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FUZZY DIAGONAL OPTIMAL ALGORITHM TO SOLVE FULLY FUZZY TRANSPORTATION PROBLEMS

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ABSTRACT

In this paper diagonal optimal algorithm is proposed to solve fully fuzzy transportation problems using the approach of Diagonal optimal method. In this proposed method the Fuzzy optimal solution of a fuzzy Transportation problem is obtained by using optimal diagonal method [17]. Yager's ranking technique is used to order the fuzzy numbers. This method can be applied to solve all kinds of fuzzy transportation problem such as unbalanced fuzzy TP, fuzzy Degeneracy problem, fuzzy TP with prohibited routes and many more.

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INTRODUCTION

Transportation problem (TP) plays a vital role in minimizing the transportation cost in logistics and supply chain management. The objectives of the Transportation problems are transporting the commodity from various sources of supply to various sinks of demand in a minimum cost. Hitchcock [13] introduced the transportation problem. Dantzig and Thapa [19] applied the simplex method to optimize the transportation cost. Charnes and Cooper [14] introduced the stepping stone method as an alternative to the simplex method. In the transportation problem, the decision parameters such as availability, requirement and the transportation cost per unit are certain to determine. Due to some uncontrollable situations, the determination of supply, demand and unit transportation cost may be imprecise. The uncertainty in determining the data can be replaced by the fuzzy notions introduced by Zadeh [1, 2] in the year 1965. If the requirement, availability and the cost per unit are represented by fuzzy parameters in a TP, then the TP is called as a Fully Fuzzy TP or TP with fuzzy environment. There are many approaches to solve a fuzzy TP and the fuzzy linear programming technique is one among them. Chanas*et al*., [4] solved fuzzy TP using parametric programming technique which not only identifies the solution, but also gives all other alternatives. Chanas et al., [5] solved fuzzy TP by converting the given problem into a bi-criterial TP with a crisp objective function. Liu Kao [3] solved the fuzzy TP using extension principle.Verma*et al* [9] proposed the fuzzy programming technique for fuzzy TP with hyperbolic and exponential membership function. Liang *etal.*, [10] used possibilistic linear programming technique for fuzzy transportation planning decisions and fuzzy linear programming to solve interactive multi objective transportation planning decision problems. Nagoorgani *et al.*, [6] proposed a parametric approach for a two stage cost minimizing fuzzy transportation problem. Pandian*et al.*, [7] solved the fuzzy TP by using fuzzy zero point method. Amit Kumar *et al.,* [11] introduced fuzzified version of least cost method, fuzzy north west corner rule and fuzzy VAM to solve fuzzy TP with generalized fuzzy numbers. Dhanasekar *et al.*, [18] proposed fuzzy hungarian- MODI algorithm to solve fully fuzzy transportation problem.

The choice of a ranking technique is important for ordering the fuzzy numbers. In this paper, Yager's ranking technique [8] is used to order the fuzzy numbers. It requires the extreme values of α-cut of fuzzy number instead of the form of membership function. In this article the Fuzzy optimal solution of a fuzzy Transportation problem is obtained by using optimal diagonal method.

In this paper, Section 2 deals with fuzzy preliminaries followed by Section 3 in which the proposed algorithm is given in detail. In Section 4, the implementation of the algorithm through example is explained. Finally, the conclusion is given in Section 5.

Section-1

1.1 Definition

The fuzzy set can be mathematically constructed by assigning to each possible individual in the universe of discourse a value representing its grade of membership in the fuzzy set [1-6]

1.2 Definition:

The fuzzy number $\tilde{A}$ is a fuzzy set whose membership function $\mu_{\tilde{A}}(x)$ satisfies the following condition [7]

a) $\mu_{\tilde{A}}(x)$is a piecewise continuous

b) $\mu_{\tilde{A}}(x)$ is a convex

c) $\mu_{\tilde{A}}(x)$is the normal (i.e)$\mu_{\tilde{A}}(x_0)$= 1

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1.3 Definition:

A fuzzy number with membership function in the form

$$\mu_{\widetilde{A}}(x) = \begin{cases} \dfrac{x-a}{b-a} & a \le x \le b \\ 1 & x = b \\ \dfrac{c-x}{c-b} & b \le x \le c \end{cases}$$

is called a triangular fuzzy number $\widetilde{A}= (a,b,c)$

1.4Definition:

