



# GENETIC EXPRESSION PROGRAMMING MODEL FOR SELECTING THE APPROPRIATE GROUND IMPROVEMENT TECHNIQUE

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## ABSTRACT

In this paper, Gene Expression Programming (GEP) models are developed to select the appropriate ground improvement technique. Ground improvement is usually used to increase the soil bearing capacity, reduce potential settlement, and mitigate liquefaction. The data used to build the GEP models was collected from 83 ground improvement projects in the UAE. Data collected from each project includes the following parameter: fine content (%), groundwater level (m), depth of improvement (m), distance to close by structures (m), and the used ground improvement method. This paper investigates five ground improvement techniques that are dynamic compaction (DC), dynamic replacement (DR), vibro compaction (VC), rapid impact compaction (RIC), and stone columns (SC). One GEP model is developed for each technique, the user will be able to input the above-mentioned parameter in each model and select the technique with higher accuracy. The developed GEP models have  $R^2$  values, for the training dataset, ranging from 0.72 to 0.95. The results showed that GEP could select the appropriate ground improvement technique with acceptable accuracy.

**Keywords:** GEP, ground improvement, DC, DR, VC, RIC, SC, expression tree.

## 1. INTRODUCTION

Ground improvement can be defined as “the process of enhancing the quality of soil” [1]. There are a variety of techniques available to improve ground conditions such as stone columns (SC), dynamic compaction (DC), short pile, rapid impact compaction (RIC), vibro compaction (VC), and dynamic replacement (DR). In essence, ground improvement is needed whenever ground conditions are poor. It enhances soil layers by different means such as mechanical and chemical methods [1].

In this regard, this paper covers the five most common methods for ground improvement: (1) Dynamic Compactions (DC); (2) Rapid Impact Compaction (RIC); (3) Vibro Compaction (VC); (4) Dynamic Replacement (DR); and (5) Stone Columns (SC).

The following sub-sections provide discussion on each of the five methods:

### A. Stone Columns (SC)

Prior to its widespread application and familiarity, the early 1960s saw the first application of stone column in the European countries [2]. The stone columns are constructed by two methods, namely: replacement, and displacement. Whenever the level of groundwater is shallow, and the soil is firm, the displacement method is preferred [3]. In this method, soil particles are displaced horizontally by a vibrating probe that works on compressed air. Baumann and Bauer in 1974 added that SC could work by compacting either sand or gravel, then injected into the clay foundation using the displacement method [4]. The replacement method applies a uniform compressive force on soil particles using holes that are built based on a combination of vibration and water jet.

Once the SC applies the load, deformation of piles will transform loads and stresses to the upper layers

of soil strata by bulging the piles into underneath soil layers [3].

The SC technique will ultimately increase the density of the soil (unit weight) as a result of consolidation from vertical piles of SC. Moreover, while consolidation takes place, paths for drainage are created allowing for more stability of soil layers by preventing liquefaction and pore pressure [1]. In line with this, Han and Ye noticed from field observations that SC can speed up the consolidation of soft clays [5]. Later in 2001, Han and Ye found that over time, SC will gain more stresses while these stresses on soil decrease. After consolidation, the stable distribution of stresses is reached [6].

Improvement of soil from the SC technique is attributed to the high stiffness of SC. Thus, it can be said that stiffness and load distribution between column and soil is crucial to the SC method. Typically, SC is appropriate for structures since they reduce settlements, increase bearing capacity, enhance drainage and stiffness. However, these advantages are not fully utilized especially when SC is used in sensitive clays. In this case, SC poses the risk of bed settlement due to the lack of lateral support. Additionally, particles of clay can decrease radial drainage [2].

### B. Rapid Impact Compaction (RIC)

When little silt and clay particles exist, which is the case of sand, the RIC is preferred. As the name implies, the RIC method depends on a dynamic compaction device to perform compaction of soil. When it comes to soil compaction, generally, there are deep compaction and surface compaction techniques. The RIC technique comes between deep and surface techniques [7].

In this technique, the ground is compacted using a hammer that weighs 9 tons. The hammer is released from a height ranging from 0.3 m to 1.2 m. The hammer is dropped on a steel paten foot with a diameter of 1.5 m; resulting in significant energy ranging from 26,486 to 105,



948 joules per drop. It can strike 40-60 blows per minute. The RIC can increase the density of loose soils down to an average depth of 5 m. The compaction of the lower levels is done first followed by the upper levels. Blowing stops when the total depth of the impact foot, defined as a specified number of blows, and the final settlement of the impact foot have been reached.

