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NUCLEAR POWER HISTORY CALCULATION USING THE BOOLEAN RULE

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ABSTRACT

This paper presents a new method for the calculation of reactivity through the nuclear power history term in the inverse equation of point kinetics. In this method, the reactivity can be written using Boole´s composite rule, which gives rise to four convolution sum terms between the nuclear power and the impulse response, with special characteristics typical of linear systems. Each convolution sum represents a filter of finite response, FIR. From the cases studied, our results provide better approximations when compared to existing methods in the literature.

Keywords: inverse equations of point kinetics, nuclear reactivity, bool´s rule, linear convolution.

INTRODUCTION

The main function of a nuclear reactor is to regulate the neutrons generated in nuclear fission chain reactions for the purpose of producing electrical energy [1]. Reactivity is one of the most important parameters used to maintain safety and proper functioning [2]. This parameter is monitored to provide control over reactor ignition tests and when programming the movement of the control rods for a given a variation in nuclear power. Most of these reactivity measurement systems are based on the inverse equation of point kinetics [3]. This is an integro-differential equation in which the integral part conserves the memory of the nuclear power history and the differential part refers to the period of the reactor.

Some preliminary studies on reactivity reported in the literature focused on discretizing the term associated with the integral of the inverse equation of point kinetics, which is known as the history of nuclear power [4-8]. In more recent works, reactivity has been calculated using methods such as the Euler-Maclaurin formula [9], the three and five point Lagrange method [10], Hamming's generalized predictor-corrector method [11] and the generalized Adams-Bashforth-Moulton predictor-corrector method [12].

In this work, a new formulation for the calculation of nuclear reactivity using the Boolean composite rule to discretize the inverse equation of point kinetics for different time steps and forms of the nuclear power term is proposed.

THEORETICAL CONSIDERATIONS

The equations of point kinetics have the following representation, [1]:

$$\frac{dP(t)}{dt} = \left[\frac{\rho(t)-\beta}{\Lambda}\right]P(t) + \sum_{i=1}^{6}\lambda_i C_i(t) \tag{1}$$

$$\frac{dC_i(t)}{dt} = \frac{\beta_i}{\Lambda}P(t) - \lambda_i C_i(t), \quad i = 1,2,\ldots 6 \tag{2}$$

With the following initial conditions:

$$P(t=0) = P_0 \tag{3}$$

$$C_i(t=0) = \frac{\beta_i}{\Lambda\lambda_i}P_0 \tag{4}$$

where $\rho(t)$ is the reactivity, $P(t)$ is the nuclear power, $C_i(t)$ is the concentration of the i-th group of delayed neutron precursors, β_i is the fraction of the i-th group of delayed neutrons, β is the total fraction of delayed neutrons which is defined as $\beta = \sum_{i=1}^{6}\beta_i$, Λ is the neutron generation time, λ_i is the decay constant of the i-th group of precursors.

Eqs. (1) and (2) of point kinetics can be derived from the neutron diffusion equation. These equations form a system of seven coupled non-linear differential equations, which describe the temporal evolution of the neutron population and the decay of delayed neutron precursors.

The inverse equation of point kinetics for the calculation of reactivity can be obtained from Eqs. (1-4). for a given power $P(t)$, this integro-differential equation has the following expression:

$$\rho(t) = \beta + \frac{\Lambda}{P(t)}\frac{dP(t)}{dt} - \frac{1}{P(t)}\times\sum_{i=1}^{6}\lambda_i\beta_i\left[\frac{\langle P_0\rangle}{\lambda_i}e^{-\lambda_i t} + \int_0^t e^{-\lambda_i(t-t')}P(t')dt'\right] \tag{5}$$

Where $\langle P_0 \rangle$ is the initial nuclear power.

Eq. (5) is the basis for the construction of reactivity meters, it is useful for programming the movement of control rods and the safe operation of the nuclear reactor. The integral term in Eq. (5) is known as the nuclear power history and can be written as

$$\int_0^t e^{-\lambda_i (t-t')} P(t')dt' = \int_0^t h_i(t-t')P(t')dt' = y(t) \quad (6)$$

In the following section, we present the theoretical underpinnings of the proposed method which is based on the Boolean composite rule, which in turn is a quadrature form of the Newton-Cotes method that allows the numerical solution of the integral term in Eq. (5).