A fuzzy number with membership function of the form

$$\mu_{\widetilde{A}}(x) = \begin{cases} \frac{x-a}{b-a} & a \le x \le b \\ 1 & b \le x \le c \\ \frac{d-x}{d-c} & c \le x \le d \\ 0 & \text{otherwise} \end{cases}$$

is called a Trapezoidal fuzzy number $\widetilde{A}=(a,b,c,d)$

1.5 Operations on trapezoidal number and triangular number:

Addition: $(a_1,b_1,c_1,d_1) + (a_2,b_2,c_2,d_2) = (a_1+a_2,b_1+b_2,c_1+c_2,d_1+d_2)$

$(a_1,b_1,c_1) + (a_2,b_2,c_2,) = (a_1+a_2,b_1+b_2,c_1+c_2)$

Subtraction: $(a_1,b_1,c_1,d_1) - (a_2,b_2,c_2,d_2) = (a_1-d_2,b_1-c_2,c_1-b_2,d_1-a_2)$

$(a_1,b_1,c_1) -(a_2,b_2,c_2,) = (a_1-c_2,b_1-b_2,c_1-a_2)$

1.6 Definition:

Yager'sRanking $Y(\widetilde{a}) = \int_0^1 .5(a_U^\alpha + a_L^\alpha)d\alpha$ where

$a_L^\alpha = Lower\alpha - cut$, $a_U^\alpha = Upper\alpha - cut$.

If $Y(\widetilde{s}) \le Y(\widetilde{i})$ then $\widetilde{s} \prec \widetilde{i}$. Yager's ranking technique satisfies Compensation, Linearity and additive properties [4].

1.7 Definition:

Fully Fuzzy transportation problem is defined by

$$\min \widetilde{Z} \approx \sum_{i=1}^{m}\sum_{j=1}^{n}\widetilde{c}_{ij}\widetilde{x}_{ij}$$

subject to $\sum_{j=1}^{n}\widetilde{x}_{ij} \approx \widetilde{s}_i$ for i=1,2,3…m.

$\sum_{i=1}^{m}\widetilde{x}_{ij} \approx \widetilde{d}_j$ for j=1,2,3…n, for all $\widetilde{x}_{ij} \ge 0$ where i=1,2,3..m and j=1,2…n.

The above problem can also be depicted as follows:

$$\begin{array}{c} \\ Source1 \\ Source2 \\ ::: \\ Sourcem \\ Demand \end{array} \begin{array}{c} \begin{array}{ccccc} \text{Destination1} & \text{Destination2} & \text{….} & \text{Destination n} & \text{Supply} \end{array} \\ \left(\begin{array}{cccc} \widetilde{c_{11}} & \widetilde{c_{12}} & \cdots. & \widetilde{c_{1n}} \\ \widetilde{c_{21}} & \widetilde{c_{22}} & \cdots. & \widetilde{c_{2n}} \\ \cdots & \cdots. & \cdots. & \cdots. \\ \widetilde{c_{m1}} & \widetilde{c_{m2}} & \cdots. & \widetilde{c_{mn}} \\ \widetilde{d_1} & \widetilde{d_2} & \cdots\cdots & \widetilde{d_n} \end{array}\right) \begin{array}{c} \widetilde{s_1} \\ \widetilde{s_2} \\ \cdots \\ \widetilde{s_m} \\ \\ \end{array} \end{array}$$

Where $\widetilde{c}_{ij}$ = the cost per unit in transporting from ith place to jth place.

PROPOSED ALGORITHM

The new algorithm is as follow:

a) Locate the two cells that have minimum cost and next to minimum cost in each row, then find their difference (Penalty) along the side of the table against the corresponding row.

b) Locate the two cells that have minimum cost and next to minimum cost in each column, then find their difference (Penalty) below the table against the corresponding column.

c) Locate the maximum penalty. If it is along the side of the table, make maximum assignment to the cell having minimum cost in that row. If it is below the table, make maximum assignment to the cell having minimum cost in that column. Continue in the same manner until all assignments are made.

d) If the penalties corresponding to two or more rows/columns are equal, find the element-wise difference between first and third minimum value. Identify the maximum among them and assign the minimum cost among them. This step gives the initial solution.

Remarks: For finding the optimal solution, we will follow step 5 and 6.

e) Write these Fuzzy assigned costs on the top of the column of original assignment problem. Let $\widetilde{a}_j$ be the assigned cost for column. Subtract $\widetilde{a}_j$ from each entry of of $\widetilde{c}_j$ the corresponding column of assignment matrix.

f) Construct a rectangle in such a way that one corner contains negative Fuzzy penalty and remaining two corners are allocated to the assigned cost values in corresponding row and column. Calculate the sum of extreme cells of unassigned diagonal, say $\widetilde{d}_{ij}$. Locate $\forall\widetilde{d}_{ij} \prec \widetilde{0}$. Identify the most negative $\widetilde{d}_{ij}$ and exchange the assigned cell of diagonals. Continue the process until all negative penalties are resolved.

