



## MEASUREMENT OF INSTANTANEOUS AND AVERAGE VALUES OF PHASE ANGLE USING PRECISION VOLTMETER

Oleh Velychko and Valentyn Isaiev

State Enterprise "All-Ukrainian State Research and Production Center for Standardization, Metrology, Certification and Consumer Rights Protection", Ukraine

E-Mail: [black2001w@gmail.com](mailto:black2001w@gmail.com)

### ABSTRACT

The influence of the main factors of distortion of the measurement results of the shift angle between two phase voltages reproduced by a three-phase ac calibrator is analyzed in the work. For the developed measurement procedure of the phase angle using the precision voltmeter method, the sequence of operations is considered depending on the instantaneous or average values of the measurand. Since the distribution of observation results was defined as symmetric and bimodal, the value of the measurand should be the mean within the interval of the variation. The Monte Carlo simulation gave results that, with small scattering, coincide with the measured value. The introduction of the correction for the instantaneous measurement did not significantly change the result because of the scattering during the whole oscillation period.

**Keywords:** phase angle, precision voltmeter, beta distribution, Monte Carlo simulation, measurement uncertainty.

### 1. INTRODUCTION

The shift angle between two voltages is an important characteristic in the three-phase power supply system and significantly affects the voltage unbalance, which is an indicator of the quality of electricity [1]. The phase angle measurement can be performed using a phase meter [2], voltmeter or other means. In the field of applied metrology, it is often the task of checking the metrological characteristics of the phase meters, the power network analyzers and the corresponding means of metrological support.

Previously, the method of three voltmeters was re-evaluated and an appropriate method for calibrating the phase angle between two voltages was developed [3]. A methodology for estimating an uncertainty of measurements in accordance with GUM 1995 is used as usual when calibrating measuring instruments [4]. But for cases of nonlinear mathematical models which describe the measurand, there is also an addition to the last-mentioned document concerning an application of the Monte Carlo methods [5]. Detailed versions of using the Monte Carlo methods are described in many works, including [6, 7].

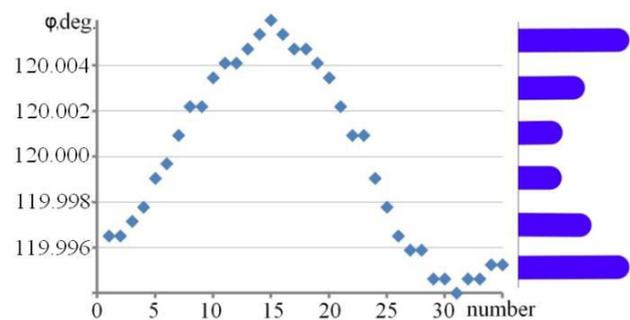
Regarding the phase angle measurement by means of a precision voltmeter, the question about the correctness of Monte Carlo simulation was not revealed in terms of the formation of the distribution law of a random event. The accuracy of the evaluation of the measurement uncertainty without taking into account the full interval of change of the measurand is also still not highlighted. Therefore, the authors propose to consider two approaches to the use of the precision voltmeter method, taking into account the oscillation period of the measurand and without regarding it.

### 2. LITERATURE REVIEW AND RESEARCH BACKGROUND

The task of measuring the phase angle is extremely relevant, taking into account the small number

of national standards of this unit in the Key Comparison Database (KCDB) [8]. A measurement of such a physical quantity with a small uncertainty can be performed using two precision multimeters in the mode of direct voltage measurement degrees [9]. In cases where synchronization is difficult to implement, one can use the methods of asynchronous phase angle measurement [10].

The phase angle measurement was also investigated using a precision phase meter with incoherent sampling, as well as an uncertainty analysis with help of Monte Carlo method was performed [11]. A comparison of the measurement results with using both a precision multimeter in the mode of measuring the alternating voltage and a precision phase meter was performed to assess the benefits of using the precision measuring instruments [12]. In the last research, the precision phase meter measured the average value of the phase angle which varied from  $119.994^\circ$  to  $120.006^\circ$ . Most of the observation results were concentrated near the lower and upper limits of the oscillation, as graphically represented in Figure-1.



**Figure-1.** Periodic oscillation of phase angle reproduced by three-phase calibrator.

It can be seen that the distribution of the observation results is similar to the symmetrical bimodal beta distribution. It is known that the beta distribution has



two characteristic parameters  $\alpha$  and  $\beta$ , and the probability density depends on a relationship between them [13].

