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MATHEMATICAL MODEL OF THE CYLINDER ROTATIONAL OSCILLATION IN AIR FLOW

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ABSTRACT

The paper describes the mathematical model of the rotational oscillations of the elastically fixed cylinder with flat stabilizer in the airflow. The cylinder has the single degree of freedom. It can rotate around axis that is perpendicular to cylinder axis and air velocity vector. The ratio of the length and the diameter of the cylinder is equal to 2. The model predicts the oscillation of the cylinder with constant amplitude. Two cases are considered. In the first case, the cylinder has only stabilizer. In the second case, the cylinder has not a stabilizer and is supported by elastic holder. Predictions of the mathematical model are verified in the wind tunnel experiments.

Keywords: cylinder, rotational oscillation, mathematical model.

INTRODUCTION

We study the oscillations of cylinders around an axis perpendicular to the free-stream velocity vector and the axis of the cylinder. The moment of aerodynamic or elastic forces acting on the cylinder with stabilizer returns the cylinder to an equilibrium position.While the moment of aerodynamic force depends on the flow velocity, the moment of elastic force does not depend on this velocity. It was found that a cylinder, the length of which is equal to two diameters, oscillates with steady amplitude in the air flow [1, 2].

In case the aerodynamic forces acting on the body depend only on the instantaneous angles of attack and sideslip, the quasistationary approximation is good for describing of the oscillations of elastically fixed bodies [3, 4, 5, 6].

However, the quasistationary approximation is not applicable to the rotational oscillations of bluff bodies, since the aerodynamic forces in this case depend not only on the instantaneous angles of attack and sideslip, but on the derivatives of these angles with respect to time too [7, 8]. Previously, oscillations of cylinders mounted on an elastic holder equipped with springs [9, 10] were studied.In this work the mathematical simulation of rotational oscillations of a cylinder with the stabilizer and with elastic holder and an experiment in the wind tunnel are described.

MATHEMATICAL MODEL OF ROTATIONAL OSCILLATIONS

The equation of motion of a cylinder elastically fixed in a flow has the form:

$$I_z\ddot{\beta} + r\dot{\beta} + SL\frac{\rho v^2}{2}m_{\beta}\beta + k\beta = SL^2\frac{\rho v^2}{2}m_{\dot{\beta}}\left(1 - \delta\beta^2 - \delta_1\frac{L}{v}\beta\dot{\beta} - \delta_2\frac{L^2}{v^2}\dot{\beta}^2\right)\dot{\beta} \quad (1)$$

Where I_z is the moment of inertia; r is the coefficient corresponding to the viscous friction; β is the angle of inclination, ρ is the air density; v is the velocity of the incoming flow. L is the length of the cylinder; $S$is the characteristic area that is equal to the area of the base of the cylinder; m_{β} and $m_{\dot{\beta}}$ are the aerodynamic derivative coefficients of the force moment; k is the spring rate;δ, δ_1, δ_2 are the coefficients of series expansion. The dot above the symbol denotes differentiation with respect to time. After introducing of new parameters:

$$\omega^2 = \frac{k_1}{L^2}v^2 + \frac{k}{I_z}, \quad k_1 = \frac{\rho}{2I_z}SL^3 m_{\beta},$$
$$\mu = \frac{\rho}{2I_z}SL^3 m_{\dot{\beta}}, \quad k_* = \frac{r}{\mu I_z},$$

one can obtain:

$$\ddot{\beta} + \omega^2\beta = \mu\left(\frac{v}{L} - k_* - \delta\frac{v}{L}\beta^2 - \delta_1\beta b\dot{\beta} - \delta_2\frac{L}{v}\dot{\beta}^2\right)\dot{\beta} = \mu\dot{f}(\beta,\dot{\beta}) \quad (2)$$

If parameter μ is smaller then oscillations of the angle β gets close to harmonic ones. Solution of the equation (2) is obtained by the method of Krylov-Bogoliubov [11].

