{a_2}\{P21e^{Q21}erfc(R21)\}+\{S21e^{T21}erfc(U21)\}+\frac{1}{a_2^2}$$
$$\{e^{P22}erfc(Q22)+\{e^{R22}erfc(S22)\} \quad (16)$$

4 RESULTS AND DISCUSSIONS

For better understanding of the problem influence of different physical parameters, exact solutions were found using the technique of Laplace transform. Increase of angle of inclination causes decrease in velocity which was shown in Figure-1. Figure-2 depicts that raise in Soret number causes a raise in velocity. This is due to the fact increase of Soret number causes a raise in mass buoyancy force, which results in raise in velocity profiles.The falling velocity of the fluid is caused by the increase of Casson parameter which was shown in Figure-3. Figure-4 shows that increase in magnetic parameter causes in fall in velocity of the fluid. Because magnetic force opposes the fluid flow. Increase of chemical reaction parameter accelerates the fluid particles and hence increases fluid velocity, which was shown in Figure-5. Increase of Schmidt number lowers mass diffusion and hence decreases in velocity. This was shown in Figure-6. Figure-7 reveals that increase in prandtl number decreases fluid velocity.

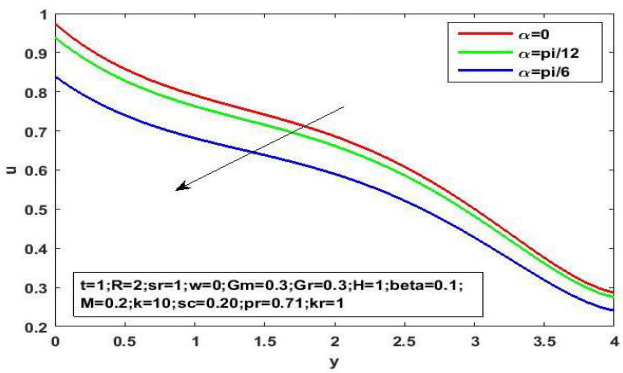

Figure-1. Contribution of angle of inclination on velocity.

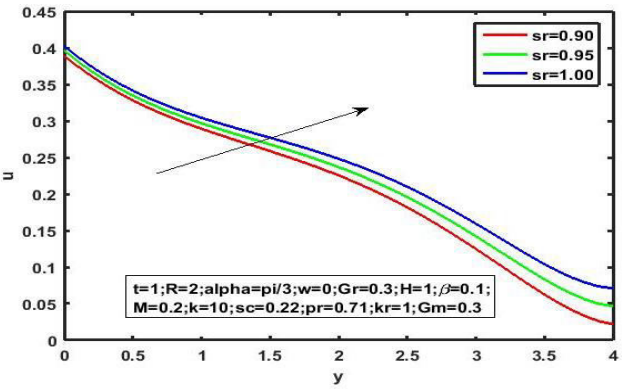

Figure-2. Influence of Soret number on velocity.

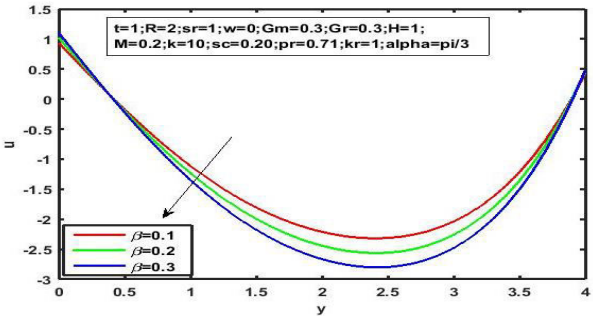

Figure-3. Influence of Casson parameter on velocity.

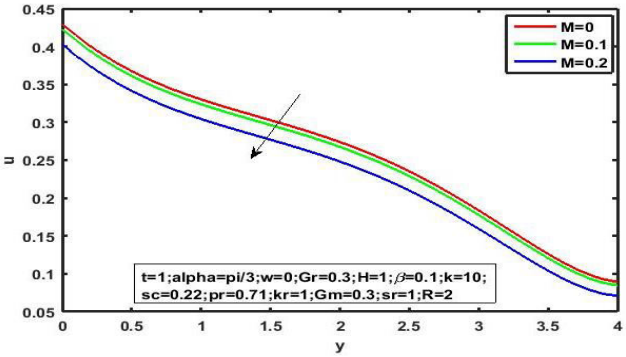

Figure-4. Effect of M on velocity.

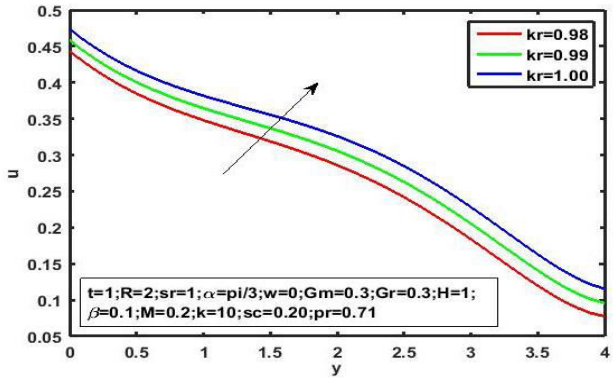

Figure-5. Effect of chemical reaction parameter on velocity.