It is known that symmetric bimodal distribution has  $\alpha$  and  $\beta$  parameters which are equal to each other, and if  $\alpha$  and  $\beta$  equal to 0.5 then a high event concentration is along the edges of the interval of possible values, and if  $\alpha$  and  $\beta$  equal to 1.0 then events are all distributed throughout the interval. Thus, the range of values of  $\alpha$  and  $\beta$  from 0.5 to 1 is interesting for this study. The expected value of a random variable of a symmetric bimodal beta distribution will always be in the middle of the interval of possible values (for this case, 120.000°). As for the variance, the formula for its definition has the form [14]:

$$Var = \alpha \cdot \beta / [(\alpha + \beta)^2 \cdot (\alpha + \beta + 1)]. \tag{1}$$

If we analyze the change of the variance in the range of values of  $\alpha$  and  $\beta$  from 0.5 to 1.0 using formula (1), then we can obtain a number of values of this characteristic in accordance with Table-1.

**Table-1.** Change of the variance in the range of  $\alpha$  and  $\beta$  values from 0.5 to 1.0.

$\alpha = \beta$	Variance Var	$S = \sqrt{Var}$	Associated standard deviation in degrees
0.5	0.063	0.250	0.0048
0.6	0.068	0.261	0.0050
0.7	0.073	0.270	0.0051
0.8	0.077	0.277	0.0053
0.9	0.080	0.283	0.0054
1.0	0.083	0.289	0.0055

From Table-1, it follows that there is no significant effect, what values of  $\alpha$  and  $\beta$  are actual in the specified range, since the difference between the extreme values of associated standard uncertainties is only 0.0007°, and the contribution of this factor is near 15 %.

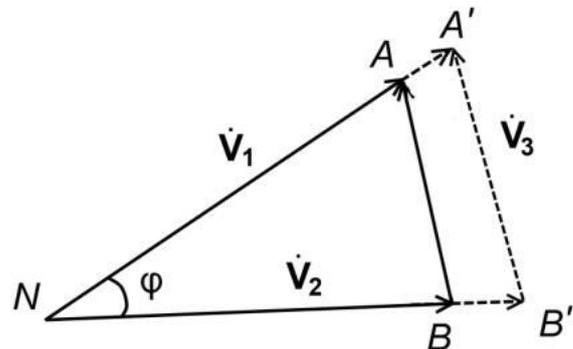
Bearing in mind that the interval of possible values of the reproducible phase angle is limited to the lower and upper limits, and by making sure by observing its invariability one can define the interval of uncertainty limited by these values, in other words, the expanded uncertainty can be equated to 0.012° for the average value of the reproducible phase angle. In [12], the type A standard uncertainty was estimated at 0.0061° when the measurand of 120° was measured by a precision phase meter, and with the type B standard uncertainty equal to about 0.0058°, the expanded uncertainty became 0.017° with a coverage factor which was equal 2. Obviously, this is not enough, since the lower and upper limits of the phase angle oscillation are known with the expanded uncertainty equal to 0.011° when using the precision phase meter [15].

### 3. MATERIALS AND METHODS

#### 3.1 Uncertainty sources when measuring phase angle by means of precision voltmeter

When measuring the angle between two phase voltages, which are reproduced by a three-phase calibrator, the influence of two main sources of uncertainty (the periodic oscillation of the output voltages, and the periodic oscillation of the phase angle) must be taken into account using the method of precision voltmeter [16]. Probably these oscillations are caused by oscillations in the power supply, although this aspect has not yet been investigated. The first factor has a significant effect on the type A standard uncertainty which is reported by the calibration certificate and can vary 6 or 10 times [15]. For a better understanding of the first factor, it is worth considering Figure-2.

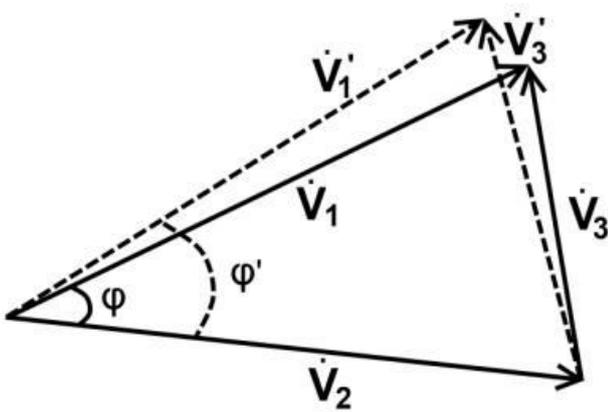
Figure-2 shows that the amplitudes of the first and second phase voltages vary, respectively, from point A to point A' and from point B to point B'.



**Figure-2.** Oscillations of measured voltages in a vector representation.

In this case, the vector  $\vec{V}_3$  shifts in parallel, and the angle between the voltage vectors  $\vec{V}_1$  and  $\vec{V}_2$  remains constant. The parallel displacement of the vector  $\vec{V}_3$  is due to the effect of the synchronous change of the output phase voltages of the three-phase calibrator, since the increase in the voltage in the power supply must, at the same time, be reflected in the increase of all output voltages. During monitoring with the help of two Fluke 8845A multimeters simultaneously connected to calibrator terminals of two phase voltages, a synchronous increase and a synchronous decrease of both phase voltages were recorded.