Falkner and others in 2010 had theoretically investigated the RIC using computer simulations to examine the delivered energy into the soil and the propagation of waves [8]. Further examinations were carried by experimental test to verify the theoretical results, resulting in verification of the initial theoretical investigations. The RIC method has been applied in many cases, for instance, Simpsons and colleagues in 2008 used the RIC method in a reclaimed site in California, USA [9]. Further discussion on RIC method can be found in [10].

### C. Vibro Compaction (VC)

The VC technique is very effective when enhancing the density of granular soils with appropriate gradation that has limited fine content (less than 5%). The required depth is penetrated by a vibroflot which is assisted by water jetting. Once the design depth is reached, the water jetting is gradually removed by stopping at uniform intervals. This ensures suitable levels of compaction are attained at each depth. In the VC technique, soil particles are densified by the radial vibrations. The soil is then characterized as having greater density and enhanced mechanical properties, such as shear strength, stiffness, and bearing capacity [10].

### D. Dynamic Compaction (DC)

Menard and Broise introduced this method in 1975 [11]. The simple application, value, and significant reachable depth have made the DC method a popular ground improvement technique. This method is applicable to any type of soil but can be more beneficial when used in sandy and granular soils. However, attention is needed when used on saturated clay and fine-grained soils.

The DC technique is applied by impacting the ground frequently via a large pounder by free-fall dropping onto grid points from a height of 10-40 m. The pounder weighs 6-40 tons to increase the compaction effort and the bearing capacity. As Lutenege noted, this will decrease the collapsibility of losses when improving a specified depth [12].

In spite of its simple application, challenges still exist. For instance, the required depth to be improved is sometimes hard to assess. In this case, parameters are developed to evaluate the necessary depth by carrying out an experimental study to identify the operational parameters. In fact, such experimental assessments will reduce the cost of applying DC. However, the challenge here is attributed to the fact that many parameters can vary significantly and in some cases will lead to no results [13, 14, 15].

The civil engineering projects are familiarized with the DC technique. Such projects include highways, airports, the coal industry, and structural buildings [16].

For instance, Bo and others in 2009 examined DC at the Changi East Reclamation in Singapore to enhance a sandy fill [17]. Data from the site were extracted to evaluate the efficiency of densification and the impact of different factors on the success of DC treatment.

### E. Dynamic Replacement (DR)

The DR technique was first presented by Menard and Broise in 1975 to enhance the properties of fine grain soils through dynamic driving of granulated attachments into the soil [2]. This method is an extension to the Dynamic Compaction (DC) in that the energy is applied to deliver the granular material, which is the fill material, down into the compressible soils. This will result in a reinforced column, made of soil, called pillar, with a diameter of 2-2.5 m and can be reached down to a depth of 7 m. Such a pillar can be formed by releasing a 15-25-ton weight from 10-25 m height. In order to increase the penetration and stamping of the weight, the DR uses larger loads than DC.

The replacement ratio from this method can be up to 25% with each pillar having a load carrying capacity of 150 tons. This numerous weight can be used to improve the nearby and underlying layers by transmitting the energy generated from this weight. Tarawneh and others in 2017 stated that the DR is superior over the DC and SC techniques since it can "produce large diameter dynamic replacement inclusions with high internal shear resistance" [18].

The advantages of DR can be summarized as follow [17]:

- a) Improving the bearing capacity of the soil.
- b) Applicability for a variety of soil types.
- c) Increase the consolidation rate of fine materials.
- d) Significant reduction of settlements especially for post-construction.
- e) Increasing productivity rates.
- f) Using a variety of materials to construct the pillar, such as sand, gravel, concrete ruins, and dredged materials.

## 2. GENETIC PROGRAMMING

Genetic programming (GP) is an optimization method generating programs with an aim of solving problems by mimicking the biological growth of living creatures [19]. Friedberg conducted the earliest applications of GP in 1958 [20]. Later in 1985; genetic algorithms (GAs) and tree-like structures were used to evolve and develop solution programs [21].

