



## SECOND LAW ANALYSIS OF MAGNETIZED MICROPOLAR FLUID ON HORIZONTAL INNER ROTATING CYLINDER WITH CHEMICAL REACTION AND CROSS DIFFUSIONS EFFECTS

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### ABSTRACT

This Research paper discusses the second analysis of Magnetohydrodynamics (MHD), a dissipative micropolar fluid by the annulus of horizontal internal rotating cylinders in the presence of chemical reactions and cross diffusions. The fluid movement is produced at a steady angular velocity from the inner rotation of the cylinder. The flow of heat and mass transfer equations are solved by RK4 shooting technique. The impact of different geometric parameters on azimuthal velocity, micro-rotation, temperature, concentration, Bejan and entropy generation numbers are shown through graphs. The results show that the entropy generation number decelerates with an increase in the temperature difference near the inner pipe, while it increases as the concentration difference parameter increases. The Brinkmann number increases the profiles of the temperature and reduces the concentration.

**Keywords:** micropolar fluid, MHD, hyper stick condition, soret and dufour parameter, joule heating, entropy.

### Nomenclature

$v$	Tangential velocity
$P$	Fluid pressure
$N_l$	Micro-rotation vector
$\sigma$	Electrical conductivity
$\mu, \kappa$	Viscosity coefficients
$\beta, \gamma$	Gyro viscosity coefficients
$\alpha = \frac{K}{\rho C_p}$	Thermal diffusivity
$K$	Thermal conductivity
$K_T$	Thermal diffusion ratio
$D$	Diffusion coefficient
$C_s$	Concentration Susceptibility
$T_m$	Mean temperature
$k_l$	Rate of reaction.
$\rho$	Density
$Be$	Bejan number
$Br (=Ec.Pr)$	Brinkman number
$B_0$	Magnetic flux
$N_s$	Entropy generation number,
$S_G$	Local volumetric entropy
$Pr$	Prandtl number
$Ha$	Hartmann number
$Sc$	Schmidt number
$N_H$	Entropy generation, Heat transfer
$N_F$	Entropy generation, fluid friction
$N_M$	Entropy generation, magnetic field
$N_D$	Entropy generation, mass transfer across finite concentration difference.
$C_p$	Specific heat
$\lambda$	Diffusive constant parameter
$T_d$	Temperature difference number.
$C_d$	Concentration difference number.
$\phi$	Irreversibility ratio

$c$	Parameter of the coupling vorticity
$s$	Couple stress parameter
$\delta$	Another rheological parameter
$Re$	Modified Reynolds number
$Df$	Dufour number,
$Cr$	Cemical reaction parameter
$Sr$	Soret number.

### INTRODUCTION

The fluid studies involving the horizontal coaxial internal rotating cylinders have received the subject of great interest because of the importance of numerous industrial applications. In the design of electric machines, oil drilling, diabatic flow in an annulus (Becker and Kaye [1]), swirl nozzles, porous bearings, airborne applications, combustion chambers and heat generated in a electric generator. Different researchers have examined the problem of Newtonian/non-Newtonian flow bounded by coaxial rotating cylinders. Coney and Shaarawi [2] are the first to study the boundary layer flow at the entrance between two horizontal cylinders with internal rotation. Mishra *et al* [3] examined the flow of Bingham plastic between two cylinders. Richard *et al.* [4] investigated the viscous flow through the annular gap between vertical inner rotating cylinders with an imposed pressure gradient. Maia and Gasparetto [5] studied flow of power law in a circular pipe. Ravanchi *et al.* [6] examined the viscoelastic flow of annulus with internal rotation. They identified that the increase in weissenberg number increases the slope of the velocity near the inner cylinder.

The magnetohydrodynamic (MHD) study has received more interest for its wide range of industrial applications in geophysics, MHD pumps, fire engineering, geothermal energy extraction, nuclear reactors. Kumari and Nath [7] examined the unsteady free convection



through coaxial cylinders, partially or completely packed with porous material. They identified that the nusselt number of the external cylinders increases as the Darcy parameter increases. The Magnetized Newtonian flow through the straight cylinder with external cylinder rotation is examined by Mozayyeni and Rahimi [8]. Many authors have investigated the analysis of cross-diffusion effects in horizontal or vertical cylinders. Although, according to the author's knowledge, a small number of studies have been reported with cross diffusions on rotating horizontal cylinders in the event of a chemical reaction. Sheikhzadeh *et al.* [9] numerically investigated  $Al_2O_3$ -water nano fluid through the internal rotating coaxial cylinder. From this, it is clear that the Nusselt number increases with the increase of volumetric nanoparticles. Sofiane *et al.* [10] studied cross- diffusion and axial magnetic field effects through a horizontal annulus. They notified that the concentration decelerates with the increase of a magnet parameter. Sheikholeslami and Abelman [11] studied nanofluid flow through two horizontal concentrate cylinders under an applied magnetic field. They concluded that the concentration profiles increase with the increasing of Schmidt number. Ramana Murthy *et al.* [12] studied the MHD flow of couple stress fluid between coaxial rotating cylinders with the inner porous lining. Sharma and Debozani [13] studied the steady Newtonian flow through coaxial cylinders containing a permeable material with a higher order chemical reaction and a radial magnetic field.