As for the second mentioned factor, its graphical interpretation is depicted in Figure-3.



**Figure-3.** Voltage vector shifts with phase angle oscillations.

Since the phase voltage amplitude changes are synchronous, the relationship between voltage vector modules will remain constant. Fixing vector  $\vec{V}_2^g$  for convenience, considering that the phase angle  $\varphi$  periodically changes to  $\varphi'$ , one can see that the direction of the vector  $\vec{V}_3^g$  varies, as well as the module, transforming in the extreme position into the vector  $\vec{V}_3^g$ . Thus, the three-phase calibrator in the part of the reproduction of the phase angle can be characterized by the instantaneous and average value of this quantity. Consequently, one can show of the measurement uncertainty of the instantaneous or average value of the phase angle during the oscillation period, and these characteristics will be different since the measurement uncertainty of the average value will be significantly higher.

The question of the expediency of determining the instantaneous value of the phase angle lies in the plane of assessment of the possibility of providing a highly stable power supply for the three-phase calibrator, or output signals of a calibrator. Indeed, only under this condition, the redistribution of voltage inside the device will reach a quasistationary state, and the stability of the output signals will be determined by internal noises and pulsations.

In addition to those sources of uncertainty described in Figure-2 and Figure-3, one must remember the influence factors described in [12], that is, the scattering of multimeter readout for a short observation time and uncertainty of measurement when calibrating the multimeter, temperature correction, etc.

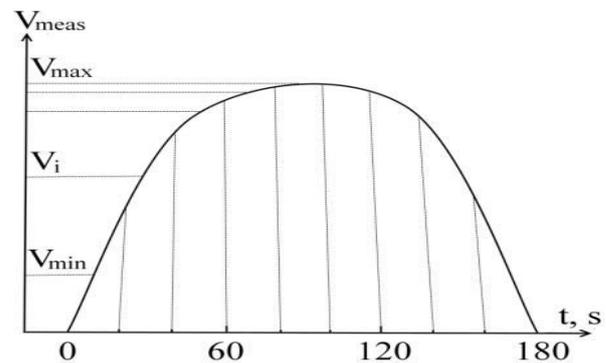
### 3.2 Problem points of measurement procedure

When determining the phase angle in accordance with [3], the measurement information is derived from three input quantities (the RMS values of the first and second phase voltages, the RMS value of the voltage between the two phases), and is calculated by the formula:

$$\varphi = \frac{180}{\pi} \cdot \arccos \left( \frac{\bar{V}_1}{2 \cdot \bar{V}_2} + \frac{\bar{V}_2}{2 \cdot \bar{V}_1} - \frac{\bar{V}_3^2}{2 \cdot \bar{V}_2 \cdot \bar{V}_1} \right), \quad (2)$$

where  $\bar{V}_1$ ,  $\bar{V}_2$ ,  $\bar{V}_3$  are mean of corresponding voltage measured by means of a precision voltmeter.

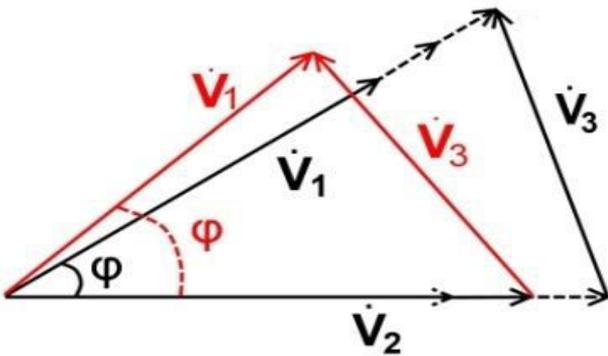
Since only one precision voltmeter is used to measure these voltages, information is obtained sequentially. In this case, the measurement time of one input quantity is about 20 seconds, and the entire measurement cycle lasts about 60 seconds. Hence, the problem point is the violation of the relationship between the measured values, comparing with the stationary state of output voltages of a calibrator. This problem can be graphically interpreted in Figure-4.



**Figure-4.** Possible measurement results with oscillating voltage.

Here, it is shown the change of a certain measurable RMS value from the moment labeled 0 s when the lower limit has been reached to a certain upper limit at a time of 90 s, and a gradual decrease again to the lower limit at the moment labeled 180 s from the beginning of the oscillation. Thus, the period of oscillation is 3 minutes, and it is possible to measure the average values of the input quantities 9 times according to the equation (2).