Overtime; GP and GA have shown capabilities in solving and developing computer programs. With a few modifications, GP can make use of GA's operators. The GA solution is presented in the form of strings of



numbers; while the GP produces tree structures as a solution outcome [22]. To evaluate the outcome of each GP; a fitness function is examined. In other words, the objective function that GP optimizes is the fitness function.

### 3. GENE EXPRESSION PROGRAMMING (GEP)

In 2001, Ferreria combined genetic algorithms (GA) and genetic programming (GP) to construct a problem-solving technique called Gene Expression Program (GEP) [23]. This technique makes use of a natural phenotype/genotype system. The GA facilitates the development of linear programs (individual or chromosomes) of specified length and expression trees (ET) as extracted from GP. In GEP, some terms are commonly used during the development and interpretation of models such as; chromosomes, genes, and ET. A chromosome in GEP is made of genes. Chromosomes are exposed to genetic changes allowing GEP progression. It should be noted that genes are exposed to operations such as mutation and recombination to make proper changes on them. Mutation means that any component of the gene's head or tail is randomly replaced by an element from the function or terminal set.

When two chromosomes are combined and fragmented at the same point, then recombination takes place. Moreover, the combination-fragmentation process aims to change the chromosomes' components to the merging point, downward.

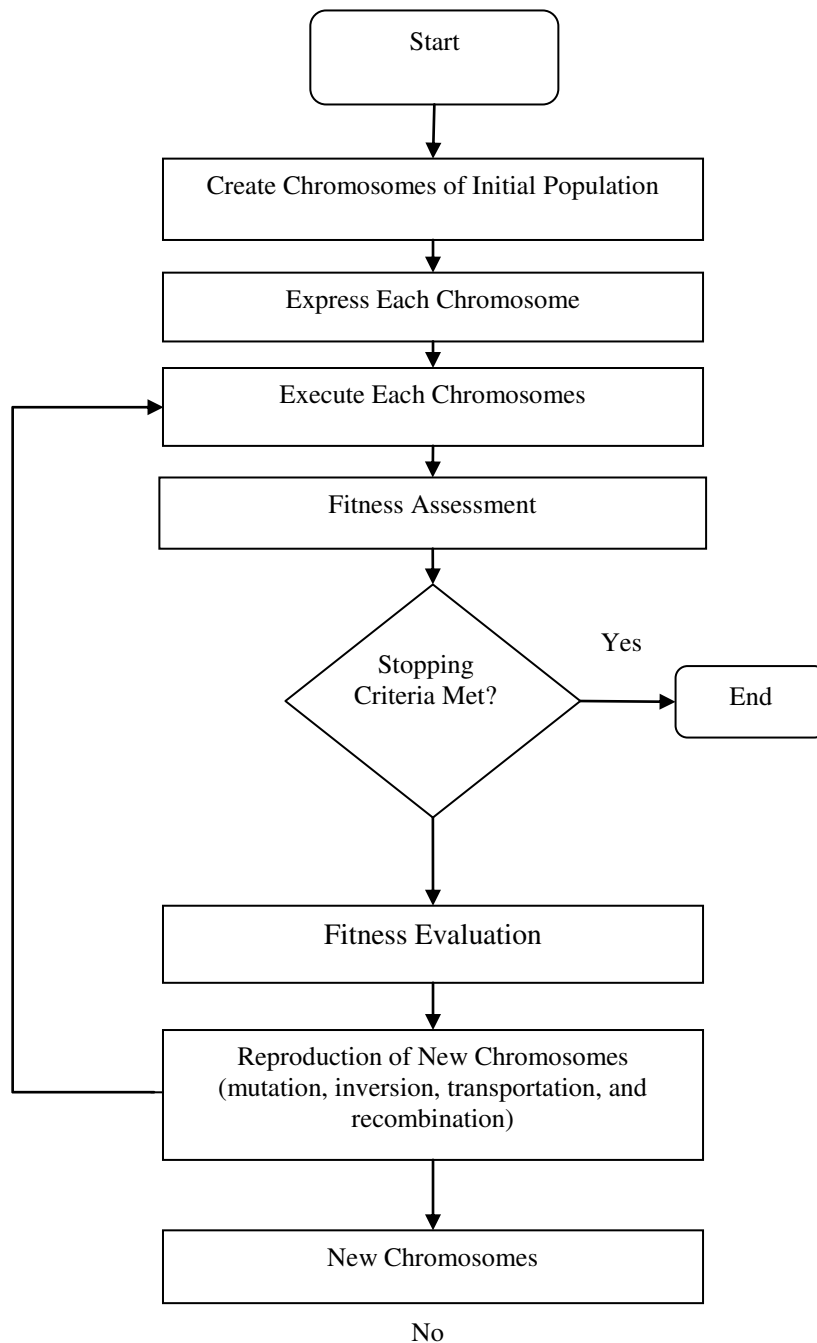
A gene is composed of head and tail. The head is in charge of holding functions (e.g., +, -), while the tail is made up of terminals. Terminals in the GEP language describes the input variables (represented as d1, d2, d3...).