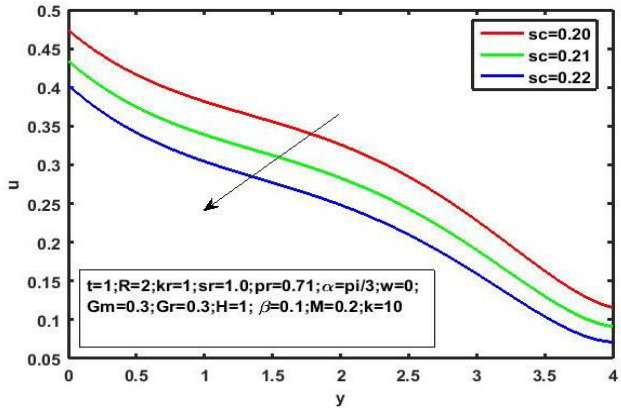

Figure-6. Variation in velocity profiles with respect to Schmidt number.

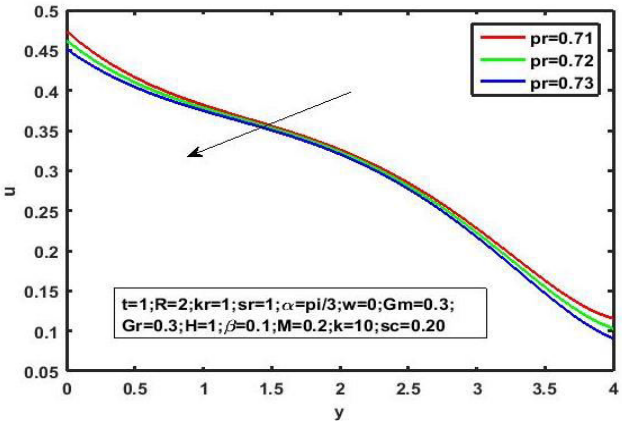

Figure-7. Influence of Prandtl number on velocity profiles.

Figure-8 depicts variation in temperature profiles with respect to Prandtl number. Increase of prandtl number decreases the thermal boundary layer thickness of the fluid and hence decreases in temperature. The increase of heat source parameter rises temperature of the fluid. That was shown in Figure-9. Figure-10 shows that increase of Radiation parameter causes a fall in temperature profiles. This is due to decrease in Roseland absorption.

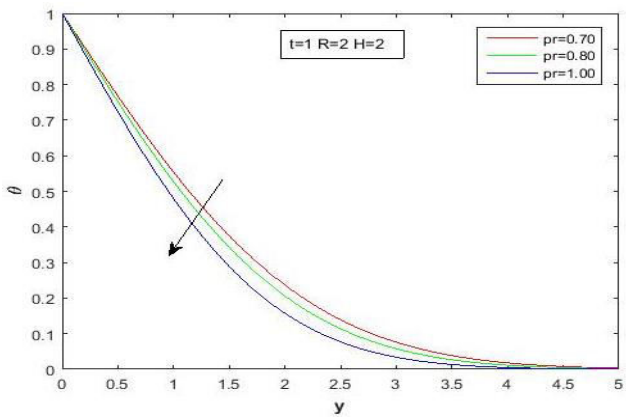

Figure-8. Influence of Prandtl number on temperature profiles.

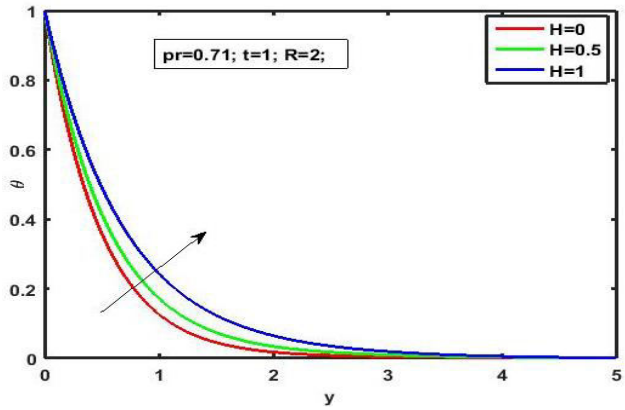

Figure-9. Variation of temperature profiles with respect to Heat source.

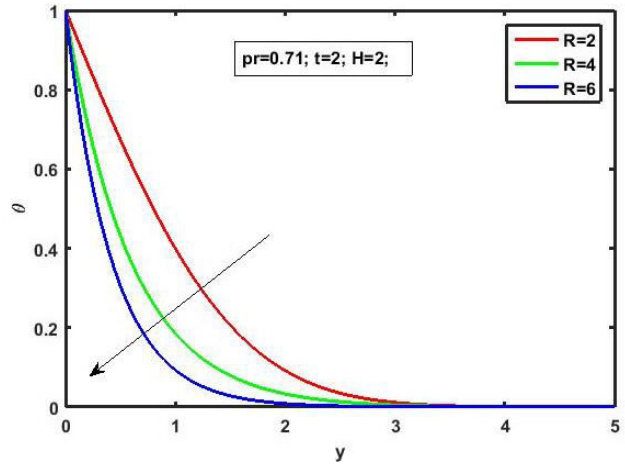

Figure-10. Influence of Radiation parameter on temperature profiles.