In recent years, the investigation on non-Newtonian flows has improved considerably due to repeated industrial processes in technical applications. Eringen [14] initiated the micropolar fluid theory, in which fluid particles underwent for translation and internal rotation. This theory allows for two separate vectors, velocity and microrotation vectors associated with each fluid particle. Hence angular momentum equation will not disappear here as it does in viscous fluid theory. The velocity vector takes place due to translatory motion, although the gyration vector appears due to a micro-rotatory motion. Micropolar fluid can exhibits couple stresses, subcutaneously the usual force stresses and can have micro-inertia. These fluids consist of elongated molecules such as polymer suspensions, ferrofluid, bubbling liquids and blood of animals, etc., which can be classified as micropolar fluids. Ariman *et al.* [15] studied the Poiseuille and Couette flow involving co-rotating cylinders. Comprehensive list of studies on micropolar fluids can be seen in [16-19]. Very recently Gajjela *et al.* [20] studied Soret, Dufour and chemical reaction effects in a magnetized micropolar rotating annular flow.

In the ongoing past, numerous specialists have been invited to analyze the entropy analysis in the study of thermodynamics with the motivation behind the entropy generation minimization (EGM). The fluid and thermal energy processes are on a very basic level irreversible, which is an extremely basic event of numerous thermodynamic processes. The second law analysis is associated with any heat exchange was initially examined by Bejan [21], who utilized EGM as the principle device.

Bejan [22] examined the thermodynamic analysis using different flow problems. The MHD flow with the entropy generation of various fluids was analyzed by many researchers [23-28].

Motivated by the above studies, we examine the thermodynamic analysis of MHD micropolar fluid with the effects of Soret, Dufour, Joule heating and first order chemical reactions in horizontal coaxial cylinders of infinite length. The resulting coupled nonlinear differential equations are solved by shooting with the Runge-Kutta approach. The response of different physical flow parameters on transverse velocity, micro-rotation, heat and mass transfer and entropy distributions has been studied through graphs.

### MATHEMATICAL MODELING

Consider a steady, incompressible flow of a micropolar fluid between horizontal inner rotating infinite cylinders and magnetic field applied Z-direction as shown in Figure-1. Additionally, the effects of chemical reaction, diffusion-thermo (Dufour) and thermal-diffusion (Soret) is considered. The internal and external cylinders preserved at a temperature  $T_1$  and  $T_2$ , concentrations  $C_1$  and  $C_2$  respectively. The viscous heating effects in the thermal equation are maintained. The governing flow equations of MHD, mass and heat transfer of a micropolar fluid as follows:

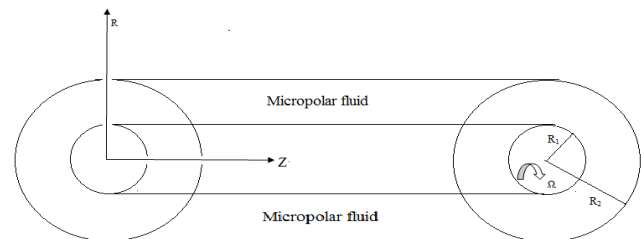


Figure-1. Physical diagram of concentric inner rotating annulus.

$$\frac{\partial P}{\partial R} = \rho \frac{V^2}{R} \quad (1)$$

$$-\kappa \frac{\partial N_1}{\partial R} + (\mu + \kappa) \left( \frac{\partial^2 V}{\partial R^2} + \frac{1}{R} \frac{\partial V}{\partial R} - \frac{V}{R^2} \right) - \sigma B_0^2 V = 0 \quad (2)$$

$$-2\kappa N_1 + \kappa \left( \frac{\partial V}{\partial R} + \frac{V}{R} \right) + \gamma \left( \frac{\partial^2 N_1}{\partial R^2} + \frac{\partial N_1}{\partial R} \right) = 0 \quad (3)$$

$$\frac{K}{\rho C_p} \left( \frac{\partial^2 T}{\partial R^2} + \frac{1}{R} \frac{\partial T}{\partial R} \right) + \frac{1}{\rho C_p} \left[ \mu \left( \frac{\partial V}{\partial R} - \frac{V}{R} \right)^2 + 4\kappa \left( \frac{1}{2} \left( \frac{\partial V}{\partial R} + \frac{V}{R} \right) - N_1 \right)^2 + \beta \left( \frac{\partial N_1}{\partial R} \right)^2 \right] + \frac{DK_T}{C_s C_p} \left( \frac{\partial^2 C}{\partial R^2} + \frac{1}{R} \frac{\partial C}{\partial R} \right) = 0 \quad (4)$$



$$D \left( \frac{\partial^2 C}{\partial R^2} + \frac{1}{R} \frac{\partial C}{\partial R} \right) + \frac{DK_T}{T_m} \left( \frac{\partial^2 T}{\partial R^2} + \frac{1}{R} \frac{\partial T}{\partial R} \right) - k_1 C = 0 \quad (5)$$

Following the analysis in [29], the associated boundary conditions are

$$\begin{aligned} (i) & V = R\Omega, N_1 = \Omega, T = T_1, C = C_1 \text{ at } R = R_1 \\ (ii) & V = 0, N_1 = 0, T = T_2, C = C_2 \text{ at } R = R_2 \end{aligned} \quad (6)$$