Therefore, a standard gene in GEP is noted as follows:  $+.-./dl./d1.d2.d3.d4.d1.d2$ , where "." is splitting between symbols; and d0, d1, and d2 are variables recognized as terminals.

The gene head is expressed by functions and italic symbols, whereas; the bold, black symbols describe the tail. The expression just provided above is known as K-expression or Karva notation (Ferreira, 2001).

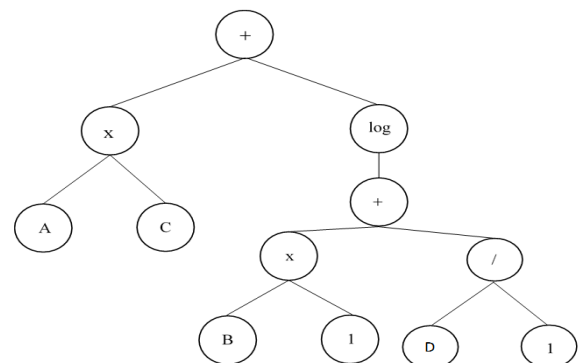
The construction of the GEP model is explained in Figure-1, as adopted from [7]. The development starts with building the population of computer programs based on a random selection from a predefined set of functions and terminals. Elementary mathematical operators such as (+, -, x, /) are included in functions, whereas; terminal include constants (logical and numerical), or variables. When implementing a program (chromosome), its checked in terms of fitness. This is done by the fitness function which is responsible for measuring the competency of a chromosome with the population. After that, chromosomes are subject to improvement based on the fitness measure. In other words, chromosomes with high levels of fitness are given higher chances of being reselected, and those with low fitness levels will have a low chance of reselection.

Mutation and recombination, as described earlier, are then applied to improve the selected programs. This implies genetic modifications for the selected programs. A new set of chromosomes are then produced with new features and characteristics to remove the existing population. The new population of chromosomes will go through the same process until stopping criteria, which has already been defined, is met.



**Figure-1.** GEP Model Progress Flow Chart (Tarawneh, 2018).

In order to graphically represent the interactions between the gene's constituents, a tree is used to represent such interactions. This tree is called the Expression Tree (ET) and serves as a solution map. In other words, the output of the GEP model is expressed by different means such as graphical, mathematical, and programming languages (C++). For instance, Figure-2 shows an ET for a GEP model output. This ET can also be converted to the k-expression, as stated earlier.



**Figure-2.** ET for typical GEP model output.



The karva notation for the ET displayed in Figure-2 can be expressed as shown in equation (1) below:

$$ET = A \times C + \log((B \times 1) + (D/1)) \quad (1)$$

The ET is translated into Karva notation by reading the ET from left to right; starting by roots, reporting nodes located at the same level, and reaching the deepest level.

Therefore, this paper will develop a GEP, with a process similar to the one described above, to select the most suitable ground improvement method (amongst the five methods described previously).

#### 4. DATA AND MODEL DEVELOPMENT

The data used to build the GEP model was collected from 83 ground improvement projects in the UAE. Data collected from each project includes fine content (%), groundwater level (m), depth of improvement (m), distance to close by structures (m), and the used ground improvement method. This paper investigates five ground improvement methods, as follows: (1) RIC, (2) DC, (3) VC, (4) DR, and (5) SC.

In order to investigate the relationship between the dependent (ground improvement method) and

independent variables (fine content, groundwater level, depth of improvement, and distance to close by structures), a regression analysis was developed. In essence, the model is prepared to predict the most likely ground improvement method that is suitable for a given set of independent variables. To use the model, ground improvement method is treated as the dependent variable. In other words, if someone is considering to improve the ground conditions, and he/she has five options, the model predicts which method is the most likely to be chosen for ground improvement based on the independent variables.