New variables amplitude A and phase $\psi$are introduced:

$\beta = A\cos\psi, \quad \psi = \omega t + \varphi.$

Amplitude A and shift of the phase φ are slow variables.For determination of derivatives of the amplitude and the phase with respect to time; the method of Krylov-Bogolyubov uses formulas [11]:

$$\frac{dA}{dt} = -\frac{\mu}{2\pi\omega}\int_0^{2\pi} f(A\cos\psi, -A\omega\sin\psi)\sin\psi\ d\psi,$$
$$\frac{d\psi}{dt} = \omega - \frac{\mu}{2\pi A\omega}\int_0^{2\pi} f(A\cos\psi, -A\omega\sin\psi)\cos\psi\ d\psi.$$

After calculation of the integrals, we obtain the equations (3) and (4):

$$\frac{dA}{dt} = A\frac{\mu}{2}\left(\frac{v}{L} - k_* - \delta\frac{v}{4L}A^2 - \delta_2\frac{3L}{4v}\omega^2 A^2\right), \quad (3)$$

$$\frac{d\psi}{dt} = \omega - \frac{\delta_1}{8}\mu\omega A^2. \quad (4)$$

Case 1. Cylinder with stabilizer: In this case, elastic force is absent. Coefficient k is equal to zero. The frequency ω is proportional to the gas velocity$v$:

$$\frac{v}{L} = \frac{\omega}{\sqrt{k_1}}. \quad (5)$$

For steady oscillation with constant amplitude, the derivative of the amplitude with respect to time in the left part of the equation (3) is equal to 0. Equating the expression in parentheses on the right side of equation (3) to zero and taking into account formula (5), one can get the dependence of the oscillation amplitude$A$ on air velocity:

$$A^2 = \frac{4}{\delta + 3k_1\delta_2}\left(1 - \frac{k_* L}{v}\right). \quad (6)$$

Our mathematical model predicts that the square of oscillation amplitudeis a linear function of $1/v$. the angular frequency of the oscillation is the derivative of the phase ψ with respect to time (4). The Strouhal number Shcalculated on the base the frequency of the oscillation is

$$Sh = \frac{\dot{\psi}}{2\pi}\frac{L}{v} = \frac{\sqrt{k_1}}{2\pi}\left(1 - \frac{\delta_1}{8}\mu A^2\right) = a + \frac{b}{v}. \quad (7)$$

Thus, mathematical model predicts that the Strouhal number is a linear function of $1/v$ too.

Case 2. Cylinder without stabilizer and with elastic holder: In this case, the aerodynamic derivative coefficient of the force moment is small but does not equal to 0:

$$\left|\frac{k_1}{L^2}v^2\right| \ll \frac{k}{I_z}, \qquad k_1 < 0.$$

The elastic holder allows the cylinder to be set in an equilibrium position at a certain angle of inclinationβ_0. The equation of the motion is

$$I_z\ddot{\beta} + r\dot{\beta} + SL\frac{\rho v^2}{2}m_\beta\beta + k(\beta - \beta_0) =$$
$$SL^2\frac{\rho v^2}{2}m_{\dot{\beta}}\left(1 - \delta\beta^2 - \delta_1\frac{L}{v}\beta\dot{\beta} - \delta_2\frac{L^2}{v^2}\dot{\beta}^2\right)\dot{\beta}. \quad (8)$$

Under action of airflow, the angle of equilibrium $\beta_0$slightly changes. Let the angle under action be β_1.

$$\beta_1 = \beta_0\frac{k}{I_z\omega^2} \approx \beta_0\left(1 - \frac{I_z}{k}\frac{k_1}{L^2}v^2\right). \quad (9)$$

Equation of the motion (8) one can transform to the equation (10):

$$\ddot{\beta} + \omega^2(\beta - \beta_1) = \mu\left(\frac{v}{L} - k_* - \delta\frac{v}{L}\beta^2 - \delta_1\beta\dot{\beta} - \delta_2\frac{L}{v}\dot{\beta}^2\right)\dot{\beta}. \quad (10)$$

Solution of the equation (10) is obtained by the method of Krylov-Bogoliubov [11]. In this case we consider that the angular oscillation is produced around the equilibrium angle β_1. Amplitude A and phase $\psi$are introduced:

$$\beta = \beta_1 + A\cos(\psi), \qquad \psi = \omega t + \varphi.$$

After applying of the Krylov-Bogoliubov method we receive the expressions (11) and (12):

$$\frac{dA}{dt} = A\frac{\mu}{2}\left(\frac{v}{L} - k_* - \delta\frac{v}{4L}A^2 - \delta\frac{v}{L}\beta_1^2 - \delta_2\frac{3L}{4v}\omega^2A^2\right), \quad (11)$$

$$\frac{d\psi}{dt} = \omega - \frac{\delta_1}{8}\mu\omega A^2. \quad (12)$$

The expression in the parentheses concludes the term that is proportional to $1/v$. If the air velocity is high, then this term can be ignored. Thus for high velocity one can obtain equation (13) for steady oscillation.

$$\frac{A^2}{4} + \beta_0^2 \approx \frac{A^2}{4} + \beta_1^2 = \frac{\left(1 - \frac{k_* L}{v}\right)}{\delta}. \quad (13)$$

When deriving the formula for the oscillation frequency, we assume that the parameter δ_1 is negligible:

$$\Omega = \frac{d\psi}{dt} \approx \omega_0\left(1 + \frac{k_1}{L^2{\omega_0}^2}v^2\right) = a + bv^2. \quad (14)$$

Thus, we received that in the case 2 the angular oscillation frequency is a linear function of the square of air velocity.

