IDENTIFICATION OF ELASTIC PARAMETERS OF LAMINATED CARBON FIBER PLATES USING EXPERIMENTAL MODAL ANALYSIS

Mikhail Nikhamkin¹, Sergey Semenov¹, Vadim V. Silberschmidt² and Danil Solomonov¹
¹Perm National Research Polytechnic University, Perm, Russia
²Wolfson School of Mechanical and Manufacturing Engineering, Loughborough University, Leicestershire, United Kingdom
E-Mail: sergey.semyonov@mail.ru

ABSTRACT
The aim of this work is to create an experimental-numerical technique for elastic parameters identification of laminated polymer composites using experimental modal analysis. The object of this research is laminated carbon fiber reinforced composite based on an equally strong carbon fabric and epoxy binder. Experimental determination of natural frequencies and corresponding vibration modes was performed using 3D scanning laser vibrometry. The finite element calculation and quasi-random search optimization technique can be effectively used to solve the identification problem. The evaluation by independent experiments showed that the error of the natural frequencies, determined from the obtained elasticity characteristics, lies within 5%. The described technique can be recommended for determining of elasticity parameters necessary for calculating the modal characteristics of structure elements made from laminated carbon fiber.

Keywords: polymer composites, CFRP, elastic parameters identification, modal analysis, 3D laser vibrometry, elastic modulus, shear modulus, Poisson’s ratio.

1. INTRODUCTION
Carbon fiber Reinforced Plastic (CFRP) composites have now a wide field of applications in aerospace industry, aircraft engines, sport equipment and others due to the possibility of reducing the weight of the structure without reducing the strength [1-3]. For example, aircraft Boeing 787 consists of 50% of the composite materials by weight.

Strength calculations of structures of CFRP are often complicated by the lack of reliable data on mechanical characteristics, in particular, elastic modules and Poisson’s ratios. The problem consists, on the one hand, in the greater (in comparison with isotropic materials) number of parameters entering into the model of the elastic behavior of the material, and on the other hand, these parameters depend on a wide range of factors (fiber and binder materials, reinforcement scheme, technological factors) [1, 2]. Given in a literature sources data about characteristics of materials is often contradictory and requires additional verification for carrying out responsible calculations.

The classical direct methodologies usually require a significant number of experimental diagrams to determine values of the elastic parameters. This requires carrying out intensive and expensive tests and a large number of specimens [1, 4].

An approach based on solving the inverse coefficient problem of solid mechanics and may be used to determine the elasticity parameters of FRP. It represents a reconstructing the elasticity parameters using experimental data on displacement fields, deformation, etc. [5-7]. In particular, experimental data on natural frequencies and modes are successfully used in [8-11] to identify the elasticity parameters. This became possible due to the active development of methods of experimental modal analysis in recent years [12, 13].

The aim of this work is to create an experimental-numerical technique for elastic parameters identification of laminated polymer composites using experimental modal analysis.

2. OBJECT OF RESEARCH
The object of this research is laminated carbon fiber reinforced composite based on an equally strong carbon fabric and epoxy binder. This material is widely used in aviation [3].

Investigation was provided on specimens which represents as rectangular plates with length 250 mm and width 24.6 mm (see Figure-1).

Figure-1. Specimen - carbon fiber plastic specimen.

The same specimens are recommended by ASTM standard for polymer composite materials mechanical properties determination in tensile and fatigue tests [18, 19]. Three specimens with difference thickness and laying schemes were investigated (Table-1).
3. EXPERIMENTAL MODAL ANALYSIS Technique and Results

Experimental modal analysis (EMA) is performed to obtain natural frequencies and vibration modes of the specimens. These data is required for further numerical model identification. Modern experimental modal analysis instruments allow obtaining the natural frequencies with high accuracy and natural modes with high resolution.

3D laser scanning vibrometry method was used in the research. This method is based on representation of studied object as an oscillating system with finite number \((n)\) of degrees of freedom (DOF). Experimental determination of system natural frequencies and corresponding vibration modes is performed using analysis of transfer function matrix \([H]\), every element of which represents results of individual frequency characteristic measurement as relation \([12, 13, 15]\):

\[
H_{ij}(\omega) = X_i(\omega)/F_j(\omega) \quad i, j=1...n
\]

where \(X_i(\omega)\) - response frequency function as a speed or acceleration for i DOF on an impact \(F_j(\omega)\), corresponding to j DOF, \(\omega\) - frequency.

Main advantage of the method of scanning laser vibrometry is the non-contact measurement of vibrations. This feature makes it possible to exclude the negative influence of sensor masses. The preparation of object by sensors is not required also. Another advantage is the high spatial resolution, which is provided by a small distance between the scanning points (2-3 mm). Three-component scanning laser vibrometer Polytec PSV400-3D was used for the experimental modal analysis [16-18].