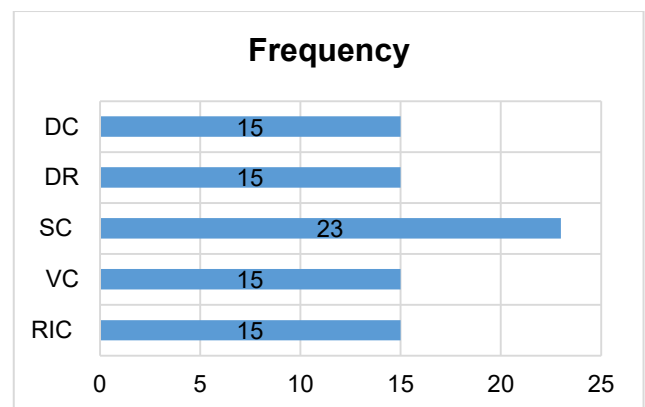
The dependent variable, in nature, is discrete. Therefore, and given the aim of this paper, this research develops a multinomial logistic regression model to predict the ground improvement method. The multinomial logistic regression is used when a discrete outcome is foreseen. There are several advantages of using a multinomial model in that it can establish the relationships between the set of independent variables and the dependent variable when the dependent variable has more than two outcomes. In GEP language, this model is called the multiclass classification model.

Table-1 describes the statistics of the variables used to develop the model.

**Table-1.** Descriptive Statistics of Model Variables.

Model Variables	Mean	Standard Deviation	Min.	Max.	Range
Fine content	27.31%	21.72%	3	9	6
The depth of Improvement (m)	7.35	4.03	1	19	18
Ground Water Level (m)	4.43	2.47	0	0	0
Distance to Close by Structure (m)	36.51	34.76	4	80	76

Additionally, Figure-3 depicts the number of observations recorded for each ground improvement method (the dependent variable). The dataset was partitioned into two sets to ensure the robustness of the model that are training and validation. The training dataset is used to fine-tune the model to develop the GP that builds the relationship between the independent and dependent variables. 79.31% of data (65 data points) were used to train the model, while the remaining data is used to validate and test the model. The ratio of such splitting between training and validation follows the guidelines suggested by Ferreira [23].



**Figure-3.** Frequency of Ground Improvement Methods.

In other words, training takes place first, followed by validation checks on the model. It should be noted that the training set includes minimum and maximum data points since artificial intelligence executes better in interpolation than extrapolation [24].

In order to facilitate the evolution of GEP development, some parameters needed to be set. Table-2



shows parameter settings for the GEP model. The parameter settings were established based on the previous study by Ferreira [23] and supported by a later study by Alvai and others [19].

**Table-2.** Parameter Settings for the GEP Model.

Parameter	Settings
Number of Chromosomes	30
Number of Genes	3
Head Size	8
Linking Function	Addition
Function Set	+, -, x, /, power, exp, √
<b>Complexity Increase</b>	
Generations without Change	2000
Number of Tries	3
Max. complexity (Genes)	5
<b>Genetic Operators</b>	
Mutation Rate	0.00138
Inversion Rate	0.00546

Before training the model, variables must have low interdependency. The correlation between variables must be investigated to avoid poor performance of the GEP model. Positive/negative correlation coefficients with high values at both ends will result in misinterpretation of the impact of independent variables on the dependent variable. All possible pairs were examined in their correlation using R-value. This was evaluated using the built-in functions of Microsoft Excel. The R-value for all pairs of variables has shown very low correlation. Therefore, all variables have been utilized for model training and validation. Most pairs show R-values less than 0.3. Smith stated, if R-value is greater than 0.8; then a strong correlation exists [25].

GEP model is developed based on several runs to optimize the modeling parameters. This will secure the robustness and generalization of the model. Parameters of interest, those to be optimized, include the number of chromosomes, functional set, structure of chromosomes, fitness function, and linking function [23].

In order to derive the optimal parameters; subsequent runs were deployed. The program is run multiple times; during each run, the value of one parameter is changed while the remaining parameters are fixed. The model is set to stop whenever the maximum fitness is reached; implying no more significant improvement to the model. Table-3 shows the optimal parameters used to develop the model.