If no monitoring of the oscillation voltage state and the agreement of the beginning of the measurement of each input quantity is performed, then the measurement may take place at an arbitrary moment of the periodic oscillation of the measurand, that is, at any second from 0 to 180. Let us suppose, the first phase voltage is started to be measured when the minimum oscillation limit is reached, then the average measured value will correspond to  $V_{min}$ , the second phase voltage will be measured in the range from 20 to 40 s, and the second mean will be somewhat larger, and the difference between these voltages will be measured in the interval from 40 to 60 seconds, and the third mean will be further increased. The described procedure of the successive measurement of the three increasing input quantities, as well as the distortion of the phase angle calculated by the formula (2), are shown in Figure-5.



**Figure-5.** Measurement result distortion in determining the phase angle on the basis of measured voltages.

The simultaneous changes in the measured voltages and the corresponding phase angle are shown in Figure-5 in black colors, and the measured successively oscillation voltages and the corresponding distorted phase angle are depicted in red colors.

However, the measurement of input quantities can be initiated at a time of 90 seconds according to Figure-4, and in this case, the displacement of a phase angle will occur in the opposite direction, that is, in the decreasing direction.

In order to avoid the indicated distortion of the actual value of the reproducible phase angle, one can observe the oscillations of each measured voltage for determining their minimum and maximum limits during, for example, 10 periods of oscillation which may last about 30 minutes. If no noticeable drift of these voltages

has been observed, then one can estimate the phase angle for the minimum, average and maximum values of the measured voltages and compare the results of determining the phase angle with each other. Herewith, the obtained values will be different depending on the instantaneous value of the phase angle (see Figure-3).

**4. RESULTS AND DISCUSSIONS**

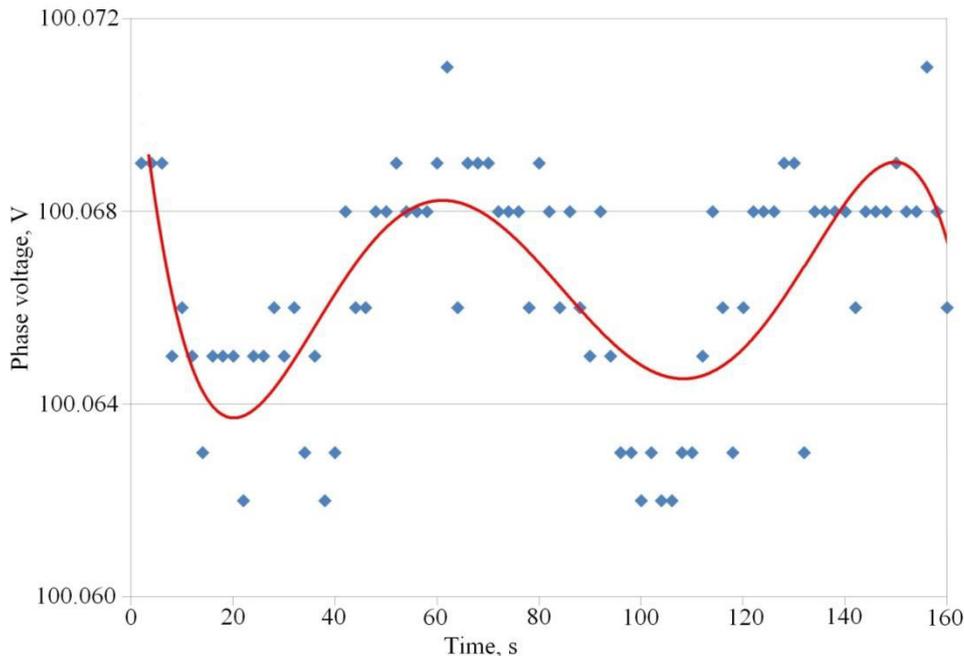
**4.1 Experimental verification of assumptions about oscillations of output voltage of three-phase calibrator**

An assumption of an oscillatory nature of the change of the measured voltages was verified using two Fluke 8845A multimeters while simultaneously measuring two output signals of the calibrator. During the investigation of the three-phase calibrator Resource-K2, the output voltage of one of the phases changed in time as shown in Figure-6.

It is evident that approximately two periods of voltage fluctuations occurred during about 160 s (see Figure-6). The voltage trend line equation generated by this software has the form:

$$V = 3 \cdot 10^{-13} \cdot t^6 - 4 \cdot 10^{-10} \cdot t^5 + 8 \cdot 10^{-8} \cdot t^4 - 6 \cdot 10^{-6} \cdot t^3 + 2 \cdot 10^{-4} \cdot t^2 - 2.2 \cdot 10^{-3} \cdot t + 100.07 \tag{3}$$

where *t* represents the time.



**Figure-6.** Phase voltage oscillation.

The calculated coefficients are valid only for the specified small interval of a time. If one could apply a sequential measurement of three voltages according to the

scheme indicated in [12] for a time interval of about one second when there is no significant change in a voltage amplitude then it could be possible to construct three trend



lines and calculate the instantaneous values of the phase angle. And if the changes in the instantaneous values would occur, this circumstance would indicate the variation of the phase angle of the calibrator in accordance with Figure-3.