Remarks: If any $\widetilde{d}_{ij} \approx \widetilde{0}$, then exchange the cells of diagonals at the end.

g) In each assignment cell allocate the least possible amount. For example if the assigned cell is (i,j) then $\widetilde{x_{ij}} = \min(\widetilde{s_i}, \widetilde{d_j})$in the cell ($i,j$) and cross out the i^{th} row if $\widetilde{s_i} \prec \widetilde{d_j}$or j^{th} column if $\widetilde{s_i} \succ \widetilde{d_j}$. Cross out both the row and column if $\widetilde{s_i} \approx \widetilde{d_j}$.

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h) Repeat this procedure until all the requirements are satisfied.

Section-3

Example 1

Consider the following Triangular transportation problem

$$\begin{array}{c|cccc|c} & \text{Destination1} & \text{Destination2} & \text{Destination3} & \text{Destination4} & \text{Supply} \\ \hline Source1 & (5,10,15) & (5,10,20) & (5,15,20) & (5,10,15) & (10,15,20) \\ Source2 & (5,10,20) & (5,15,20) & (5,15,20) & (10,15,20) & (5,10,15) \\ Source3 & (5,10,20) & (10,15,20) & (10,15,20) & (5,10,15) & (20,30,40) \\ Source4 & (10,15,25) & (5,10,15) & (10,20,30) & (10,15,25) & (15,20,25) \\ Demand & (25,30,35) & (10,15,20) & (5,15,20) & (10,15,25) & \end{array}$$

Applying the proposed algorithm

$$\begin{array}{c|cccc|c} & \text{Destination1} & \text{Destination2} & \text{Destination3} & \text{Destination4} & \text{fuzzy penalty} \\ \hline \text{Source1} & (5,10,15) & (5,10,20) & (5,15,20) & (5,10,15) & (-10,0,10) \\ \text{Source2} & (5,10,20) & (5,15,20) & (5,15,20) & (10,15,20) & (-15,5,15) \\ \text{Source3} & (5,10,20) & (10,15,20) & (10,15,20) & (5,10,15) & (-10,0,15) \\ \text{Source4} & (10,15,25) & (5,10,15) & (10,20,30) & (10,15,25) & (-5,5,20) \\ \text{fuzzy penalty} & (-10,0,15) & (-10,0,15) & (-15,0,15) & (-10,0,10) & \end{array}$$

Out of all the Fuzzy penalty the fourth row is having maximum fuzzy penalty. Choose the minimum in that row. Strike of the corresponding row and column.

$$\begin{pmatrix} (5,10,15) & (5,10,20) & (5,15,20) & (5,10,15) \\ (5,10,20) & (5,15,20) & (5,15,20) & (10,15,20) \\ (5,10,20) & (10,15,20) & (10,15,20) & (5,10,15) \\ (10,15,25) & (\mathbf{5},\mathbf{10},\mathbf{15}) & (10,20,30) & (10,15,25) \end{pmatrix}$$

Repeating the same procedure for the remaining matrix

$$\begin{array}{c|ccc|c} & \text{Destination1} & \text{Destination3} & \text{Destination4} & \text{fuzzy penalty} \\ \hline \text{Source1} & (5,10,15) & (5,15,20) & (5,10,15) & (-10,0,10) \\ \text{Source2} & (5,10,20) & (5,15,20) & (10,15,20) & (-15,5,15) \\ \text{Source3} & (5,10,20) & (10,15,20) & (5,10,15) & (-10,0,15) \\ \text{fuzzy penalty} & (-10,0,15) & (-15,0,15) & (-10,0,10) & \end{array}$$

Since the row 2, row 3 and column 1 has maximum fuzzy penalty, we can choose any one. We choose column 1.Choose the minimum in that column. Strike of the corresponding row and column.

Repeating the same procedure for the remaining matrix

$$\begin{array}{c|cc|c} & \text{Destination3} & \text{Destination4} & \text{fuzzy penalty} \\ \hline Source2 & (5,15,20) & (10,15,20) & (-15,5,15) \\ Source3 & (10,15,20) & (5,10,15) & (-10,0,15) \\ fuzzy\ penalty & (-15,0,15) & (-10,0,10) & \end{array}$$

Since the row 2 and column 2 has maximun fuzzy penalty, we can choose any one. We choose column 2.Choose the minimum in that column. Strike of the corresponding row and column.