PROPOSED METHOD

The reactivity of the system is calculated from the discrete version of the nuclear power history given by Eq. (5).

Suppose that $x_k = x_0 + hk$ are equidistant nodes, $f_k = f(x_k)$ for each $k = 0,1,2,3,...,M$ and h is the time step. If $f \in C^6[a,b]$ then it is possible to prove, using the Lagrange interpolator polynomial, that

$$\int_{x_0}^{x_4} f(x)\,dx = \frac{2h}{45}(7f_0 + 32f_1 + 12f_2 + 32f_3 + 7f_4) + E[f] \quad (7)$$

Where

$E[f] = -\frac{8h^7}{945} f^{(6)}(c)$, represents the maximum error at the point $c \in [x_0, x_4]$.

Using Eq. (7) for a more general interval $[a,b]$ we get:

$$\int_a^b f(x)\,dx \approx B_n(f) =$$

$$= \frac{14T}{45}\left[f(x_0) + f(x_n) + 2\sum_{k=1}^{\frac{n}{4}-1} f(x_{4k}) + \frac{32}{7}\sum_{k=0}^{\frac{n}{4}-1}\left(f(x_{4k+1}) + f(x_{4k+3})\right) + \frac{12}{7}\sum_{k=0}^{\frac{n}{4}-1} f(x_{4k+2}) \right] \quad (8)$$

Where n indicates the number of samples in the interval [a, b] given by $n = (b-a)/h$.

Taking into account Eq. (8), Eq. (6) in its discrete version can be written as

$$y[n] = \frac{14T}{45}\left[h_i[n]P[0] + h_i[0]P[n] + 2\sum_{k=1}^{\frac{n}{4}-1} h_i[n-4k]P[4k] + \frac{32}{7}\sum_{k=0}^{\frac{n}{4}-1} h_i[n-4k-1]P[4k+1] + \right.$$

$$\left. + \frac{32}{7}\sum_{k=0}^{\frac{n}{4}-1} h_i[n-4k-3]P[4k+3] + \frac{12}{7}\sum_{k=0}^{\frac{n}{4}-1} h_i[n-4k-2]P[4k+2] \right] \quad (9)$$

Where $P[n]$, $h_i[n]$ and $y[n]$ are discrete time functions for the continuous variables $P(t)$ $h_i(t)$ and $y(t)$, respectively. The index n must be a multiple of 4. Eq. (9) is characterized by having four summations, where each of them represents a FIR filter, that is, the linear convolution between the nuclear power $P[n]$ and the response to the finite impulse $h_i[n]$, which is characteristic of a linear time invariant system, LTI [13]. Replacing Eq. (9) into Eq. (5) gives

$$\rho[n] = \beta + \frac{\Lambda}{P[n]}\frac{dP(t)}{dt} - \frac{\langle P_0 \rangle}{P[n]}\sum_{i=1}^{6}\beta_i e^{-\lambda_i nT} -$$
$$-\frac{14T}{45P[n]}\sum_{i=1}^{6}\lambda_i\beta_i\left(h_i[n]P[0] + h_i[0]P[n]\right) -$$
$$-\frac{14T}{45P[n]}\left[2\sum_{i=1}^{6}\lambda_i\beta_i h_i[4n] * P[4n] + \frac{32}{7}\sum_{i=1}^{6}\lambda_i\beta_i\left(h_i[4n+1] * P[4n+1] + h_i[4n+3] * P[4n+3]\right) + \right.$$
$$\left. + \frac{12}{7}\sum_{i=1}^{6}\lambda_i\beta_i h_i[4n+2] * P[4n+2]\right] \quad (10)$$

Where the sign * denotes a liner convolution.

In the following section we present some of the results obtained in the simulations for the calculation of reactivity with the proposed method, which is given by Eq. (10).