Two variants of oscillations were recorded during the experimental verification of the assumption of synchronous change of both phase voltages with the help of two Fluke 8845A multimeters for a phase angle of 120°

for phase voltages of 100 V. These are shown in Figures 7 and 8. For convenience, the voltage deviation has been expressed as the difference between the smallest measured value of the measurand and the current value. It can be seen in Figure-7 that the directions of change of each of the phase voltages were approximately opposite for the fixed case. This means that the oscillations of the voltage difference were not according to Figure-2, but according to Figure-9.

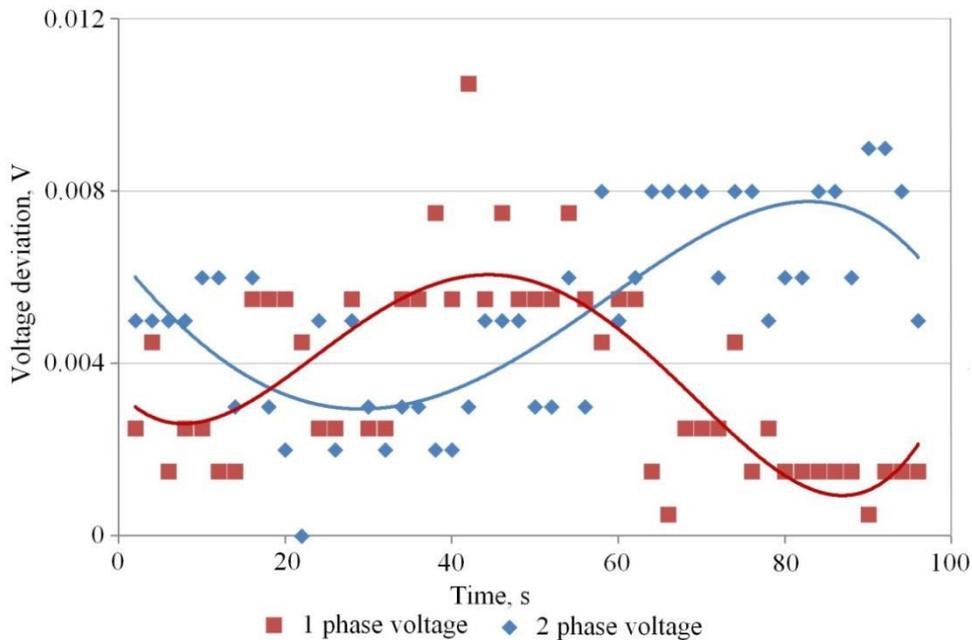


Figure-7. Phase voltage oscillation with both unstable phases.

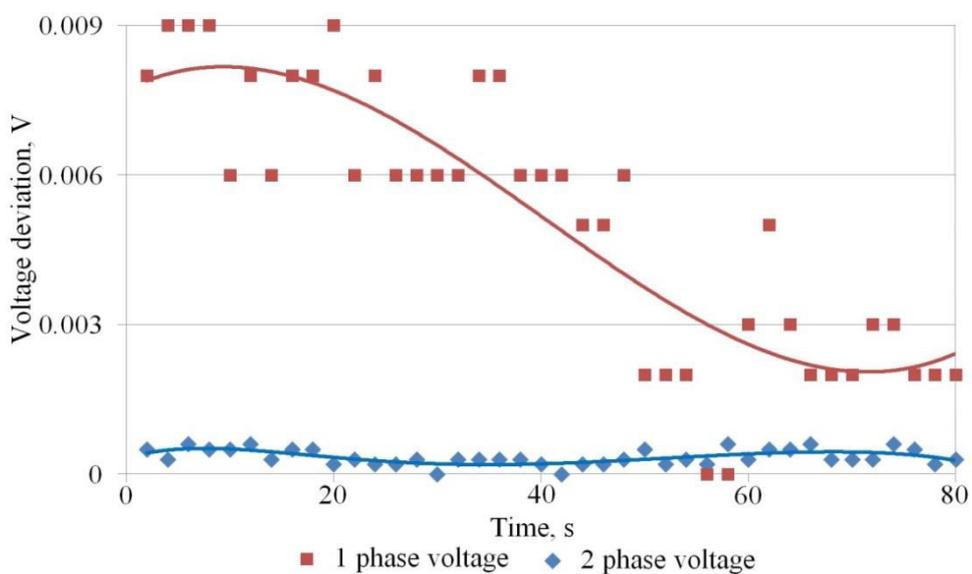
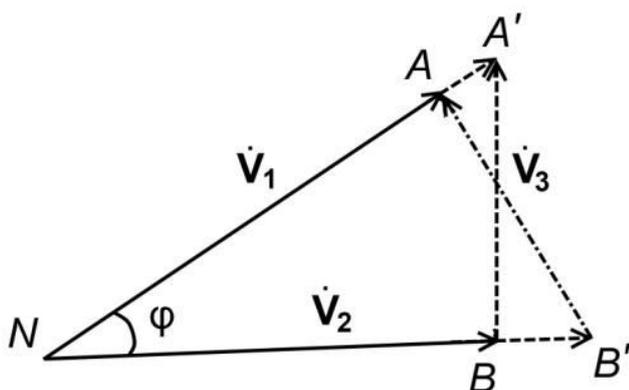