The specimen was fixed on a rigid metal frame in compliant elastic suspensions (Figure-2). Such scheme is often used in the experimental modal analysis, because it simulates free boundary conditions. It is convenient for reproduction in subsequent calculations. The oscillations were excited by acoustic oscillator. It was driven with an electrical harmonic signal varying in time according to a law with constant amplitude and increasing frequency from 20 to 6400 Hz. Five experiments with different parameters scanning grid (varying from 25 to 165 nodes) were performed. The scan grid had 165 nodes.

In investigated frequency range 20-6400 Hz, 11 eigenmodes of oscillations were found from specimens #1 and #2, and 8 eigenmodes for the specimen #3. The dispersion of the eigen frequencies (\(i\) - number of eigenmode) was in the range of 0.57% (by the variation ratio). The proper modes of oscillation was approximately the same for all specimens (Table-2), the corresponding eigenfrequencies are given in Table-4.

4. NUMERICAL MODAL ANALYSIS Technique and Results

Modal analysis was performed using finite element method (FEM). When the damping is neglected, the natural vibrations of the finite element model with \(n\) degrees of freedom are described in the matrix form by the equation \([20]\):

\[
[M](\ddot{u}) + [K][u] = 0,
\]

where \([K]\) and \([M]\) are the stiffness and mass matrices, and \([u]\) is the displacement vector at the nodes of the finite element model.

The problem of natural frequencies and vibration modes determining reduces to the problem of the eigenvalues \(\omega_k\) and the vectors \([u_0]_k\), which turn the determinant to zero:

\[
det|[K] - \alpha^2[M]| = 0
\]

Solution of the problem was accomplished by the finite element method in the ANSYS Workbench package. It was assumed that the specimen was free of restrictions.
on movement (free boundary conditions). The finite-element model consists of eight-node shell elements.

Table-2. The eigenmodes and the eigenfrequencies \( (f_i) \) of specimen#1 obtained experimentally.

<table>
<thead>
<tr>
<th>i</th>
<th>Eigenmodes</th>
<th>( f_i, \text{Hz} )</th>
<th>i</th>
<th>Eigenmodes</th>
<th>( f_i, \text{Hz} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>322</td>
<td>7</td>
<td></td>
<td>3256</td>
</tr>
<tr>
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<td></td>
<td>890</td>
<td>8</td>
<td></td>
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<tr>
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<td>1592</td>
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<td></td>
<td>4968</td>
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<td></td>
<td>5547</td>
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<td>5</td>
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<td></td>
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<td>6</td>
<td></td>
<td>2859</td>
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</tr>
</tbody>
</table>

The model of material behavior is linear elastic orthotropic layered composite with different orientation of layers. It is assumed that there is a plane stress state. The elastic behavior of each monolayer is characterized by four parameters: the elastic moduli along the bases \( E_{11} \) and across the base \( E_{22} \), the shear modulus \( G_{12} \) and the Poisson's ratio \( \nu_{12} \). Axis directions are shown in Figure-3. All layers of CFRP, oriented in different ways, are united in one element. The characteristic size of the elements was chosen from the results of estimating the convergence of calculating the natural frequencies results. A series of calculations with the dimensions of the elements from 5 mm to 1.25 mm showed that for an element size of 2.5 mm (about 1000 elements for a whole model), the values of the first 8 natural frequencies converge with an error of 0.3%. The model of the specimen material is a laminated composite with orthotropic linear elastic layers. It was assumed that the differently oriented layers of the composite were deformed together without slipping.

5. METHOD AND RESULTS OF IDENTIFICATION OF ELASTIC MATERIAL CHARACTERISTICS

The task of identification problems is to find the mentioned four elasticity constants of the laminate monolayer: \( E_{11}, E_{22}, G_{12} \) and \( \nu_{12} \):

\[
E = [E_{11}, E_{22}, G_{12}, \nu_{12}]^T
\]

were \( E \) is the vector identified parameters, index T means the transpose operation.

To determination of elasticity constants the results of experimental modal analysis (natural frequencies and vibration modes) of the specimen were used. The problem of identification of the elastic parameters is considered as an optimization problem [28, 29] with the objective function:

\[
I(E) = \sum_{i=1}^{n} \alpha_i \left( \frac{f_{ci}}{f_{ei}} \right)^2 \rightarrow \min
\]

where: \( f_{ci} \) and \( f_{ei} \) are the calculated and experimental values of the \( i \)-th natural frequency, \( \alpha_i \) - weighting coefficients.