Figure-11 shows the influence of chemical reaction parameter on concentration profiles. When chemical reaction increases distribution of concentration decreases, which in turn results in increase in transport phenomena and hence reduction in concentration distribution. Schmidt number gives the relative effectiveness of momentum to the mass diffusion of species concentration, which gives a decrease in concentration. This was shown in Figure-12. Increase in Soret number causes raise molecular diffusion and hence rise in concentration of the fluid. This was depicted in Figure-13.

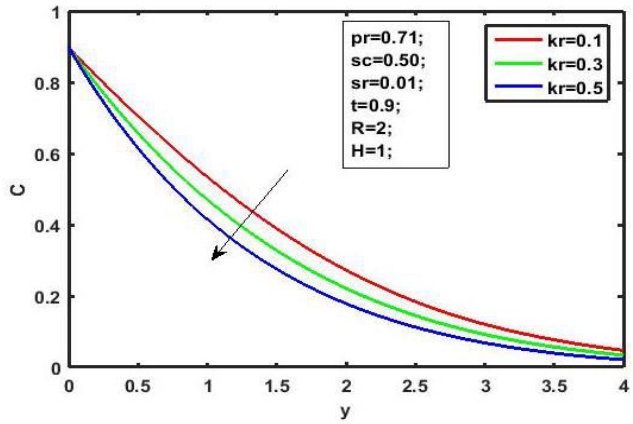

Figure-11. Effect of Chemical reaction parameter on Concentration profiles.

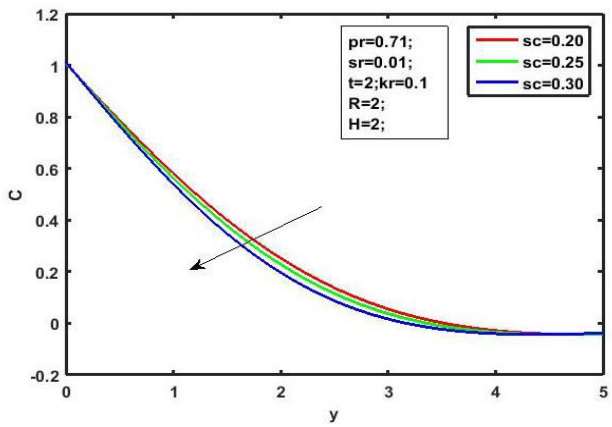

Figure-12. Variation in Concentration profiles With respect to Schmidt number.

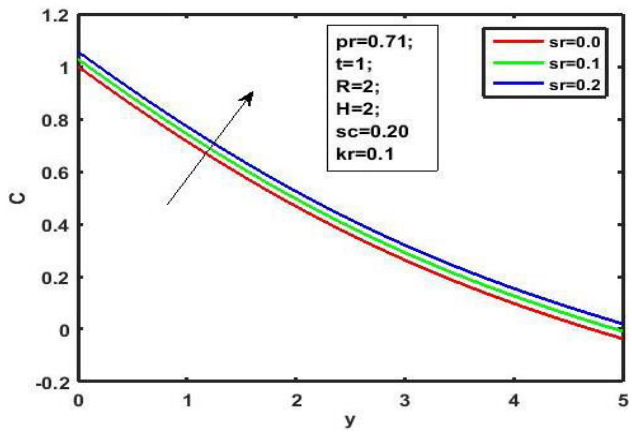

Figure-13. Variation in Concentration profiles With respect to Soret number.

The physical quantities of engineering interest are Skin friction, Nusselt number and Sherwood number.

Skin friction

Skin friction is rate of shear stress at the plate due to the flow, given by $sf = -\left(1+\frac{1}{\beta}\right)\frac{\partial u}{\partial y} at\ y=0$

Nusselt number

Rate of heat transfer at the plate is Nusselt number, described by $Nu = \frac{\nu}{U_0\left(T'-T_\infty\right)}\frac{\partial T'}{\partial y'} at\ y=0$

Sherwood number

Sherwood number gives the rate of mass transfer given by $sh = -\frac{\partial C}{\partial y}$.

The influence of different physical parameters on Skin friction, Nusselt number and Sherwood number were shown in Table-1, Table-2 and Table-3.

Table-1.

α	kr	sc	Sr	**t**	β	M	R	sf
$\frac{\pi}{3}$	0.8	0.22	1	1	0.1	0.2	2	0.152908
$\frac{\pi}{3}$	0.9	0.22	1	1	0.1	0.2	2	0.139809
$\frac{\pi}{3}$	1	0.22	1	1	0.1	0.2	2	0.128184
0	1	0.22	1	1	0.1	0.2	2	0.142919
$\frac{\pi}{6}$	1	0.22	1	1	0.1	0.2	2	0.127949
$\frac{\pi}{3}$	1	0.22	1	1	0.1	0.2	2	0.114559
$\frac{\pi}{3}$	1	0.22	1	1	0.1	0.2	2	1.646373
$\frac{\pi}{3}$	1	0.22	1	1	0.3	0.2	2	1.528085
$\frac{\pi}{3}$	1	0.22	1	1	0.5	0.2	2	1.419823
$\frac{\pi}{3}$	1	0.21	1	1	0.5	0.2	2	0.151664
$\frac{\pi}{3}$	1	0.22	1	1	0.5	0.2	2	0.136160
$\frac{\pi}{3}$	1	0.23	1	1	0.5	0.2	2	0.122258