Figure-8. Phase voltage oscillation with second stable phase



**Figure-9.** Vector representation of phase voltage oscillation with both unstable phases.

Figure-9 represents this event in the vector form, that is, at the moment of reaching the minimum value of the amplitude of the first phase voltage at a point  $A$ , the amplitude of the second phase voltage reached a maximum at a point  $B'$  and vice versa. Consequently, if the values of these voltages would be successively measured by a multimeter for 5 or 10 seconds, a distortion would occur in accordance with Figure-5.

It can be seen in Figure-8 that for another fixed case, the amplitude of the oscillation of the second phase was negligibly small, and the change of the first phase had a similar character to the previous event. In this case, the distortion of the phase angle measurement results caused by the oscillations of the output voltages of the calibrator would have a less expressed character comparing to the case where the change in phase voltages would occur simultaneously in one direction.

It was also interesting to study the voltage oscillation between the first and second phases at a phase shift of  $6^\circ$  (see Figure-10). The figure shows that the oscillation had approximately the same period as previously considered events. But the peak to peak deviation of the oscillation amplitude (which was equal to approximately  $0.003\text{ V}$ ) decreased unproportionally to the amplitude of the measured voltage. But in the context of the current study, we were more interested in the

magnitude of oscillation for determining the parameters of the Monte Carlo simulation. It should be noted that the dip of the voltage in the interval between the 80th and 100th second can significantly distort the phase angle measurement result. Therefore, this moment needs a further clarification.

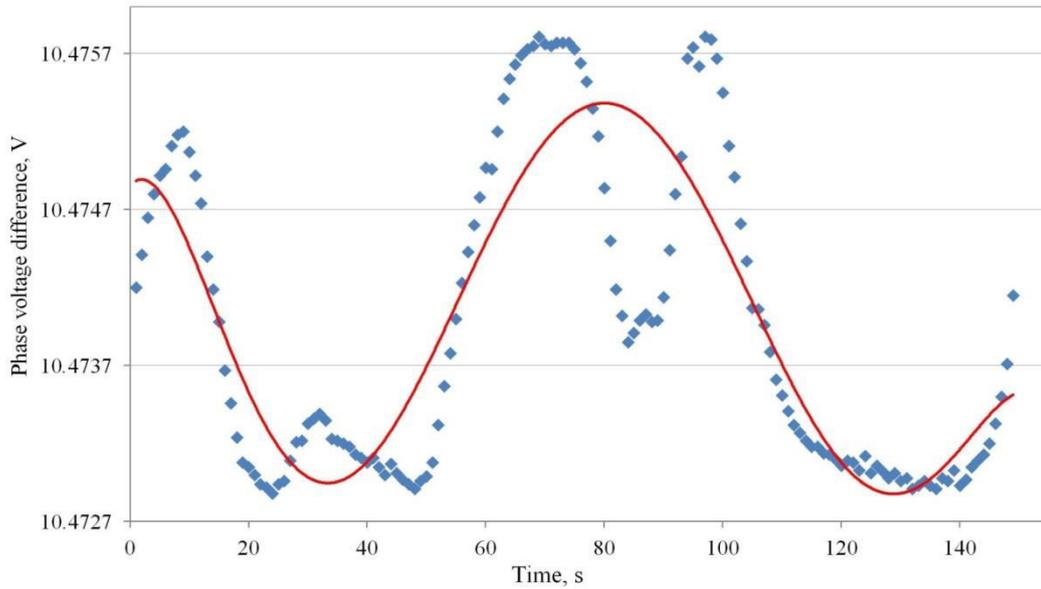
#### 4.2 Comparison of measurement and simulation results

The expediency or inappropriateness of the use of GUM 1995 [4] for estimating the uncertainty of measurement of the phase angle by means of precision voltmeter has been discussed in [16]. In order to verify the correctness of measurement uncertainty estimation when using a precision voltmeter method, a simulation in the Microsoft Excel 2007 software environment was performed by generating random numbers with the RANDBETWEEN function.