The basic equation together with boundary conditions, Eq. (2) to (6), which are currently ending up dimensional less form:

$$-\frac{c}{\eta} \frac{\partial N}{\partial r} + \left( \frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} - \frac{v}{r^2} \right) - Ha^2 (1-\eta)^{-2} v = 0 \quad (7)$$

$$-2sN + s\eta \left( \frac{\partial v}{\partial r} + \frac{v}{r} \right) + \left( \frac{\partial^2 N}{\partial r^2} + \frac{1}{r} \frac{\partial N}{\partial r} \right) = 0 \quad (8)$$

$$\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} + Br \left[ \left( \frac{\partial v}{\partial r} - \frac{v}{r} \right)^2 + \frac{4c}{1-c} \left( \frac{1}{2} \left( \frac{\partial v}{\partial r} + \frac{v}{r} \right) - \frac{N}{\eta} \right)^2 + \delta \left( \frac{\partial N}{\partial r} \right)^2 \right] \quad (9)$$

$$+ Pr D_f \left( \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} \right) = 0$$

$$\left( \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} \right) + Sc Sr \left( \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} \right) - (K_1 + Cr \phi) = 0 \quad (10)$$

$$\begin{aligned} (i) & v = N = \theta = \phi = 1 \text{ at } r = \eta \\ (ii) & v = N = \theta = \phi = 0 \text{ at } r = 1 \end{aligned} \quad (11)$$

where

$$v = \frac{V}{R_1 \Omega}, r = \frac{R}{R_2}, N = \frac{N_1}{\Omega}, \eta = \frac{R_1}{R_2}, p = \frac{P}{\rho \Omega^2 R_1^2}, \theta = \frac{T - T_2}{T_1 - T_2}, \phi = \frac{C - C_2}{C_1 - C_2}$$

$$, c = \frac{\kappa}{\kappa + \mu}, Ha = B_0 (R_2 - R_1) \sqrt{\frac{\sigma}{\mu + \kappa}}, s = \frac{\kappa R_2^2}{\gamma}$$

$$S = \frac{\beta}{\mu R_1^2}, Br = \frac{\mu \Omega^2 R_1^2}{K (T_1 - T_2)}, Df = \frac{DK_T (C_1 - C_2)}{\nu C_S C_P (T_1 - T_2)},$$

$$Pr = \frac{\mu C_P}{K}, Sc = \frac{\nu}{D}, Cr = \frac{k_1 R_1^2}{D}, K_1 = \frac{C_2 R_2^2}{D (C_1 - C_2)}, Sr = \frac{DK_T (T_1 - T_2)}{\nu T_m (C_1 - C_2)}$$

The dimensional less coefficient of skin-friction, Nusselt as well as Sherwood numbers at inner and outer pipes are taken as

$$C_f = \frac{2\eta}{Re} \left( \frac{\partial v}{\partial r} - (1-c) \frac{v}{r} - \frac{c}{\eta} N \right), Nu = -\eta \frac{\partial \theta}{\partial r}, Sh = -\eta \frac{\partial \phi}{\partial r} \quad (12)$$

$$\text{Where } Re = \frac{\rho \Omega R_1^2}{\mu + \kappa}$$

**SECOND LAW ANALYSIS**

In existence of Joule Heating, the volumetric entropy generation number can be expressed as

$$S_G = \frac{K_T}{T_1^2} (\nabla T)^2 + \frac{\mu}{T_1} \Phi + \frac{J^2}{\sigma T_1} + \frac{RD}{C_1} (\nabla C)^2 \quad (13)$$

In Eq.(13),  $\Phi$  is the viscous heating,  $J$  is current density and  $R$  is ideal gas constant.

$$\begin{aligned} S_G = & \frac{K (\Delta T)^2}{(R_2 T_1)^2} \left( \frac{\partial \theta}{\partial r} \right)^2 + \frac{\mu R_1^2 \Omega^2}{T_1 R_2^2} \left\{ \left( \frac{\partial v}{\partial r} - \frac{v}{r} \right)^2 + \frac{4c}{1-c} \left[ \frac{1}{2} \left( \frac{\partial v}{\partial r} + \frac{v}{r} \right) - \frac{N}{\eta} \right]^2 \right. \\ & \left. + \delta \left( \frac{\partial N}{\partial r} \right)^2 \right\} + \frac{\sigma R_1^2 \Omega^2 B_0^2}{T_1} v^2 + \frac{RD}{C_1} \left( \frac{\Delta C}{R_2} \frac{\partial \phi}{\partial r} \right)^2 \end{aligned} \quad (14)$$

Here,  $\Delta T$  is Temperature difference and  $\Delta C$  is Concentration difference. The dimensionless form of Entropy generation  $Ns$  the equation (14) can be given as

$$\begin{aligned} Ns = & \frac{1}{K} \left( \frac{R_2 T_1}{\Delta T} \right)^2 S_G \\ S_G = & \left( \frac{\partial \theta}{\partial r} \right)^2 + \frac{Br}{T_d} \left\{ \left( \frac{\partial v}{\partial r} - \frac{v}{r} \right)^2 + \frac{4c}{1-c} \left[ \frac{1}{2} \left( \frac{\partial v}{\partial r} + \frac{v}{r} \right) - \frac{N}{\eta} \right]^2 \right. \\ & \left. + \delta \left( \frac{\partial N}{\partial r} \right)^2 \right\} + Ha^2 (1-\eta)^{-2} \frac{Br}{T_d} v^2 + \lambda \left( \frac{C_d}{T_d} \right) \left( \frac{\partial \phi}{\partial r} \right)^2 \end{aligned} \quad (15)$$

$$Ns = N_H + N_F + N_M + N_D \quad (15)$$

Where

$$T_d = \frac{\Delta T}{T_1}, C_d = \frac{\Delta C}{C_1} \text{ and } \lambda = \frac{RDC_1}{K}$$

**BEJAN NUMBER**

Bejan number is described to be fraction of entropy generation due to heat transfer.

$$Be = \frac{N_H}{Ns} \quad (16)$$

**NUMERICAL SOLUTION**

The combined collection of nonlinear differential Eqs.(7)-(10) with boundary conditions(11) formed a second- order boundary value problem and was resolved by shooting with the Runge-Kutta scheme through an initial value problem. The convergence region in our counts was set by a minimum of  $10^{-6}$  [Ref. Odelu *et al.*[27].