Table-2.

t	R	Pr	Nu
1	2	0.70	0.028204
1	2	0.80	0.028168
1	2	1.00	0.028098
1	2	0.70	0.134813
1	4	0.70	0.117124
1	6	0.70	0.101708

Table-3.

t	sc	kr	sr	sh
1	0.20	0.1	0.01	0.025233
1	0.22	0.1	0.01	0.025155
1	0.20	0.1	0.01	0.022667
1	0.20	0.1	0.02	0.022607
1	0.20	0.1	0.01	0.022258
1	0.20	0.2	0.01	0.022104

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APPENDIX

A1=$-y\sqrt{a_5}\sqrt{a_4+iw)}+(a_4+iw)t$, B1=$\frac{y\sqrt{a_5}}{2\sqrt{t}}-\sqrt{(a_4+iw)t}$

A2=$y\sqrt{a_5}\sqrt{a_4+iw)}+(a_4+iw)t$, B2=$\frac{y\sqrt{a_5}}{2\sqrt{t}}+\sqrt{(a_4+iw)t}$

A3=$-y\sqrt{a_5}\sqrt{a_4-iw)}+(a_4-iw)t$, B3=$\frac{y\sqrt{a_5}}{2\sqrt{t}}+\sqrt{(a_4-iw)t}$

A4=$y\sqrt{a_5}\sqrt{a_4-iw)}+(a_4-iw)t$, B4=$\frac{y\sqrt{a_5}}{2\sqrt{t}}+\sqrt{(a_4-iw)t}$

$A5=-y\sqrt{a_5}\sqrt{a_4+a_7}+(a_4+a_7)t$

B5=$\frac{y\sqrt{a_5}}{2\sqrt{t}}-\sqrt{(a_4+a_7)t}$

$D5=^{-y\sqrt{a_5}}\sqrt{(a_4+a_7)}+(a_4+a_7)(t-1)$

E5=$\frac{y\sqrt{a_5}}{2\sqrt{(t-1)}}-\sqrt{(a_4+a_7)\,(t-1)}$

$A6=y\sqrt{a_5}\sqrt{a_4+a_7}+(a_4+a_7)t$

B6=$\frac{y\sqrt{a_5}}{2\sqrt{t}}+\sqrt{(a_4+a_7)t}$

$D6=^{y\sqrt{a_5}}\sqrt{(a_4+a_7)}+(a_4+a_7)(t-1)$

E6=$\frac{y\sqrt{a_5}}{2(t-1)}+\sqrt{(a_4+a_7)\,(t-1)}$

A7=$-y\sqrt{a_5}\sqrt{a_4}+a_4t$

B7=$\frac{y\sqrt{a_5}}{2\sqrt{t}}-\sqrt{a_4t}$

A8=$y\sqrt{a_5}\sqrt{a_4}+a_4t$, B8=$\frac{y\sqrt{a_5}}{2\sqrt{t}}+\sqrt{a_4t}$

D8=y$\sqrt{a_5}\sqrt{a_4}+a_4(t-1)$

E8=$\frac{y\sqrt{a_5}}{2(t-1)}+\sqrt{a_4(t-1)}$

A9=$-y\sqrt{a_5}\sqrt{a_4}$

C9=$t-\frac{y\sqrt{a_5}}{2\sqrt{a_4}}$

$A_{10}=e^{y\sqrt{a_5}\sqrt{a_4}}$

C10=$t+\frac{y\sqrt{a_5}}{2\sqrt{a_4}}$

$F_{10}=t-1+\frac{y\sqrt{a_5}}{2\sqrt{a_4}}$

$A_{11}=-y\sqrt{a_5}\sqrt{a_4+a_9}+(a_4+a_9)t$

$B_{11}=\frac{y\sqrt{a_5}}{2\sqrt{8}}-\sqrt{(a_4+a_9)t}$

$D_{11}=-y\sqrt{a_5}\sqrt{a_4+a_9}+(a_4+a_9)(t-1)$

$E_{11}=\frac{y\sqrt{a_5}}{2\sqrt{(t-1)}}-(a_4+a_9)(t-1)$

F11=$t-1-\frac{y\sqrt{Pr}}{2\sqrt{a_4}}$

$A_{12}=y\sqrt{a_5}\sqrt{a_4+a_9}+(a_4+a_9)t$

$B_{12}=\frac{y\sqrt{a_5}}{2\sqrt{t}}+(a_4+a_9)t$

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C12= $t+\frac{y\sqrt{Pr}}{2\sqrt{a_1}}$