EXPERIMENTAL VERIFICATION OF THE MATHEMATICAL MODEL FOR THE CASE1

The tests are carried out in the wind tunnel of St. Petersburg State University. The wind tunnel has open test section. The diameter of the outlet circular cross section of the nozzle is 1.5 m. The flow velocity in the test section varies from 0 to 40 m/s.

The dimensions of the cylinder to be tested are shown in the Figere-1. In the first experiment, the possibility of representing the aerodynamic torque as a linear function of the angle of inclination $\beta$is studied. The moment is measured using an aerodynamic balance. In this experiment, the model is immobile.

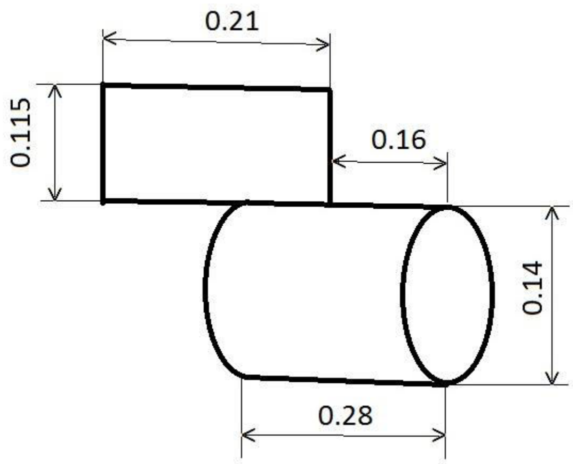

Figure-1. The dimensions of the cylinder with the stabilizer. The dimensions are given in metres.

Thus, only the component of the moment represented by the third term in the left side of equation (1) was measured.

In Figure-2 there is a plot of the moment coefficient m_z as a function of the inclination angle β.

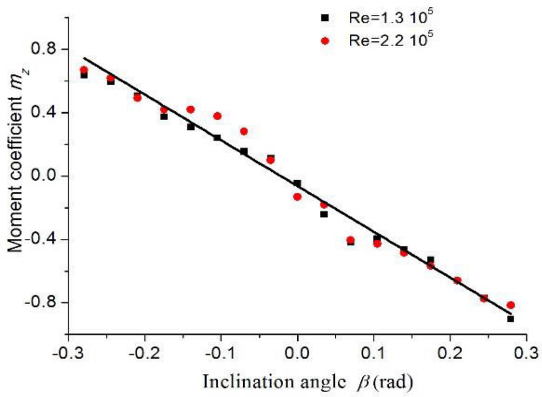

Figure-2. The dependence of the moment coefficient on the angle of inclination.

The dependence is close to linear; the slope is almost independent on the Reynolds number. The experimental data allowed us to determine the parameter $m_\beta = 2.85$. The error determined by the scatter of data is 5%.

The scheme of the second experiment is shown in Figure-2.

A cylinder *1* with a stabilizer *4*is installed in the test section of the wind tunnel so that it could only rotate around the vertical axis *6* passing through the geometric center of the cylinder. The suspension is implemented with steel wires with a diameter of 0.3 mm, a length of more than 1 m. The wires are located vertically above and below the cylinder and are attached to the cylinder surface at two diametrically opposite points.

The transverse movements of the cylinder are eliminated with the help of thin steel spacer. Lubrication is applied to the points of contact of the wires with the spacers. A laser pointer*5*is attached to the aft end of the cylinder. The light beam from the pointer is directed downwards and, when the cylinder oscillates, it intersects the surface of the photodiode *2*, which is fixed in two positions.