**Table-3.** Optimal Parameters Used to Develop the Model.

<b>Chromosomes</b>	<b>30</b>
Genes	3
Head Size	8
Tail Size	9
Gene Size	26
Linking Function	Addition

## 5. RESULTS AND DISCUSSIONS

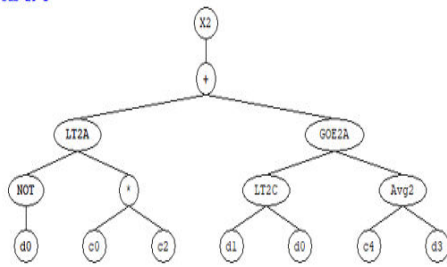
For each case of the improvement methods; a multiclass classification is built. The GEP was deployed using Gene Xpro Tools 5.0 program. Each ground improvement method is treated by a separate logistic model that is based on a binary dependent variable. Therefore, in accordance with the dependent variable, 5 different models are developed. In this regard, the algorithm is trained for each class (ground improvement method). Therefore; for five different ground improvement methods; five models are presented.

As stated earlier, the GEP model is presented in the form of expression trees as depicted in Figure-2. Since the model predicts one method among the five available methods, the GEP model results are expressed in five different ETs. Under each model's ET three sub-ETs are developed. In fact, this number of sub-ETs is in accordance with parameter settings in Table-2. In other words, since the model has three genes, this implies three different sub-ETs that are linked by addition function. Additionally, ETs can be converted into K-expression (Karva notation). Figure-4 presents the results of the GEP model. The performance of all five models is evaluated in Table-4 for both training and testing observations. The evaluation is based on accuracy and R-square. For more details on the resulting ETs for the current model, the Karva notation tables are presented in the appendix.