$D_{12}=y\sqrt{a_5}\sqrt{a_4+a_9}+(a_4+a_9)(t-1)$

$E_{12}=\frac{y\sqrt{a_5}}{2\sqrt{(t-1)}}+\sqrt{(a_4+a_9)(t-1)}$

F12=$t-1+\frac{y\sqrt{Pr}}{2\sqrt{a_4}}$

$A_{13}=y\sqrt{a_5}\sqrt{(a_2+a_4)}+(a_2+a_4)t$

$B_{13}=\frac{y\sqrt{a_5}}{2\sqrt{t}}-(a_2+a_4)t$

$D_{13}=-y\sqrt{a_5}\sqrt{(a_2+a_4)}+(a_2+a_4)(t-1)$

$E_{13}=\frac{y\sqrt{a_5}}{2\sqrt{t-1}}-\sqrt{(a_2+a_4)(t-1)}$

$A_{14}=y\sqrt{a_5}\sqrt{a_2+a_9}+\sqrt{a_2+a_9})t$ $B_{14}=\frac{y\sqrt{a_5}}{2\sqrt{t}}+(a_2+a_9)t$

$D_{14}=y\sqrt{a_5}\sqrt{(a_2+a_4)}+(a_2+a_4)(t-1)$

$E_{14}=\frac{y\sqrt{a_5}}{2\sqrt{t-1}}+\sqrt{(a_2+a_4)(t-1)}$

$A_{15}=-y\sqrt{a_5}\sqrt{(a_4+a_{11})}+(a_4+a_{11})t$

$B_{15}=\frac{y\sqrt{a_5}}{2\sqrt{t}}-\sqrt{(a_4+a_9)t}$,

$D_{15}=-y\sqrt{a_5}\sqrt{(a_2+a_{11})}+(a_2+a_{11})(t-1)$,

$E_{15}=\frac{y\sqrt{a_5}}{2\sqrt{t-1}}-\sqrt{(a_4+a_9)(t-1)}$

$A_{16}=y\sqrt{a_5}\sqrt{(a_4+a_{11})}+(a_4+a_{11})t$

$B_{16}=\frac{y\sqrt{a_5}}{2\sqrt{t}}+\sqrt{(a_4+a_9)t}$

$D_{16}=y\sqrt{a_5}\sqrt{(a_4+a_{11})}+(a_4+a_{11})(t-1)$,

$E_{16}=\frac{y\sqrt{a_5}}{2\sqrt{t-1}}+\sqrt{(a_4+a_9)(t-1)}$

$A17=-y\sqrt{Pr}\sqrt{a_1}+a_{1^t}$

B17= $\frac{y\sqrt{Pr}}{2\sqrt{t}}-\sqrt{a_1t}$

D_{17}=-$y\sqrt{pr}\sqrt{a_1}+a_1(t-1)$

$E17=\frac{y\sqrt{Pr}}{2\sqrt{t-1}}-\sqrt{a_1(t-1)}$

$A18=y\sqrt{Pr}\sqrt{a_1}+a_{1^t}$

B18= $\frac{y\sqrt{Pr}}{2\sqrt{t}}+\sqrt{a_1t}$

$D18=y\sqrt{Pr}\sqrt{a_1}+a_1(t-1)$

E18=$\frac{y\sqrt{Pr}}{2\sqrt{t-1}}+\sqrt{a_1(t-1)}$

A19=$-y\sqrt{Pr}\sqrt{a_1}$

A20=$y\sqrt{Pr}\sqrt{a_1}$

A21 $=-y\sqrt{Pr}\sqrt{(a_1+a_7)}+(a_1+a_7)t$

B21=$\frac{y\sqrt{Pr}}{2\sqrt{t}}-\sqrt{(a_1+a_7)t}$

E21=$\frac{y\sqrt{Pr}}{2\sqrt{t-1}}-\sqrt{(a_1+a_7)(t-1)}$

$D21=^{-y}\sqrt{Pr}\sqrt{(a_1+a_7)}+(a_1+a_7)(t-1)$

$A22=^{y\sqrt{Pr}}\sqrt{(a_1+a_7)}+(a_1+a_7)t$

B22=$\frac{y\sqrt{Pr}}{2\sqrt{t}}-\sqrt{(a_1+a_7)t}$

$D22=^{y}\sqrt{Pr}\sqrt{(a_1+a_7)}+(a_1+a_7)(t-1)$

E22=$\frac{y\sqrt{Pr}}{2\sqrt{t-1}}+\sqrt{(a_1+a_7)(t-1)}$

$A_{23}=e^{-y\sqrt{s_c}\sqrt{k_2}+\sqrt{krt}}$

$B_{23}=\frac{y\sqrt{s_c}}{2\sqrt{8}}$ -$\sqrt{krt}$

$D_{23}=e^{-y\sqrt{s_c}\sqrt{k_2}+k_r(t-1)}$

$E_{23}=\frac{y\sqrt{a_5}}{2\sqrt{t-1}}-k_r(t-1)$

$A_{24}=e^{y\sqrt{s_c}\sqrt{k_2}+\sqrt{krt}}$

$B_{24}=\frac{y\sqrt{s_c}}{2\sqrt{8}}+\sqrt{krt}$

$D_{24}=e^{y\sqrt{s_c}\sqrt{k_2}+k_r(t-1)}$

$E_{24}=\frac{y\sqrt{s_c}}{2\sqrt{t-1}}+k_r(t-1)$