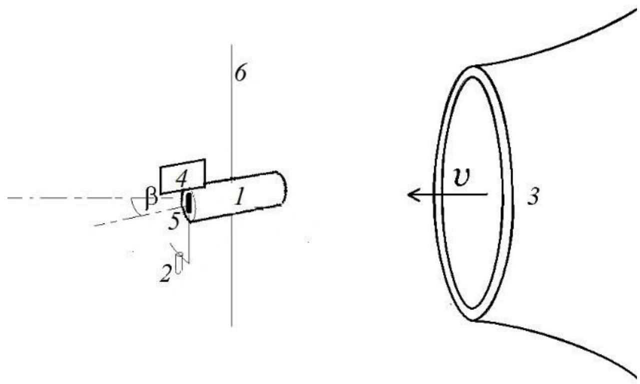

Figure-3. Record of the signal at damped oscillations. *1* - cylinder, *2* - photodiode, *3* - confusor of the wind tunnel, *4* - stabilizer, 5 - laser pointer, *6* - axis of rotation.

The photodiode signal is recorded with a Velleman-PCS500A PC-oscilloscope. The oscilloscope is connected to a computer, on which the signal's dependence on time is recorded in a file. The signal from the photodiode is read at a frequency of 1250 Hz. One measurement file contains 4095 reads. Two examples of the dependence of the signal on the readout number are shown in Figure-4 and Figure-5. These examples differ in the location of the photodiode. When the photodiode is located in the center, the laser beam intersects the surface of the photodiode twice in a period at regular intervals. When the photodiode is displaced by some distance, the beam also crosses the photodiode twice, but the time between adjacent intersections takes on two different values in turn.

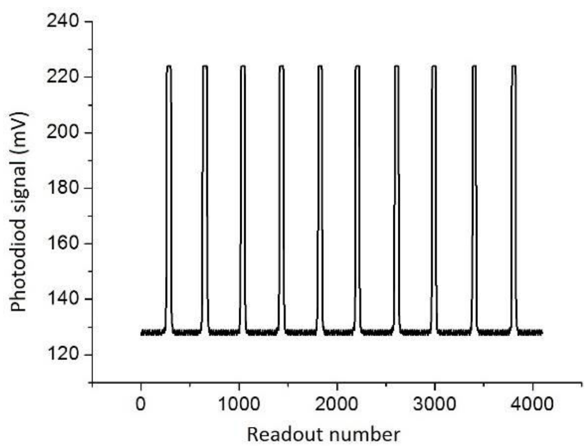

Figure-4a. Dependence of the signal from the readout number. The photodiode is in the center.

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Both options allow us to measure the oscillation period T and determine the Strouhal number using the formula $\mathrm{Sh} = D/(Tv)$.

The amplitude of oscillations was determined in two ways. An explanation of determination of the amplitude is shown in Figure-6. To find the amplitude from the data obtained when the photodiode is located in the center, it is necessary to determine time intervalΔt during which the beam crosses the photodiode surface having the shape of a circle with a diameter of d.

If the photodiode is diplaced by the angle X, the maximum τ between adjacent pulses must be determined. The amplitude was calculated using the following formulas:

$$A = \frac{d/2}{\sin(2\pi\Delta t/T)}, \quad A = \frac{X}{\sin(\pi\tau/T - \pi/2)}.$$

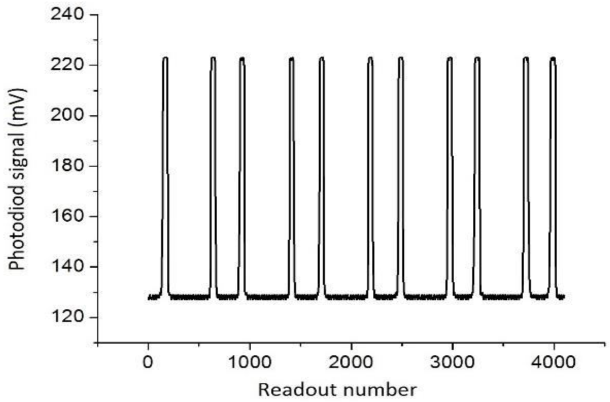

Figure-4b. Dependence of the signal from the readout number. The photodiode is displaced from the center.

Both methods of calculation yielded close amplitude values.