**RESULTS AND DISCUSSIONS**

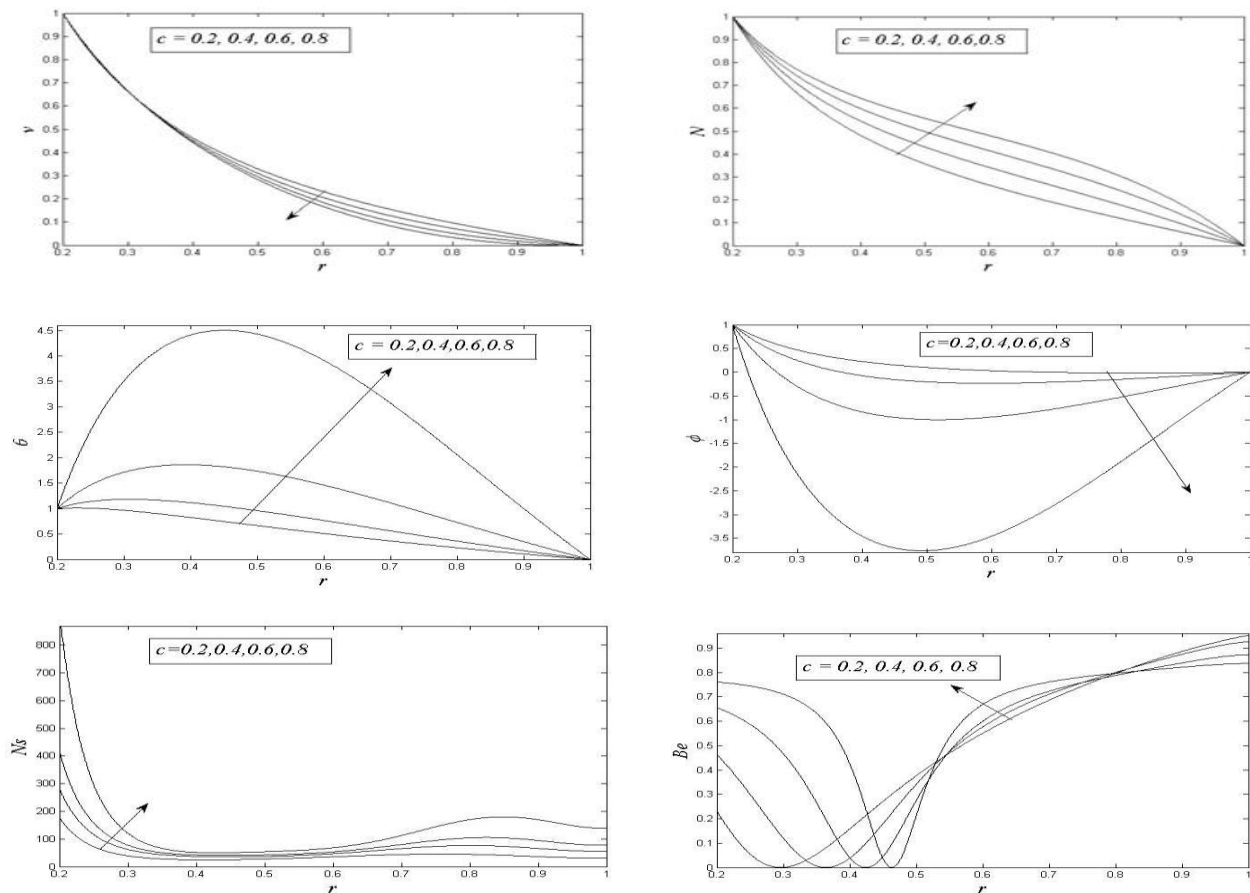
The results obtained for velocity, micro- rotation, concentration, temperature, Bejan and entropy number, heat and mass transfer rates for various flow and



geometric parameters are explained in detail and presented in graphical form and tables.

The effect of coupling number ' $c$ ' at  $Ha = 1$ ,  $s = 2$ ,  $\eta = 0.2$ ,  $Pr = 0.7$ ,  $Br = 0.35$ ,  $\delta = 0.05$ ,  $Sc = 0.5$ ,  $Df = 1.5$ ,  $Sr = 0.9$ ,  $T_d = 1$ ,  $C_d = 0.25$ ,  $\lambda = 0.9$  on velocity ( $v$ ), micro-rotation ( $N$ ), temperature ( $\theta$ ), concentration ( $\phi$ ), entropy ( $Ns$ ) and Bejan number ( $Be$ ) is shown in Figure-2. It is identified that  $c$  increases the azimuthal velocity and decelerates the concentration, while micro-rotation, temperature and entropy increase, but the number of Bejan fluctuates throughout the region. The influence of Hartmann number ' $Ha$ ' at  $c = 0.4$ ,  $s = 1.5$ ,  $\eta = 0.2$ ,  $Br = 0.7$ ,  $Pr = 0.5$ ,  $\delta = 0.1$ ,  $Sc = 0.45$ ,  $Df = 0.8$ ,  $Sr = 0.9$ ,  $Cr = 1$ ,  $T_d = 1$ ,  $C_d = 0.25$ ,  $\lambda = 0.9$  on  $v$ ,  $N$ ,  $\theta$  and  $\phi$  are presented in Figure 3. For flow profiles, the Hartmann number characterizes the proportion of electromagnetic force to viscous force. It is recognized that the magnetic field allows a damping reaction at azimuthal velocity by generating drag force that resists motion, leads to velocity and Bejan numbers decrease near the inner cylinder and  $Be$  reaches a minimum value at  $r = 0.3$ . As  $Ha$  increases micro-rotation, entropy, temperature profiles decrease. The concentration ( $\phi$ ) profiles increase with an increase in  $Ha$  values within the annular region. Figure-4 displays the effect of