$A_{25}=e^{-y\sqrt{s_c}\sqrt{k_2}}$

$A_{26}=e^{y\sqrt{s_c}\sqrt{k_2}}$

$A_{27}=e^{-y\sqrt{s_c}\sqrt{k_2+a_9}+(k_2+a_9)t}$

$B_{27}=\frac{y\sqrt{s_c}}{2\sqrt{t}}-\sqrt{(k_2+a_9)t}$

$D_{27}=e^{-y\sqrt{s_c}\sqrt{k_2+a_9}+(k_2+a_9)(t-1)}$

$E_{27}=\frac{y\sqrt{s_c}}{2\sqrt{(t-1)}}-\sqrt{(k_2+a_9)(t-1)}$

$A_{28}=e^{y\sqrt{s_c}\sqrt{k_2+a_9}+(k_2+a_9)t}$

$B_{28}=\frac{y\sqrt{s_c}}{2\sqrt{t}}+\sqrt{(k_2+a_9)t}$

$D_{28}=e^{y\sqrt{s_c}\sqrt{k_r+a_9}+(k_r+a_9)(t-1)}$

$E_{28}=\frac{y\sqrt{s_c}}{2\sqrt{(t-1)}}+\sqrt{(k_2+a_9)(t-1)}$

$A_{29}=e^{-y\sqrt{s_c}\sqrt{a_2+k_2}+(a_2+k_2)t}$

$B_{29}=\frac{y\sqrt{s_c}}{2\sqrt{t}}$ -$\sqrt{(a_2+k_2)t}$

$D_{29}=e^{-y\sqrt{s_c}\sqrt{a_2+k_2}+(a_2+k_2)(t-1)}$

$E_{29}=\frac{y\sqrt{s_c}}{2\sqrt{t}}-\sqrt{(a_2+k_2)(t-1)}$

$A_{30}=e^{y\sqrt{s_c}\sqrt{a_2+k_2}+(a_2+k_2)t}$

$B_{30}=\frac{y\sqrt{s_c}}{2\sqrt{t}}+\sqrt{(a_2+k_2)t}$

$D_{30}=e^{y\sqrt{s_c}\sqrt{a_2+k_2}+(a_2+k_2)(t-1)}$

$E_{30}=\frac{y\sqrt{s_c}}{2\sqrt{t}}+\sqrt{(a_2+k_2)(t-1)}$

$A_{31}=-y\sqrt{pr}\sqrt{(a_1+a_{11})}+(a_1+a_{11})t$ $B_{31}=\frac{y\sqrt{pr}}{2\sqrt{t}}-\sqrt{(a_1+a_{11})t}$

$D_{31}=-y\sqrt{pr}\sqrt{(a_1+a_{11})}+(a_1+a_{11})(t-1)$

$E_{31}=\frac{y\sqrt{pr}}{2\sqrt{t-1}}-\sqrt{(a_1+a_{11})(t-1)}$

$A_{32}=y\sqrt{pr}\sqrt{(a_1+a_{11})}+(a_1+a_{11})t$ $B_{32}=\frac{y\sqrt{pr}}{2\sqrt{t}}+\sqrt{(a_1+a_{11})t}$

$D_{32}=y\sqrt{pr}\sqrt{(a_1+a_{11})}+(a_1+a_{11})(t-1)$,

$E_{32}=\frac{y\sqrt{pr}}{2\sqrt{t-1}}+\sqrt{(a_1+a_{11})(t-1)}$

$D_{33}=-y\sqrt{pr}\sqrt{(a_1+a_2)}+(a_1+a_2)(t-1)$,

$E_{33}=\frac{y\sqrt{pr}}{2\sqrt{t-1}}-\sqrt{(a_1+a_2)(t-1)}$

$D_{34}=y\sqrt{pr}\sqrt{(a_1+a_2)}+(a_1+a_2)(t-1)$, $E_{34}=\frac{y\sqrt{pr}}{2\sqrt{t-1}}+\sqrt{(a_1+a_2)(t-1)}$

P1 = $\left(t-\frac{y\sqrt{s_c}}{2\sqrt{k_r}}\right)$, Q1=$-y\sqrt{s_c}\sqrt{k_r}$, R1=$\left(\frac{y\sqrt{s_c}}{2\sqrt{t}}-\sqrt{krt}\right)$,S1=$\left(t+\frac{y\sqrt{s_c}}{2\sqrt{k_r}}\right)$ T1= $y\sqrt{s_c}\sqrt{k_r}$

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U1=$\left(\frac{y\sqrt{s_c}}{2\sqrt{t}}+\sqrt{k_r t}\right)$, P2=$\left(t-1-\frac{y\sqrt{s_c}}{2\sqrt{k_r}}\right)$ Q2=$-y\sqrt{s_c}\sqrt{k_r}$, R2=$\frac{y\sqrt{s_c}}{2\sqrt{t-1}}-\sqrt{k_r(t-1)}$

S2=$\left(t-1+\frac{y\sqrt{s_c}}{2\sqrt{k_r}}\right)$, T2= $y\sqrt{s_c}\sqrt{k_r}$