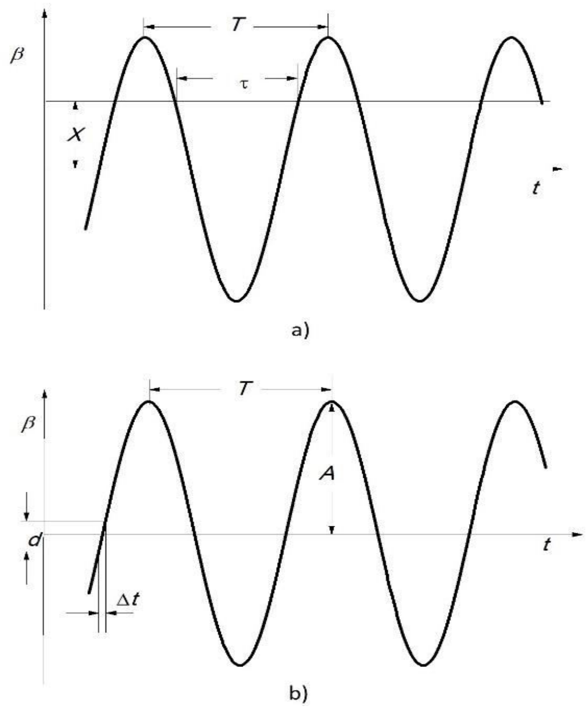

Figure-5. Explanation of determination of the amplitude. a) The photodiod is in the cenher. b) The photodiode is displaced from the center.

Figure-6 and Figure-7 show the graphs of the dependence of the square of the amplitude A^2 and the Strouhal number Shon$1/v$. As predicted by the mathematical model, these dependencies are linear functions. This indirectly indicates the validity of the assumption that the friction in the suspension is proportional to the speed of rotation of the cylinder. By approximating the data in the graph of the dependence of A^2 on $1/v$ with a straight line, we continue it to the intersection with the axis of ordinates. At the point of intersection with the axis, we obtain the value of the flow velocity, below which the oscillations damp out.

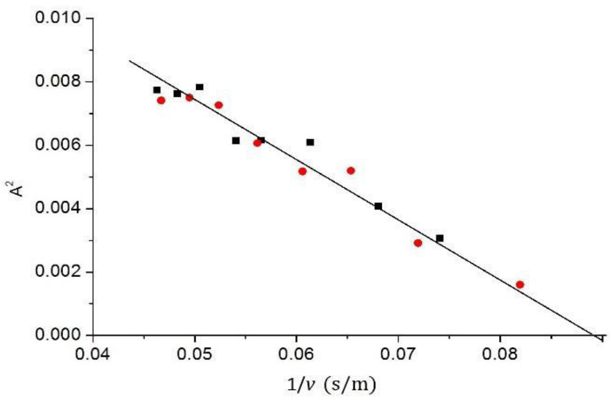

Figure-6. Dependence of the square of amplitude of oscillation on $1/v$.

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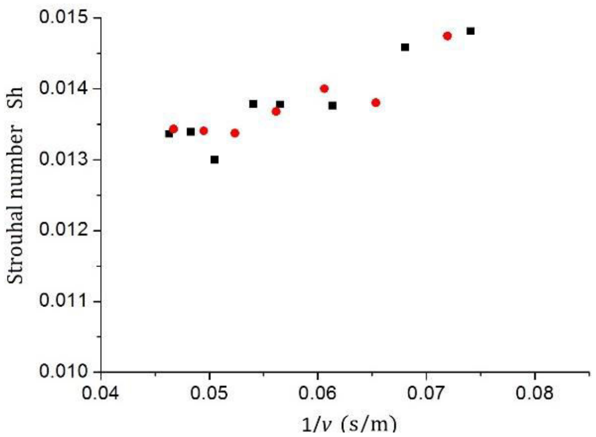

Figure-7. Dependence of Strouhal number on $1/v$.

If it would be possible to eliminate the suspension resistance, the amplitude of oscillations would not depend on the velocity of the incident flow. Of course, this is true only in the range of self-similarity of the flow with respect to Reynolds number. The mechanism of self-oscillations itself is apparently connected with the separation and attachment of the boundary layer to the surface of the cylinder.

In this paper, the cylinder and the stabilizer are treated as one body. It can be assumed that a decrease in the stabilizer area entail a decrease in the returning moment and a decrease in the oscillation frequency.

EXPERIMENTAL VERIFICATION OF THE MATHEMATICAL MODEL FOR THE CASE2

The scheme of the experiment is shown in Figure-8. The cylinder *1* without the stabilizeris fixed with the wire suspension. It could rotate around the horizontal axis *5* that is perpendicular to the mean velocity vector of the oncoming stream. A steel tail holder *4*is fixed to the aft end of the cylinder. Two steel springs *2* are attached to the holder. A semiconductor strain gauge *3* registers the tension of one of the springs. PC-oscilloscope Velleman-PCS500A transferred the signal from strain gauge to the computer. The frequency of the records was equal to 100 Hz.