Brinkman number ' $Br$ ' at  $c = 0.4$ ,  $s = 2.5$ ,  $Ha = 1.5$ ,  $Pr = 0.5$ ,  $Sc = 0.3$ ,  $\delta = 0.25$ ,  $Sr = 0.15$ ,  $Df = 1.5$ ,  $Cr = 1$ ,  $K_I = 0.01$ ,  $T_d = 1$ ,  $C_d = 0.5$ ,  $\lambda = 0.25$  on  $\theta$ ,  $\phi$ ,  $Ns$  and  $Be$ . The increase in  $Br$  increases the temperature, entropy, but decelerates the concentration. The Bejan number ( $Be$ ) has fluctuations throughout the region. Figure-5 shows the control of Schmidt number ' $Sc$ ' on  $v$ ,  $N$ ,  $\theta$ , and  $\phi$ . The increase in  $Br$  increases the temperature, entropy, but decelerates the concentration. The first decreases near the internal cylinder to  $r = 0.3$ , but increases in the external region. The effects of  $C_d$ ,  $\lambda$  and  $T_d$  on  $Ns$  and  $Be$  are displayed in Figures 6-8. It is identified that the increase in  $C_d$  and  $\lambda$  raise the  $Ns$  close to the inner cylinder and the reverse behaviour is observed with enhancing of Temperature difference parameter  $T_d$ ; The Bejan number decrease with increasing values of  $C_d$  and  $\lambda$  and attains minimum value at  $r = 0.37$  and  $Be$  enhance with increment values of  $T_d$ . The response of Dufour parameter ( $Df$ ) on  $\theta$ , Soret number ( $Sr$ ) and chemical reaction parameter ( $Cr$ ) on  $\phi$  for distinct values of alternative parameters and are displayed in Figure-9. The rise in  $Df$  increases  $\theta$  whereas  $Sr$  decelerates  $\phi$ . The concentrations ( $\phi$ ) accelerate as  $Cr$  enhances. For greater values of  $Cr$  increase molecular diffusivity at the centre of pipe.



**Figure-2.**The response of  $c$  on (a) Fluid Velocity, (b) Microrotation, (c) Temperature, (d) Concentration, (e) Entropy generation number, (f) Bejan number.

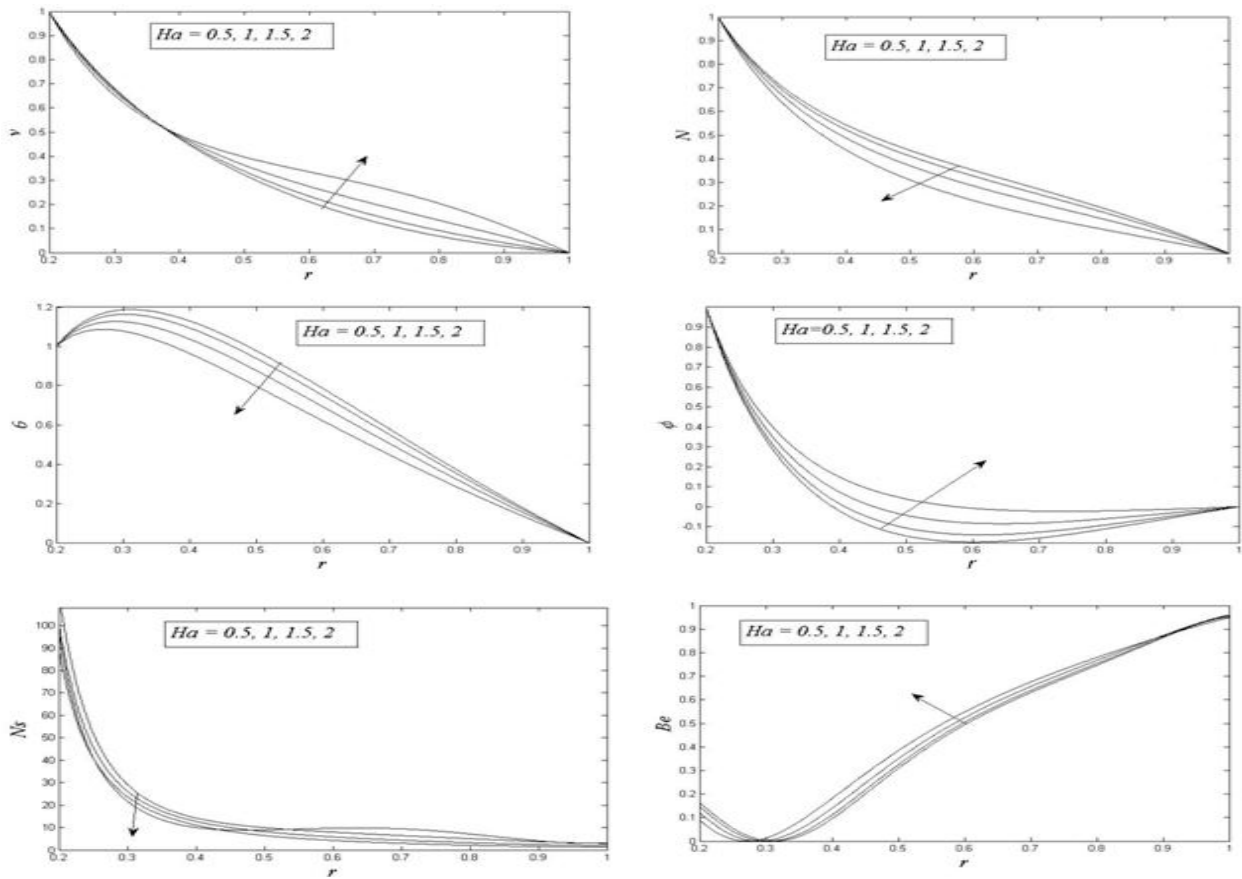


Figure-3. The response of  $Ha$  on (a) Fluid Velocity, (b) Microrotation, (c) Temperature, (d) Concentration, (e) Entropy generation number, (f) Bejan number.