P3=$-y\sqrt{S_c}\sqrt{a_2+k_r}+(a_2+k_r)t$

Q3=$\left(\frac{y\sqrt{s_c}}{2\sqrt{t}}-\sqrt{(a_2+k_r)t}\right)$ R3= $y\sqrt{s_c}\sqrt{a_2+k_r}+(a_2+k_r)t$,

S3=$\left(\frac{y\sqrt{s_c}}{2\sqrt{t}}+\sqrt{(a_2+k_r)t}\right)$

P4=$-y\sqrt{s_c}\sqrt{k_r}+k_r t$, Q4=$\left(\frac{y\sqrt{s_c}}{2\sqrt{t}}-\sqrt{k_{rt}}\right)$

R4= $y\sqrt{s_c}\sqrt{k_r}+k_{rt}$, S4=$\left(\frac{y\sqrt{s_c}}{2\sqrt{t}}+\sqrt{k_r t}\right)$

P5=$-y\sqrt{S_c}\sqrt{a_2+k_r}+(a_2+k_r)(t-1)$ Q5=$\frac{y\sqrt{s_c}}{2\sqrt{t-1}}-\sqrt{(a_2+k_r)(t-1)}$

R5= $y\sqrt{s_c}\sqrt{a_2+k_r}+(a+k_r)(t-1)$, S5=$\left(\frac{y\sqrt{s_c}}{2\sqrt{t-1}}+\sqrt{(a_2+k_r)(t-1)}\right)$

P6=$-y\sqrt{k_r}\sqrt{s_c}+\sqrt{k_r(t-1)}$,

Q6=$\left(\frac{y\sqrt{s_c}}{2\sqrt{t-1}}-\sqrt{k_r(t-1)}\right)$,

R6= $y\sqrt{s_c}\sqrt{k_r}+k_r(t-1)$

S6=$\left(\frac{y\sqrt{s_c}}{2\sqrt{(t-1)}}+\sqrt{k_r(t-1)}\right)$, P7=$-y\sqrt{s_c}\sqrt{k_r}+k_r t$,

Q7=$\left(\frac{y\sqrt{s_c}}{2\sqrt{t}}-\sqrt{k_r t}\right)$

R7= $y\sqrt{s_c}\sqrt{k_r}+k_r t$, S7=$\left(\frac{y\sqrt{s_c}}{2\sqrt{t}}+\sqrt{k_r t}\right)$ P8=$\left(t-\frac{y\sqrt{s_c}}{2\sqrt{k_r}}\right)$, Q8=$-y\sqrt{s_c}\sqrt{k_r}$

R8=$\left(\frac{y\sqrt{s_c}}{2\sqrt{t}}-\sqrt{k_r t}\right)$

S8=$\left(t+\frac{y\sqrt{s_c}}{2\sqrt{k_r}}\right)$ T8= $y\sqrt{s_c}\sqrt{k_r}$

U8=$\left(\frac{y\sqrt{s_c}}{2\sqrt{t}}+\sqrt{k_r t}\right)$ P9=$-y\sqrt{s_c}\sqrt{a_2+k_r+(a_2+k_r)t}$

Q9=$\left(\frac{y\sqrt{s_c}}{2\sqrt{t}}-\sqrt{(a_2+k_r)t}\right)$ R9= $y\sqrt{s_c}\sqrt{a_2+k_r}+(a_2+k_r)t$,

S9=$\left(\frac{y\sqrt{s_c}}{2\sqrt{t}}+\sqrt{(a_2+k_r)t}\right)$

P10=$-y\sqrt{s_c}\sqrt{k_r}+k_r(t-1)$

Q10= $\left(\frac{y\sqrt{s_c}}{2\sqrt{(t-1)}}-\sqrt{k_r(t-1)}\right)$, R10=$+y\sqrt{s_c}\sqrt{k_r}+k_r(t-1)$

S10=$\frac{y\sqrt{s_c}}{2\sqrt{t-1}}+\sqrt{k_r(t-1)}$,

P11=$\left(t-1-\frac{y\sqrt{s_c}}{2\sqrt{k_r}}\right)$, Q11=$-y\sqrt{s_c}\sqrt{k_r}$ R11=$\left(\frac{y\sqrt{s_c}}{2\sqrt{t-1}}-\sqrt{k_r(t-1)}\right)$

S11= $t-1+\frac{y\sqrt{s_c}}{2k_r}$, T11= $y\sqrt{s_c}\sqrt{k_r}$ U11=$\left(\frac{y\sqrt{s_c}}{2\sqrt{t-1}}+\sqrt{k_r(t-1)}\right)$ P12=$-y\sqrt{s_c}\sqrt{a_2+k_r}+(a_2+k_r)(t-1)$,