The assignments are

$$\begin{pmatrix} (\mathbf{5},\mathbf{10},\mathbf{15}) & (5,10,20) & (5,15,20) & (5,10,15) \\ (5,10,20) & (5,15,20) & (\mathbf{5},\mathbf{15},\mathbf{20}) & (10,15,20) \\ (5,10,20) & (10,15,20) & (10,15,20) & (\mathbf{5},\mathbf{10},\mathbf{15}) \\ (10,15,25) & (\mathbf{5},\mathbf{10},\mathbf{15}) & (10,20,30) & (10,15,25) \end{pmatrix}$$

Checking the optimum of these assignments

$$\begin{pmatrix} (5,10,15) & (5,10,15) & (5,15,20) & (5,10,15) \\ (\mathbf{5},\mathbf{10},\mathbf{15}) & (5,10,20) & (5,15,20) & (5,10,15) \\ (5,10,20) & (5,15,20) & (\mathbf{5},\mathbf{15},\mathbf{20}) & (10,15,20) \\ (5,10,20) & (10,15,20) & (10,15,20) & (\mathbf{5},\mathbf{10},\mathbf{15}) \\ (10,15,25) & (\mathbf{5},\mathbf{10},\mathbf{15}) & (10,20,30) & (10,15,25) \end{pmatrix}$$

Subtracting the each element of the column from the corresponding assignment

$$\begin{pmatrix} (\mathbf{10},\mathbf{0},\mathbf{10}) & (-10,0,15) & (-15,0,15) & (-10,0,10) \\ (-10,0,15) & (10,5,15) & (-\mathbf{15},\mathbf{0},\mathbf{15}) & (-5,5,15) \\ (-10,0,15) & (-5,5,15) & (-10,0,15) & (-\mathbf{10},\mathbf{0},\mathbf{10}) \\ (-5,5,20) & (-\mathbf{10},\mathbf{0},\mathbf{10}) & (-10,5,25) & (-5,5,20) \end{pmatrix}$$

For the non-assigned cells

$$\tilde{d}_{12} = \begin{pmatrix} (-10,0,10) & (-10,0,15) \\ (-5,5,20) & (-10,0,10) \end{pmatrix} = (-10,0,15) + (-5,5,15) = (-15,5,35) \succ \tilde{0}.$$

$$\tilde{d}_{13} = \begin{pmatrix} (-10,0,10) & (-15,0,15) \\ (-10,0,15) & (-15,0,15) \end{pmatrix} = (-15,0,15) + (-10,0,15) = (-25,0,30) \succ \tilde{0}.$$

$$\tilde{d}_{14} = \begin{pmatrix} (-10,0,10) & (-10,0,10) \\ (-10,0,15) & (-10,0,10) \end{pmatrix} = (-10,0,15) + (-10,0,10) = (-20,0,25) \succ \tilde{0}.$$

By repeating this procedure, for all the non assigned cells $\widetilde{d_{12}} \succ \tilde{0}$. So the assignments are optimum. Now allocate the units in the corresponding assignment cells.

The optimum solution is

$$\begin{array}{c} \\ \text{Source1} \\ \text{Source2} \\ \text{Source3} \\ \text{Source4} \\ \text{Demand} \end{array} \begin{array}{c} \begin{array}{ccccc} \text{Destination1} & \text{Destination2} & \text{Destination3} & \text{Destination4} & \text{Supply} \end{array} \\ \begin{pmatrix} (5,10,15)^{(10,15,20)} & (5,10,20) & (5,15,20) & (5,10,15) & (10,15,20) \\ (5,10,20) & (5,15,20) & (5,15,20)^{(5,10,15)} & (10,15,20) & (5,10,15) \\ (5,10,20)^{(-20,10,40)} & (10,15,20) & (10,15,20)^{(-10,5,15)} & (5,10,15)^{(10,15,25)} & (20,30,40) \\ (10,15,25)^{(-5,5,15)} & (5,10,15)^{(10,15,20)} & (10,20,30) & (10,15,25) & (15,20,25) \\ (25,30,35) & (10,15,20) & (5,15,20) & (10,15,25) & \end{pmatrix} \end{array}$$

The transportation cost is given by =(5,10,15)*(10,15,20)+(5,15,20)*(5,10,15)+(5,10,20)*(-20,10,40)+(10,15,20)*(-10,5,15)+(5,10,15)*(10,15,25)+(10,15,25)*(-5,5,15)+(5,10,15)*(10,15,20)=(-75,850,2750).

Section-4

CONCLUSIONS

In this paper, an efficient method using diagonal optimal approach is proposed. It can also be used to solve special type of fuzzy transportation problem like unbalanced transportation problems and transportation problem with Degeneracy.

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