The following limitations were set for identifiable parameters:

\[
 E_{11}^{\text{min}} < E_{11} < E_{11}^{\text{max}}
 E_{22}^{\text{min}} < E_{22} < E_{22}^{\text{max}}
 G_{12}^{\text{min}} < G_{12} < G_{12}^{\text{max}}
 \nu_{12}^{\text{min}} < \nu_{12} < \nu_{12}^{\text{max}}
\]

5.1. Results of Identification

Figure-3. Axis directions of plate.
It was assumed that maximal and minimal values of all parameters are determined as ±10% deviation from the initial value $E_{11}^0, E_{22}^0, G_{12}^0, \text{and } \nu_{12}^0$. The initial values were adopted on the basis of the literary data [21] (see Table-3).

In the relation (5), it is important that the calculated and experimental values of the natural frequencies correspond to the same natural frequencies. In this paper, a comparison of the calculated and experimental natural frequencies was carried out on the basis of an analysis of their animation representation. In more complex cases, it is recommended to use analysis of the MAC matrix [13].

To solve the optimization problem, the quasi-random search method was used. Weight coefficients were taken equal to one. Quasi-random search procedure included 50 realizations.

Not all experimental eigenfrequencies were included in objective function (5). Identification was carried out using only the first 8 natural frequencies of specimen#1.

### Table-3. Values of elastic parameters of material: results of identification and literary data.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$E_{11}, \text{GPa}$</th>
<th>$E_{22}, \text{GPa}$</th>
<th>$G_{12}, \text{GPa}$</th>
<th>$\nu_{21}$</th>
</tr>
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<tbody>
<tr>
<td>Literary data [21]</td>
<td>63.9</td>
<td>63.9</td>
<td>4.8</td>
<td>0.04</td>
</tr>
<tr>
<td>Results of identification</td>
<td>70</td>
<td>68</td>
<td>4.4</td>
<td>0.04</td>
</tr>
</tbody>
</table>

As the result of identification calculations, a set of elasticity characteristics was presented in Table-3. The elastic moduli $E_{11}$ and $E_{22}$ were 6-9% higher than those given in [21], and the shear modulus $G_{12}$ was 9% lower. The Poisson's ratio in the results of the calculations and in the literary data is found by the same and the same.

To estimate the accuracy of the obtained elasticity characteristics given in Table-3, the first 11 natural frequencies for all specimens were calculated by FEM (Table-4). The discrepancy between the values obtained as a result of identification and the experimental data does not exceed 4.14% for specimen#1, about 1.9% for specimen#2 and 4.9% for specimen#3. It should be noted that in the identification process, experimental data were used only for 8 natural frequencies of only one specimen#1. Other independent data confirm the accuracy of the identification procedure.

Correspondence of the natural vibration modes obtained by FEM calculation using the identified elastic parameters of Table-3 with the experimental data obtained by laser vibrometry was also checked. The proper forms shown in Table-5 are in good agreement with each other.
Table-4. Comparison of the eigenfrequencies obtained as a result of FEM calculation with experimental data.

<table>
<thead>
<tr>
<th>i</th>
<th>Specimen#1</th>
<th>Specimen#2</th>
<th>Specimen#3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_{ei}$, Hz</td>
<td>$f_{ci}$, Hz</td>
<td>$f_{ei}$, Hz</td>
</tr>
<tr>
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<td>322</td>
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<td>5547</td>
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<td>11</td>
<td>5841</td>
<td>5617</td>
<td>3.84</td>
</tr>
</tbody>
</table>

Table-5. Comparison of the natural vibration modes of the specimens: experiment and FEM calculation.

<table>
<thead>
<tr>
<th># of natural mode, i</th>
<th>Experiment</th>
<th>FEM calculation</th>
</tr>
</thead>
<tbody>
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<td>1</td>
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<td><img src="image21.png" alt="Image" /></td>
<td><img src="image22.png" alt="Image" /></td>
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</tbody>
</table>
6. CONCLUSIONS
To identify the elasticity characteristics of laminated carbon fiber plates with orthotropic layers, the technique based on solving the inverse coefficient problem and the use of experimental data on the natural frequencies and natural vibration modes of standard specimens can be recommended.

To obtaining of the experimental data on the natural frequencies and natural vibration modes the use of 3D scanning laser vibrometry may be recommended. In the present work it was possible to obtain the natural oscillation frequencies of the specimen from laminated carbon fiber plates in the frequency range up to 6400 Hz with scattering within 0.5%.

To solve the identification problem, the finite element calculation and quasi-random search optimization technique can be effectively used. The evaluation by independent experiments showed that the error of the natural frequencies, determined from the obtained elasticity characteristics, lies within 5%.

The described technique can be recommended for determining the elasticity parameters necessary for calculating the modal characteristics of structure elements from laminated carbon fiber

ACKNOWLEDGMENTS
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