Q12=$\left(\frac{y\sqrt{s_c}}{2\sqrt{t-1}}-\sqrt{(a_2+k_r)(t-1)}\right)$

R12= $y\sqrt{s_c}\sqrt{a_2+k_r}+(a_2+k_r)(t-1)$

S12=$\frac{y\sqrt{s_c}}{2\sqrt{t-1}}+\sqrt{(a_2+k_r)(t-1)}$

P13=$-y\sqrt{p_r}\sqrt{a_1+a_2}+(a_1+a_2)(t-1)$

Q13=$\left(\frac{y\sqrt{p_r}}{2\sqrt{t-1}}-\sqrt{(a_1+a_2)(t-1)}\right)$

R13= $y\sqrt{p_r}\sqrt{a_1+a_2}+(a_1+a_2)(t-1)$

S13=$\left(\frac{y\sqrt{p_r}}{2\sqrt{t-1}}+\sqrt{(a_1+a_2)(t-1)}\right)$ P14=$-y\sqrt{p_r}\sqrt{a_1}+a_1(t-1)$,

Q14=$\left(\frac{y\sqrt{p_r}}{2\sqrt{t-1}}-\sqrt{a_1(t-1)}\right)$

R14= $y\sqrt{p_r}\sqrt{a_1}+a_1(t-1)$,

S14=$\left(\frac{y\sqrt{p_r}}{2\sqrt{t-1}}+\sqrt{a_1(t-1)}\right)$, P15=$-y\sqrt{p_r}\sqrt{a_1+a_2}+(a_1+a_2)(t)$

Q15=$\left(\frac{y\sqrt{p_r}}{2\sqrt{t}}-\sqrt{(a_1+a_2)t}\right)$ R15= $y\sqrt{p_r}\sqrt{a_1+a_2}+(a_1+a_2)t$ S15=$\left(\frac{y\sqrt{p_r}}{2\sqrt{t}}+\sqrt{(a_1+a_2)t}\right)$

P16=$-y\sqrt{p_r}+\sqrt{a_1}+a_1 t$,

Q16=$\left(\frac{y\sqrt{p_r}}{2\sqrt{t}}-\sqrt{a_1 t}\right)$, R16= $y\sqrt{p_r}\sqrt{a_1}+a_1 t$, T16=$\left(\frac{y\sqrt{p_r}}{2\sqrt{t}}+\sqrt{a_1 t}\right)$

P17=$-y\sqrt{p_r}\sqrt{a_1}+a_1(t-1)$,

Q17=$\left(\frac{y\sqrt{p_r}}{2\sqrt{t-1}}-\sqrt{a_1(t-1)}\right)$,

R17= $y\sqrt{p_r}\sqrt{a_1}+a_1(t-1)$

S17=$\left(\frac{y\sqrt{p_r}}{2\sqrt{t-1}}+\sqrt{a_1(t-1}\right)$, P18=$\left(t-1-\frac{y\sqrt{p_r}}{2a_1}\right)$

Q18=$-y\sqrt{p_r}\sqrt{a_1}$, R18=$\left(\frac{y\sqrt{p_r}}{2\sqrt{t-1}}-\sqrt{a_1(t-1)}\right)$

S18=$\left(t-1+\frac{y\sqrt{p_r}}{2\sqrt{a_1}}\right)$, T18=$+y\sqrt{p_r}\sqrt{a_1}$ U18=$\left(\frac{y\sqrt{p_r}}{2\sqrt{t-1}}-\sqrt{a_1(t-1)}\right)$,

P19=$-y\sqrt{p_r}\sqrt{a_1+a_2}+(a_1+a_2)(t-1)$,

Q19=$\left(\frac{y\sqrt{p_r}}{2\sqrt{t-1}}-\sqrt{(a_1+a_2)(t-1)}\right)$

R19=$y\sqrt{p_r}\sqrt{a_1+a_2}+(a_1+a_2)(t-1)$

S19=$\left(\frac{y\sqrt{p_r}}{2\sqrt{t-1}}+\sqrt{(a_1+a_2)(t-1)}\right)$ P20=$-y\sqrt{p_r}\sqrt{a_1}$,

Q20=$\left(\frac{y\sqrt{p_r}}{2\sqrt{t}}-\sqrt{a_1 t}\right)$, R20= $y\sqrt{p_r}\sqrt{a_1}+a_1 t$

S20=$\left(\frac{y\sqrt{p_r}}{2\sqrt{t}}+\sqrt{a_1 t}\right)$, P21=$\left(t-\frac{y\sqrt{p_r}}{2\sqrt{a_1}}\right)$, Q21=$-y\sqrt{p_r}\sqrt{a_1}$,

R21=$\left(\frac{y\sqrt{p_r}}{2\sqrt{t}}-\sqrt{a_1 t}\right)$, S21=$\left(t+\frac{y\sqrt{p_r}}{2\sqrt{a_1}}\right)$

T21==$+y\sqrt{p_r}\sqrt{a_1}$, U21=$\left(\frac{y\sqrt{p_r}}{2\sqrt{t}}-\sqrt{a_1 t}\right)$,

P22=$-y\sqrt{p_r}\sqrt{a_1+a_2}+(a_1+a_2)t$

Q22=$\left(\frac{y\sqrt{p_r}}{2\sqrt{t}}-\sqrt{(a_1+a)t}\right)$, R22= $y\sqrt{p_r}\sqrt{a_1+a_2}+(a_1+a_2)t$, S22=$\left(\frac{y\sqrt{p_r}}{2\sqrt{t}}+\sqrt{(a_1+a_2)t}\right)$