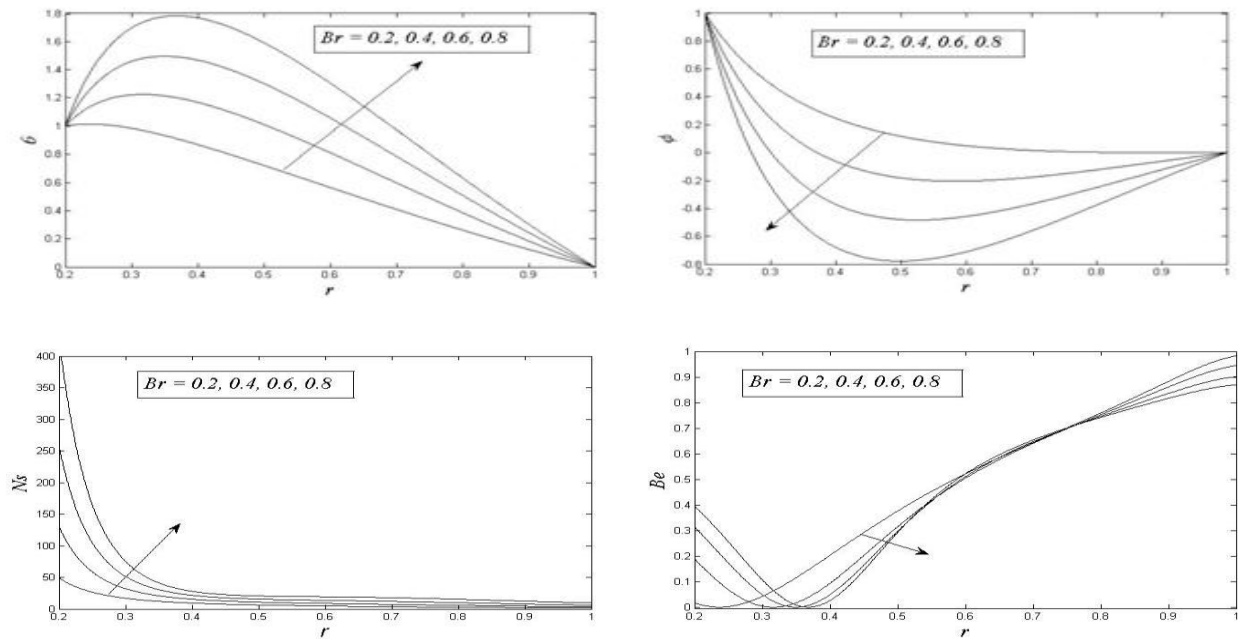


Figure-4. The influence of  $Br$  on (a) Temperature, (b) Concentration, (c) Entropy generation number, (d) Bejan number.

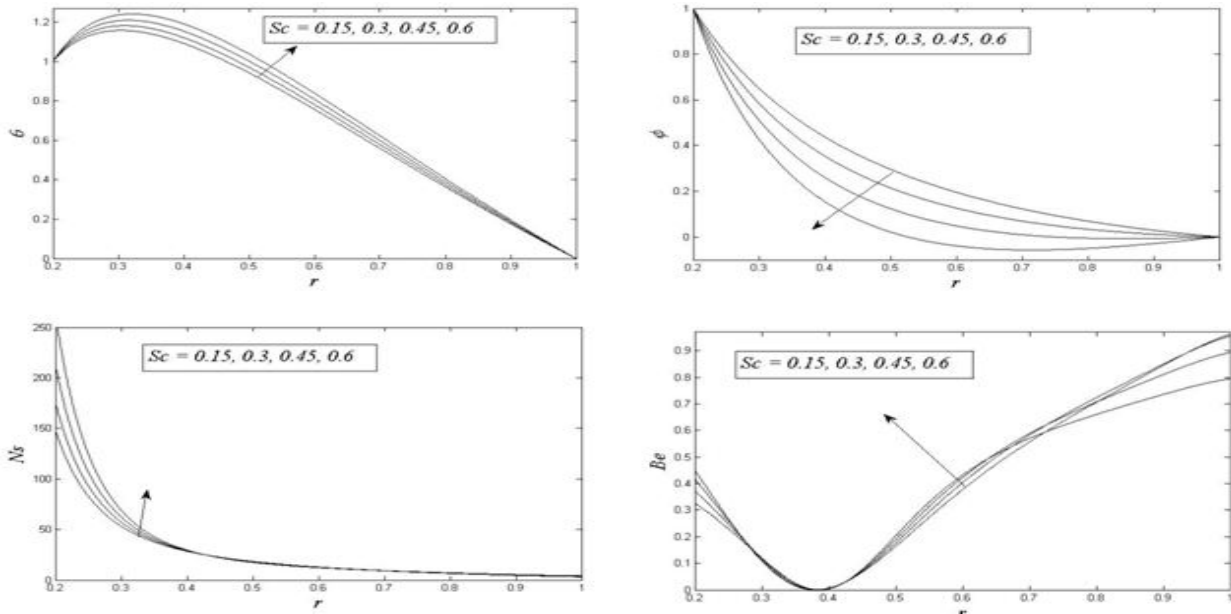


Figure-5. The response of  $Sc$  on (a) Temperature, (b) Concentration, (c) Entropy generation number, (d) Bejan number.