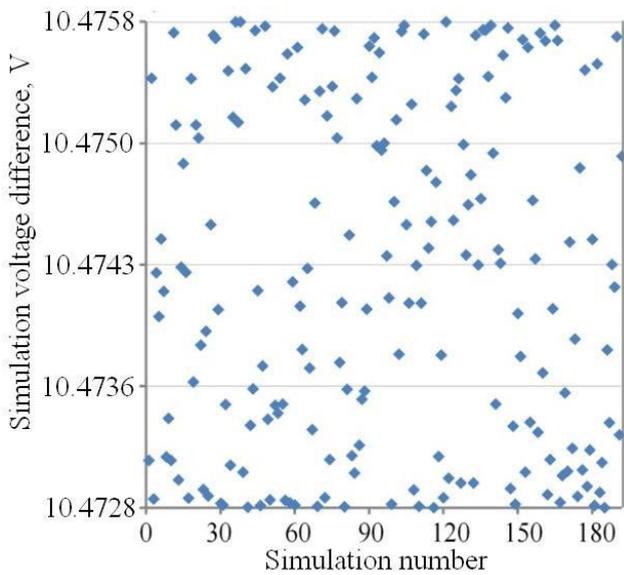
The numbers were generated within the intervals in which the measured voltages changed, i.e., in the range from  $100.0549$  to  $100.0557\text{ V}$  for the first phase voltage, from  $100.0799$  to  $100.0806\text{ V}$  for the second phase voltage, and from  $10.4786$  to  $10.4797\text{ V}$  for the difference between these voltages for the case of a phase angle of  $6^\circ$ . The BETAINV function with parameters  $\alpha$  and  $\beta$  equal to  $0.6$  was applied to achieve a reproduction of the distribution of the generated values by the law close to the real symmetric bimodal. The number of the simulated voltage values was chosen  $200$ , and an example of the received distribution for the first phase voltage is shown in Figure-11.

Figure-11 shows that the peak concentration of  $200$  values got into the lower and upper intervals of the distributed values. Within the four intervals, the values were distributed:  $35\%$  in the lower interval,  $31\%$  in the upper,  $16\%$  and  $18\%$  in the two middle intervals. The other two input quantities had a similar character.

$12$  duplications of the phase angle of  $6.0028^\circ$ ,  $5$  duplications of the phase angle of  $6.0027^\circ$  and  $3$  duplications of the phase angle of  $6.0029^\circ$  were obtained with a standard deviation of  $0.0001^\circ$  according to the results of the reusable repetition of the simulation ( $20$  times).



**Figure-10.** Voltage difference oscillation for phase angle of 6°.

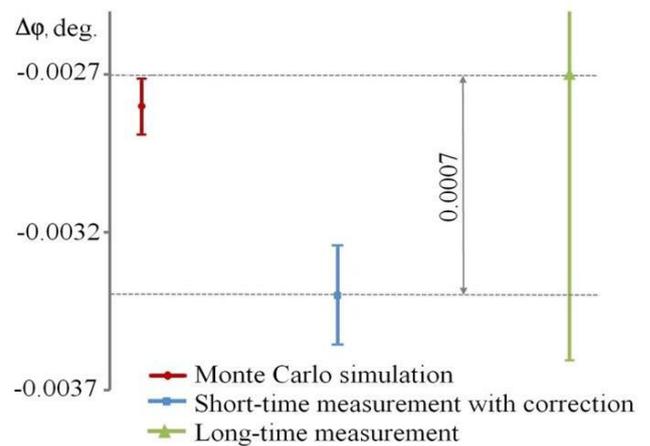


**Figure-11.** Simulating measurement results of first phase voltage.

Herewith, in accordance with the measured RMS voltages, the lower limit of the oscillation of the phase angle was calculated as  $6.0025^\circ$  by the formula (2), as well as the upper limit was  $6.0031^\circ$  with an expanded uncertainty of  $0.0007^\circ$ . The value of the phase angle was calculated with ten times measurement of the RMS voltages for a period of 3 minutes, and the value of  $6.0027^\circ$  was obtained. with a standard deviation of  $0.0009^\circ$  according to formula (2). When evaluating for a period of few tens of seconds the measurand of  $6.0034^\circ$  was obtained taking into account the corrections of Fluke 8845A precision multimeter with a standard deviation of  $0.00016^\circ$ .

A comparison of three estimates of the calibrator display error for the phase angle of  $6^\circ$ , obtained in

different ways, was made, and the corresponding contributions of the scattering of the measured or simulated voltages into the combined standard uncertainty, as shown in Figure-12.



**Figure-12.** Comparison of errors and corresponding standard deviations.

It is seen here that the uncertainty of measurement of the phase angle over the entire period of an oscillation covers all the errors obtained in other ways, both without taking into account the corrections of the precision multimeter and taking into account.

In general, the advantage of this study is to refine the result of uncertainty estimation when using the precision voltmeter method to measure a phase angle. The simulation of measurement results in the described manner gave three times greater standard deviation [16]. Although, the contribution of the type A uncertainty with a magnitude of  $0.0001^\circ$  to the combined standard uncertainty of about  $0.0004^\circ$  did not significantly affect this characteristic.