RESULTS

The results were validated through multiple numerical simulations for a nuclear reactor with the following parameters: total fraction of delayed neutrons $\beta = 0.007$, neutron generation time $\Lambda = 2\times10^{-5} s$, decay constants $\lambda_i = [0.0127,\ 0.0317,\ 0.115,\ 0.311,\ 1.4,\ 3.87]\ s^{-1}$ and delayed neutron fractions $\beta_i = [0.000266,\ 0.001491,\ 0.001316,\ 0.002849,\ 0.000896,\ 0.000182]$. The results were compared with other recent studies reported in the literature where the Euler-Maclaurin methods [9], the three and five point Lagrange methods [10], the generalized Hamming method [11] and the generalized Adams-Bashforth-Moulton (ABM) method [12] were used.

In all the experiments shown, Eq. (5) was used as the reference method, the first nuclear power considered was of the form $P(t) = P(0)\exp(\omega t)$ for different values of ω which correspond to the positive roots of the inhour equation.

We start the simulations by considering low values for $\omega = 0.00243$ and $\omega = 0.01046$ which produce reactivities of $\rho = 20$ pcm and $\rho = 70$ pcm, respectively. Tables 1-2 show a comparison of the difference in reactivities in the interval, [0, 1000] s in the first case, and [0, 800] s in the second case. It can be seen that the Boolean method produces better approximations for the calculation of reactivity when compared to the Euler-Maclaurin method for time steps of $0.01 \le h \le 0.2$ s, however, it is less precise for a time step of $h = 0.5$ s. Equivalent comparisons are obtained with the best of the five-point Lagrange methods reported in the literature. When comparing the proposed method with predictor-corrector methods such as the Hamming and ABM methods, the opposite is true, that is, the proposed method turns out to be more precise only for large step sizes since these methods have a limitation on the convergence given by $h \le 2.66667 / \lambda_i$.

Table-1. Differences in reactivity for $P(t) = P(0)\exp(\omega t)$ with $\omega = 0.00243$.

h (s)	Euler-Maclaurin	Hamming	ABM V	BOOLE
0.01	6.20×10^{-8}	2.09×10^{-11}	2.48×10^{-11}	1.31×10^{-10}
0.05	3.85×10^{-5}	5.66×10^{-8}	5.66×10^{-8}	2.01×10^{-6}
0.1	6.14×10^{-4}	1.22×10^{-6}	1.25×10^{-6}	1.22×10^{-4}
0.2	9.73×10^{-3}	3.47×10^{-5}	5.40×10^{-5}	6.30×10^{-3}
0.5	3.60×10^{-1}	9.07×10^{346}	2.48×10^{307}	5.65×10^{-1}

Table-2. Differences in reactivity for $P(t) = P(0)\exp(\omega t)$ with $\omega = 0.01046$.

h (s)	Euler-Maclaurin	5 points Lagrange	Hamming	ABM V	BOOLE
0.01	6.24×10^{-8}	2.37×10^{-5}	9.05×10^{-11}	1.06×10^{-10}	1.34×10^{-10}
0.05	3.88×10^{-5}	3.98×10^{-4}	1.57×10^{-7}	2.43×10^{-7}	2.03×10^{-6}
0.1	6.19×10^{-4}	1.14×10^{-3}	4.72×10^{-6}	5.37×10^{-6}	1.23×10^{-4}
0.2	9.80×10^{-3}	-	1.47×10^{-4}	2.30×10^{-4}	6.40×10^{-3}
0.5	3.60×10^{-1}	1.04×10^{-2}	1.30×10^{250}	3.28×10^{305}	5.69×10^{-1}

For higher values of ω in the inhour equation, Tables 3-5 show comparisons in reactivity differences for $\omega = 0.02817$ in [0, 600] s, $\omega = 0.12353$ in [0, 600] s and $\omega = 1.00847$ in [0, 600] s that produce reactivities of 140 pcm, 300 pcm and 550 pcm, respectively. It is observed that the proposed method obtains better approximations than the Euler-Maclaurin method for $0.01 \leq h \leq 0.2$ s, as in the previous case for lower ω values. When comparing the proposed method with the generalized Hamming and ABM methods, it can be seen that the proposed method increases in precision as the calculation time h increases. The error reduces even further for $\omega = 1.00847$ when compared to the ABM method for $h = 0.1$ s.