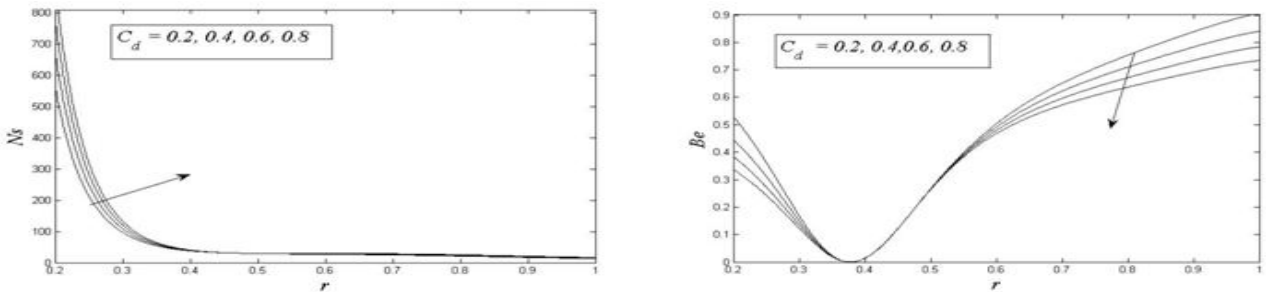


Figure-6. The response of  $C_d$  on (a) Entropy generation number, (b) Bejan number.

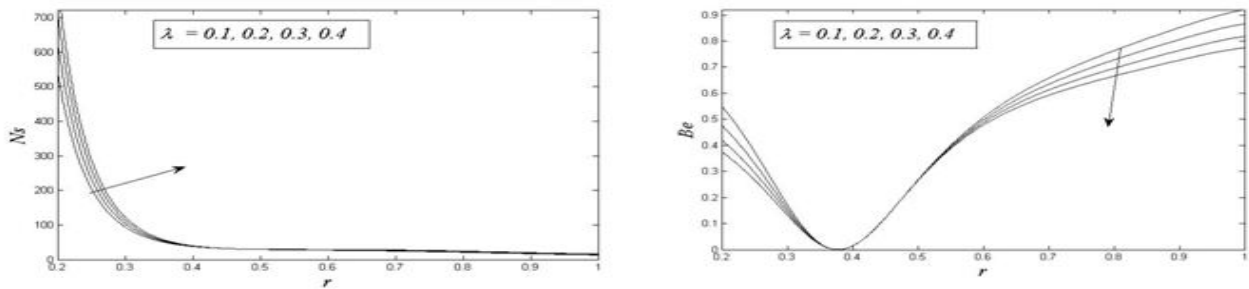


Figure-7. The response of  $\lambda$  on (a) Entropy generation number, (b) Bejan number.

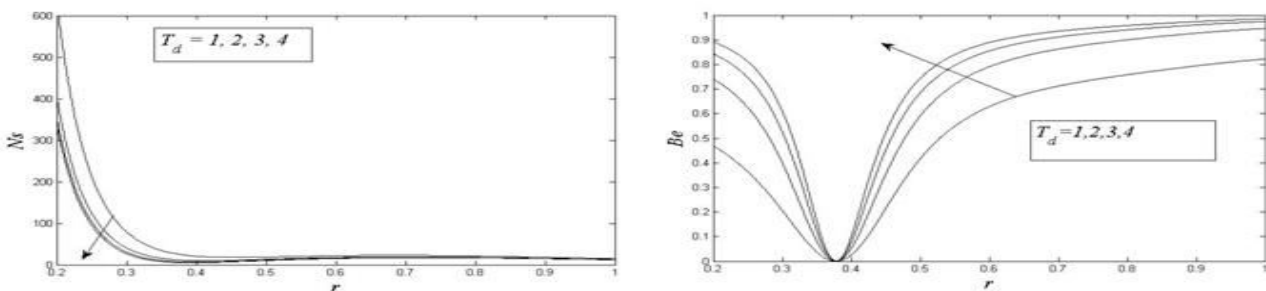
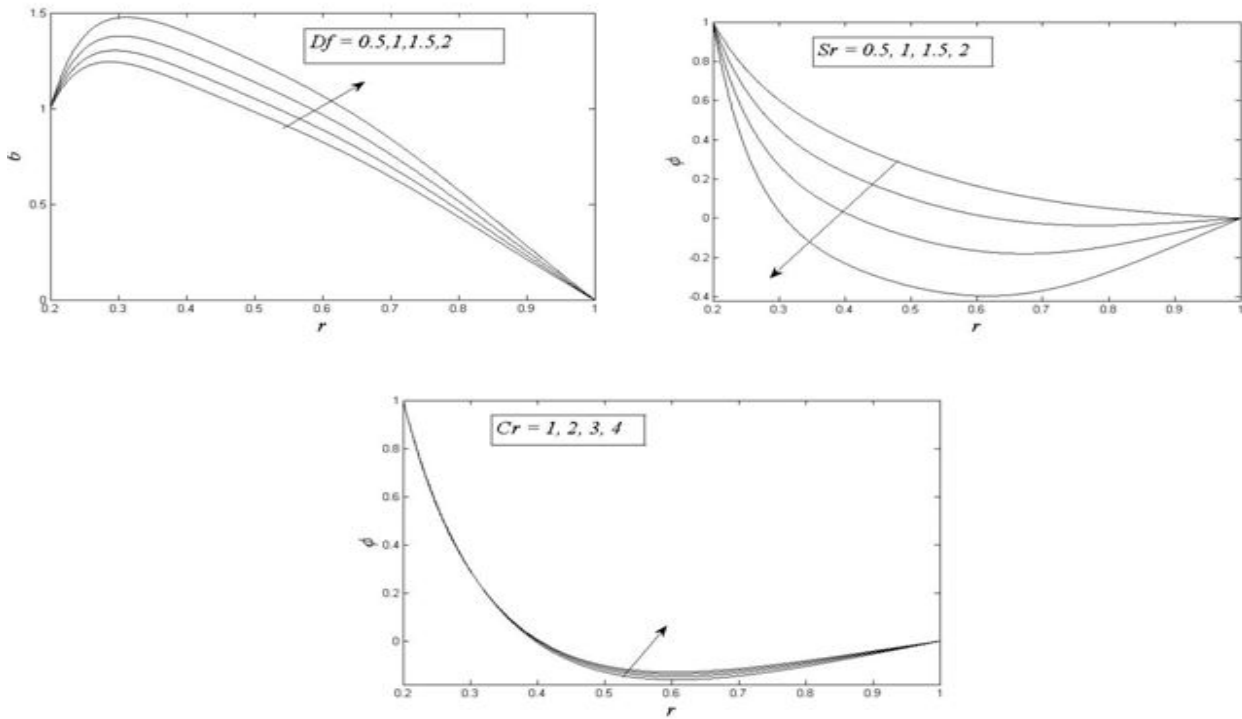