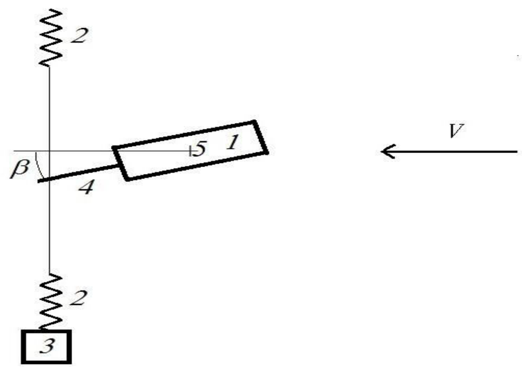

Figure-8. Scheme of the experiment. *1* - cylinder, *2* - springs, *3* - semiconductor strain-gorge, *4* - tail holder, *5* - axis of rotation.

The signal is proportional to the tension of the lower spring. Two calibration experiments are carried out. The calibration experiments and the method of processing of experimental results are described in [10].

In the Figure-9 the dependence of $A^2/4+\beta_0^2$on $1/v$ is presented. Figure-10 shows the dependence of oscillation frequencyΩ on square of air velocity v^2.

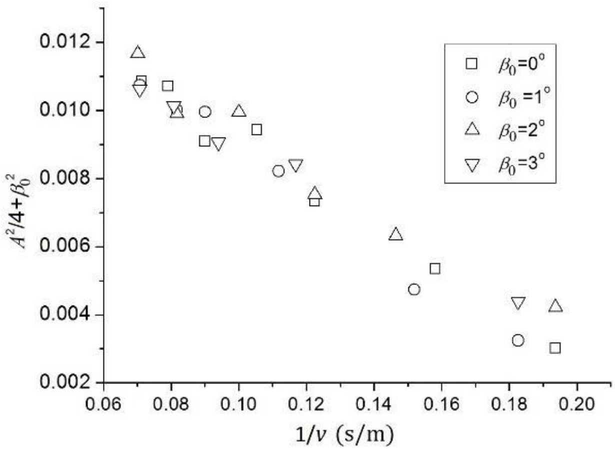

Figure-9. Dependence of $A^2/4+\beta_0^2$on $1/v$.

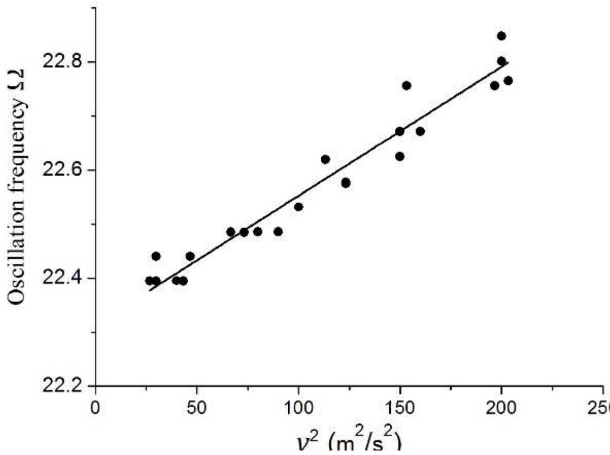

Figure-10. Dependence of Ω on v^2.

Formula (13) predicts that for high air velocity v the expression $A^2/4+\beta_0^2$ is the linear function of $1/v$. one can see in the Figure-9 that the experiment confirms this prediction. Increasing of the air velocity v from 5 m/s to 14 m/s leads to arising of oscillation frequency Ω by 1.8% only. The dependence of Ω on the square of air velocity closes to linear function according predictions of formula (14).

CONCLUSIONS

A mathematical model is proposed for describing the rotational oscillations of a cylinder with a stabilizer with an elastic holder in the airflow. Two cases are considered. In the case 1, the elastic holder is absent. The model predicts that the dependence of the square of the oscillation amplitude is a linear function of the reverse velocity of the airflow. The Strouhal number of cylinder oscillations is a linear function of the square of the amplitude and, therefore, the dependence of the Strouhal number on the inverse velocity is linear. In the case 2, the stabilizer is absent. In this case, we consider different positions of equilibrium. The model predicts that the dependence of the sum of the square of the oscillation amplitude divided by four and the square of equilibrium angle of inclination is a linear function of the reverse velocity of the airflow if the air velocity is large. For large air velocity, the oscillation frequency is proportional to the square of air velocity. A comparison of the predictions of the mathematical model with the results of experiments conducted in a wind tunnel was made. Experiments confirm the predictions.

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