Table-3. Differences in reactivity for $P(t) = P(0)\exp(\omega t)$ with $\omega = 0.02817$.

h (s)	Euler-Maclaurin	5 points Lagrange	Hamming	ABM V	BOOLE
0.01	6.33×10^{-8}	6.38×10^{-5}	2.37×10^{-10}	2.88×10^{-10}	1.37×10^{-10}
0.05	3.93×10^{-5}	1.06×10^{-3}	4.19×10^{-7}	6.56×10^{-7}	2.08×10^{-6}
0.1	6.28×10^{-4}	3.05×10^{-3}	1.25×10^{-5}	1.43×10^{-5}	1.26×10^{-4}
0.2	9.96×10^{-3}	-	3.86×10^{-4}	6.06×10^{-4}	6.50×10^{-3}
0.5	3.60×10^{-1}	2.65×10^{-2}	4.44×10^{182}	1.17×10^{301}	5.77×10^{-1}

Table-4. Differences in reactivity for $P(t) = P(0)\exp(\omega t)$ with $\omega = 0.12353$.

h (s)	Euler-Maclaurin	5 points Lagrange	Hamming	ABM V	BOOLE
0.01	6.88×10^{-8}	2.78×10^{-4}	1.07×10^{-9}	1.27×10^{-9}	1.55×10^{-10}
0.05	4.28×10^{-5}	4.50×10^{-3}	1.76×10^{-6}	2.88×10^{-6}	2.35×10^{-6}
0.1	6.83×10^{-4}	1.24×10^{-2}	6.24×10^{-5}	6.24×10^{-5}	1.42×10^{-4}
0.2	1.08×10^{-2}	-	1.50×10^{-3}	2.36×10^{-3}	7.30×10^{-3}
0.5	3.90×10^{-1}	8.29×10^{-2}	3.30×10^{78}	2.49×10^{145}	6.27×10^{-1}

Table-5. Differences in reactivity for $P(t) = P(0)\exp(\omega t)$ with $\omega = 1.00847$.

h (s)	Euler-Maclaurin	5 points Lagrange	Hamming	ABM V	BOOLE
0.01	1.41×10^{-7}	2.11×10^{-3}	9.76×10^{-9}	1.11×10^{-8}	4.36×10^{-10}
0.05	8.81×10^{-5}	2.59×10^{-2}	1.12×10^{-5}	2.43×10^{-5}	6.59×10^{-6}
0.1	1.40×10^{-3}	5.03×10^{-2}	1.89×10^{-4}	5.15×10^{-4}	3.87×10^{-4}
0.2	2.20×10^{-2}	-	3.87×10^{-3}	9.94×10^{-3}	1.83×10^{-2}
0.5	7.90×10^{-1}	5.50×10^{-1}	7.80×10^{-1}	1.74×10^{10}	1.26×10^{0}

Tables 6-7 show comparisons in of reactivity difference for $\omega = 1.023$ in [0, 100] s and $\omega = 2.345$ in [0, 80] s. It is observed that the proposed method obtains better approximations for $h = 0.01\ s$. For the other calculation steps, the differences are of the same order when compared to the generalized Hamming and ABM methods.

Table-6. Differences in reactivity for $P(t) = P(0)\exp(\omega t)$ with $\omega = 1.023$.

h (s)	Hamming	ABM V	BOOLE
0.01	9.92×10^{-9}	1.13×10^{-8}	1.61×10^{-9}
0.05	1.15×10^{-5}	2.46×10^{-5}	2.40×10^{-5}
0.1	1.91×10^{-4}	5.22×10^{-4}	1.40×10^{-3}
0.2	3.00×10^{-3}	1.00×10^{-2}	1.86×10^{-2}
0.3	1.50×10^{-1}	6.70×10^{-2}	1.42×10^{-1}

Table-7. Differences in reactivity for $P(t) = P(0)\exp(\omega t)$ with $\omega = 2.345$.

h (s)	Hamming	ABM V	BOOLE
0.01	2.83×10^{-8}	3.04×10^{-8}	1.61×10^{-9}
0.05	5.01×10^{-5}	6.35×10^{-5}	2.40×10^{-5}
0.1	1.19×10^{-3}	1.31×10^{-3}	1.40×10^{-3}
0.2	3.68×10^{-2}	2.64×10^{-2}	5.69×10^{-2}
0.3	2.70×10^{-1}	2.10×10^{-1}	3.88×10^{-1}

Tables 8-9 show further increases in the value of ω. The comparison of differences in reactivity for $\omega = 11.6442$ in [0, 60] s and $\omega = 52.80352$ in [0,10] s produces very high reactivities of 700 pcm and 800 pcm, respectively. It is observed that the proposed method obtains better approximations for all calculation times, when compared to the ABM and Euler-Maclaurin methods.