Figure-8. The response of  $T_d$  on (a) Entropy generation number, (b) Bejan number.



**Figure-9.** The influence of (a) Dufour parameter on  $\theta$  (b) Soret parameter on  $\phi$  (c) Chemical reaction  $Cr$  on  $\phi$ .

The deviation of the coupling number ( $c$ ), Brinkman number ( $Br$ ), together with the Schmidt number ( $Sc$ ) is shown in Table-1 with the set values of other parameters. Skin friction coefficient, Nusselt number ( $Nu$ ) at the internal cylinder and mass transfer at the external cylinder increases, but heat transfer at the external and mass transfer at the inner cylinder decelerates as  $c$  increases. In addition, it can be identified that the  $Nu$  at the

inner and mass transfer rates at the outer cylinders increase with an increase in  $Br$ , while  $Nu$  at the outer and  $Sh$  at the inner cylinders decelerate with the increase in  $Br$ . Finally, it can be observed that the  $Nu$  at the inner and  $Sh$  at the outer cylinders increase with an increase in  $Sc$ , while  $Nu$  at the outer and  $Sh$  at the inner cylinders decelerate with an improved  $Sc$ .

**Table-1.** Numerical outcomes for Coefficient of Skin friction Nusselt and Sherwood numbers at  $S = 1.5, \eta = 0.2, \delta = 0.05, K_1 = 0.01, Pr = 0.7, Ha = 4, \delta = 0.05, Df = 0.8, Sr = 0.9, Cr = 1, Td = 1, Cd = 0.25, \lambda = 0.9$ .

C	Br	Sc	$C_f(\eta)$	$C_f(1)$	$Nu(\eta)$	$Nu(1)$	$Sh(\eta)$	$Sh(1)$
0.1	0.5	0.45	-0.0055	-0.0415	0.3015	-0.6189	0.0254	1.2589
0.4	0.5	0.45	-0.0046	-0.0400	0.3761	-1.1373	0.0011	1.4535
0.7	0.5	0.45	-0.0037	-0.0387	0.4252	-2.7432	-0.0126	2.0765
0.3	0.2	0.45	-0.0024	-0.0202	0.2347	-0.0713	0.0481	1.0503
0.3	0.4	0.45	-0.0024	-0.0202	0.3180	-0.6446	0.0203	1.2663
0.3	0.6	0.45	-0.0024	-0.0202	0.4013	-1.2179	-0.0075	1.4824
0.3	0.5	0.1	-0.0024	-0.0202	0.3240	-0.6648	0.0700	0.8986
0.3	0.5	0.4	-0.0024	-0.0202	0.3538	-0.8866	0.0168	1.2945
0.3	0.5	0.7	-0.0024	-0.0202	0.3941	-1.2021	-0.0550	1.8579



**Table-2.** Comparison of the present numeric consequences of the velocity and that of Sofiane *et al.*[10] when  $c \rightarrow 0$ .

r	Solution of Sofiane <i>et al.</i> [10]	Present solution
0.2	1	1
0.3	0.8955	0.8954
0.4	0.7469	0.7467
0.5	0.6320	0.6318
0.6	0.5439	0.5437
0.7	0.4198	0.4195
0.8	0.2674	0.2671
0.9	0.1247	0.1242
1	0	0

The present outcomes are significant in electromagnetic flow processing, engineering and biophysical flows, sustenance conservation, aerodynamic heating, MHD energy systems, diabolic flow in arteries and artificial dialysis and MHD pumps.

### CONCLUSIONS

The numerical solutions for the second- law analysis of MHD, dissipative micropolar flow between two coaxial cylinders with chemical reaction and cross diffusion are presented by RK4 with a shooting procedure. The results obtained are excellent in agreement with previously published results (see Table-2). The following specific conclusions were drawn from this study:

- The fluid velocity increases near to inner cylinder with the raise coupling number and is pragmatic to decelerate with rise in Hartmann number.
- The temperature of the fluid rises with the enhancement in Brinkman and Dufour numbers.
- The fluid concentrations decelerate with the increase in Schmidt number and Soret parameters.
- The entropy generation number rises near the inner cylinders with increment values of the concentration difference parameter and the diffusive constant parameter, whereas it decelerates with the rise of the temperature difference parameter.
- Bejan number decelerates and reaches a minimum value at  $r = 0.37$  with an increase value of the Concentration Difference Parameter, diffusive constant parameter, and increases with the Temperature Difference Parameter.

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### CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

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