The results of the study were obtained at phase voltages of 100 V at a frequency of 50 Hz since the three-phase calibrator of AC voltage of an industrial frequency was used as an object of this study.

## 5. CONCLUSIONS

Two metrological characteristics of a three-phase calibrator have been distinguished at a phase angle reproduction: instantaneous value and average value. The uncertainty of measurement at the calibration in the part of an instantaneous phase angle would have a much lower contribution of a standard deviation, whereas, in estimating an average phase angle over the entire oscillation period, the contribution of a standard deviation may be up to 10 times greater.

The law of distribution of the phase angle values reproduced by a three-phase calibrator has been identified likely symmetric bimodal beta distribution of the probability of a random event occurring.

The influence of the periodic oscillations of the phase angle and the output voltages as sources of uncertainty in the reproduction by a three-phase calibrator has been analyzed. Such an approach can be applied in the studying other measures of the phase angle between two voltages.

A methodology for investigating the short-term stability of the reproducible phase angle with the help of a precision voltmeter, and for studying the effect of this stability on the uncertainty of measurement of the phase angle by means of a precision voltmeter, have been proposed.

## REFERENCES

- [1] Voltage characteristics of electricity supplied by public distribution system. EN Standard 50160. 2003.
- [2] Clarke K. K. and Hess D. T. 1989. Phase measurement, traceability, and verification theory and practice. 6<sup>th</sup> IEEE Conference Record. Instrumentation and Measurement Technology Conference. Washington, USA. doi: 10.1109/IMTC.1989.36856.V.
- [3] Velychko O.M. and Isaiev V.V. 2017. Some features of the calibration method of multifunctional calibrators. Scientific Digest of Odessa State Academy of Technical Regulation and Quality. 1(10): 59-64.
- [4] Evaluation of measurement data - Guide to the expression of uncertainty in measurement (JCGM 100 (GUM 1995)). 2008.
- [5] Evaluation of measurement data - Supplement 1 to the Guide to the expression of uncertainty in measurement - Propagation of distributions using a Monte Carlo method (JCGM 101:2008). 2008.
- [6] Robert C. and Casella G. 2013. Monte Carlo statistical methods. Springer Science & Business Media.
- [7] Bardenet R., Doucet A. and Holmes C. 2017. On Markov chain Monte Carlo methods for tall data. The Journal of Machine Learning Research. 18(1): 1515-1557.
- [8] The BIPM key comparison database. Kcdb.bipm.org. 2018. [Online]. Available: [https://kcdb.bipm.org/AppendixC/search.asp?met=EM & reset=1](https://kcdb.bipm.org/AppendixC/search.asp?met=EM&reset=1). [Accessed: 10-Dec-2018].
- [9] Kyriazis G. A. and de Campos M. L. R. 2005. An algorithm for accurately estimating the harmonic magnitudes and phase shifts of periodic signals with asynchronous sampling. IEEE Transactions on Instrumentation and Measurement. 54(2): 496-499. doi: 10.1109/TIM.2005.843559.
- [10] Augustyn J. and Kampik M. 2018. Improved sine-fitting algorithms for measurements of complex ratio of ac voltages by asynchronous sequential sampling. IEEE Transactions on Instrumentation and Measurement. doi: 10.1109/TIM.2018.2875901.
- [11] Šíra M. and Mašláň S. 2014. Uncertainty analysis of non-coherent sampling phase meter with four parameter sine wave fitting by means of Monte Carlo. 29th Conference on Precision Electromagnetic Measurements (CPEM 2014). Rio de Janeiro, Brazil. doi: 10.1109/CPEM.2014.6898395.
- [12] Velychko O, Isaiev V. and Kulish Y. 2018. Comparison of phase angle measurement results by means of two methods. 2018 Conference on Precision Electromagnetic Measurements (CPEM 2018). Paris, France. doi: 10.1109/CPEM.2018.8500900.
- [13] Gray L. A. and Alava M. H. 2018. A command for fitting mixture regression models for bounded dependent variables using the beta distribution. Stata Journal. 18: 51-75.
- [14] Johnson N. L., Kotz, S. I. and Balakrishnan N. 1994. Beta distributions. Continuous univariate distributions. 2nd ed. New York, John Wiley and Sons: 221-235.



- [15] Velychko O. M., Shevkun S. M., Dobroliubova M. V. and Izbash Y. M. 2015. The uncertainty estimate in the calibration of phase meters with using the state phase angle between two voltage standards. *Information Processing Systems*. 127(2): 86-88.
- [16] Velychko O. and Isaiev V. 2018. Comparison of Two Methods for Phase Angle Measurement. *IEEE Transactions on Instrumentation and Measurement*. doi: 10.1109/TIM.2018.2880000.