Table-8. Differences in reactivity for $P(t) = P(0)\exp(\omega t)$ with $\omega = 11.6442$.

h (s)	Euler-Maclaurin	ABM V	BOOLE
0.01	1.00×10^{-5}	4.26×10^{-6}	2.87×10^{-7}
0.05	6.23×10^{-3}	1.10×10^{-2}	3.60×10^{-3}
0.1	9.64×10^{-2}	2.40×10^{-1}	1.31×10^{-1}
0.2	1.37×10^{0}	3.10×10^{0}	2.28×10^{0}
0.5	3.07×10^{1}	2.80×10^{1}	2.45×10^{1}

Table-9. Differences in reactivity for $P(t) = P(0)\exp(\omega t)$ with $\omega = 52.8035$.

h (s)	Euler-Maclaurin	ABM V	BOOLE
0.01	6.77×10^{-4}	1.36×10^{-3}	2.66×10^{-4}
0.05	3.60×10^{-1}	8.60×10^{-1}	6.07×10^{-1}
0.1	4.04×10^{0}	4.80×10^{0}	4.06×10^{0}
0.2	3.04×10^{1}	1.48×10^{1}	1.34×10^{1}
0.5	2.75×10^{2}	4.35×10^{1}	4.18×10^{1}

In order to validate the proposed method, we considered other numerical experiments such as those obtained using the Lagrange method for three and five points with an exponential form of nuclear power for several values of ω and calculation times h. It can be seen in Table-10 how the proposed method is more accurate, achieving good results even for calculation times of up to $h = 1$ s.

Table-10. Differences in reactivity for $P(t) = P(0)\exp(\omega t)$ with several values of ω between the Lagrange method and the Boolean rule.

ω (s^{-1})	$t(s)$	$h(s)$	3 points Lagrange	5 points Lagrange	BOOLE
0.1176	100	1	-5.24×10^{-1}	9.46×10^{-2}	6.98×10^{0}
		0.1	-3.47×10^{-2}	1.18×10^{-2}	1.41×10^{-4}
1.176		1	-2.19×10^{1}	-6.24×10^{0}	1.36×10^{1}
		0.1	-4.51×10^{-1}	5.11×10^{-2}	4.60×10^{-4}
		0.01	-8.00×10^{-3}	2.40×10^{-3}	5.16×10^{-10}
11.76	20	0.1	-1.03×10^{1}	-3.03×10^{0}	1.36×10^{-1}
		0.01	-1.87×10^{-1}	1.07×10^{-2}	2.99×10^{-7}

Finally, other experiments performed with forms of nuclear power such as $P(t) = a + bt$ and $P(t) = a + \sin bt$, with initial condition $P(0) = \langle P_0 \rangle = a$ consisted of comparing the proposed method with the Lagrange method of three and five points reported in the literature, fora calculation time of $h = 0.1$s and with different simulation intervals. Table-11 shows how with this method we can get more accurate results.

Table-11. Differences in reactivity for linear power and harmonics forms between the Lagrange method and the Boolean rule.

Nuclear Power$P(t)$	a	b (s^{-1})	$t(s)$	3 points Lagrange	5 points de Lagrange	BOOLE
$a + bt$	1	1	10	-2.03×10^{-1}	6.15×10^{-2}	1.99×10^{-4}
$a + \sin bt$	1	$\pi/10$	180	-7.80×10^{-2}	2.79×10^{-2}	5.66×10^{-3}

CONCLUSIONS

In this work, a new formulation for the calculation of nuclear reactivity was derived using the Boolean compound integration method. The result consisted in the sum of four convolution terms between the nuclear power and the impulse response, each term being a FIR filter which allowed us to reproduce the nuclear power history, and obtain better approximations for the calculation of reactivity for different time steps. The method turned out to have a very good precision when compared with the works recently reported in the literature, such as Euler-Maclaurin, Lagrange of three and five points, Hamming and ABM.

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