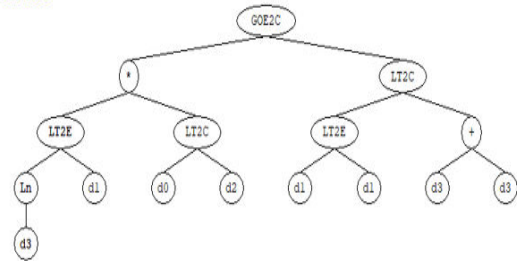




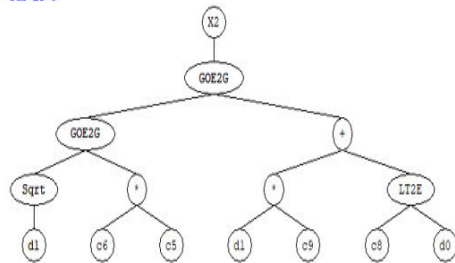
Sub-ET 1



Sub-ET 2

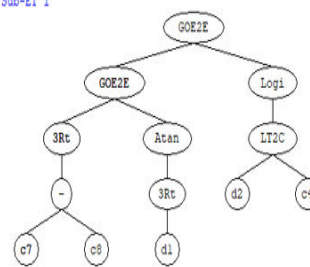


Sub-ET 3

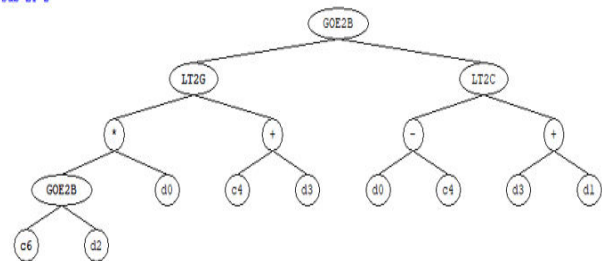


(a) VC

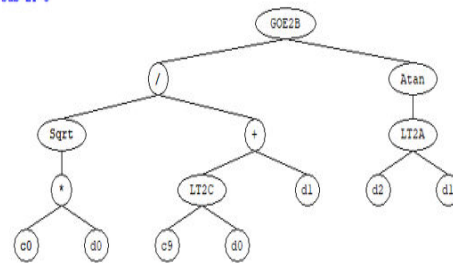
Sub-ET 1



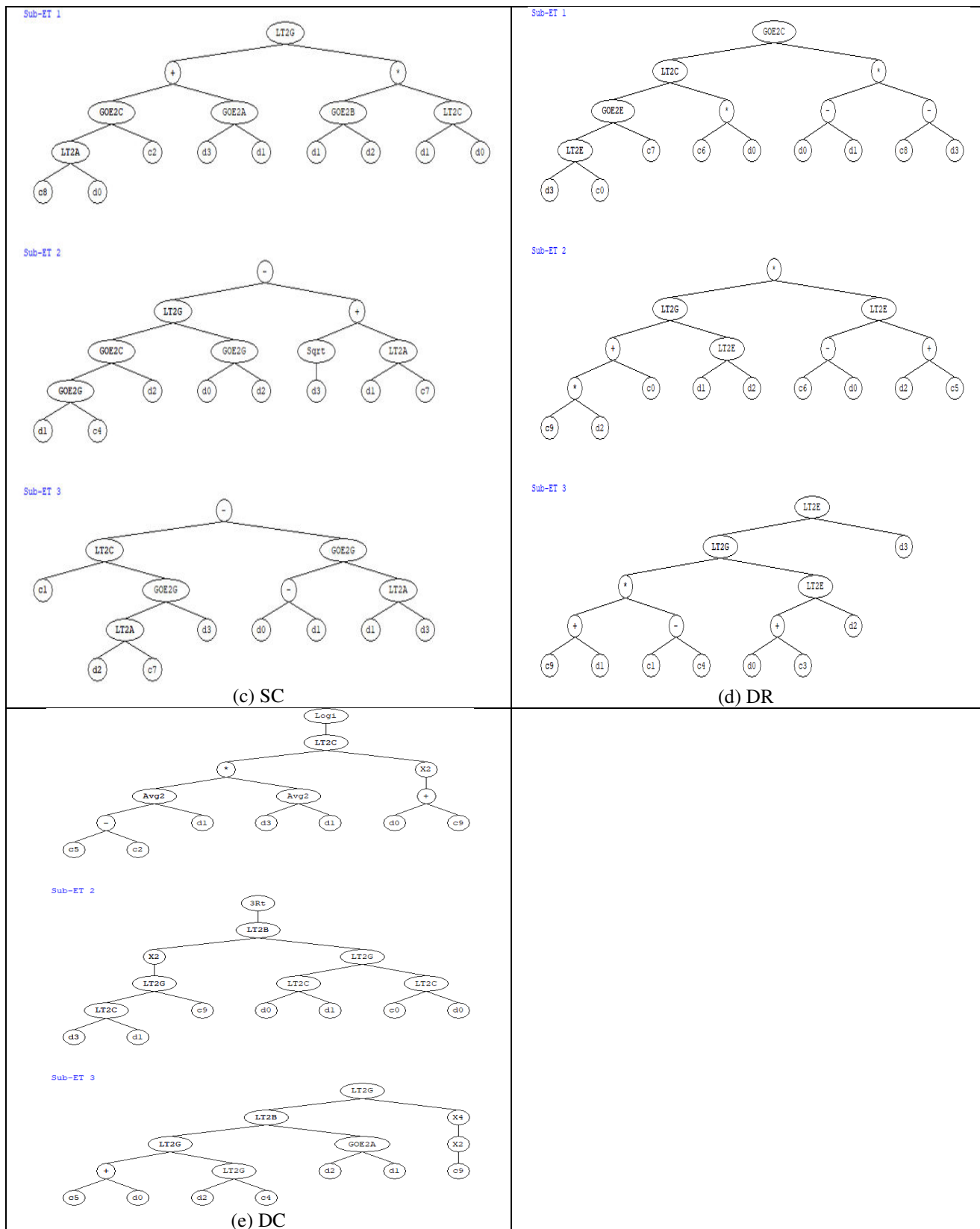
Sub-ET 2



Sub-ET 3



(b) RIC



**Figure-4.** The results of the GEP model.



**Table-4.** Performance Parameters of GEP Models.

Model	Training		Validation	
	Accuracy	R <sup>2</sup>	Accuracy	R <sup>2</sup>
VC	100%	0.95	94.4%	0.83
RIC	100%	0.79	77.8%	0.40
SC	100%	0.98	83.3%	0.43
DR	100%	0.88	83.3%	0.26
DC	100%	0.72	88.9%	0.20

Each of the five models obtained for a specific class (ground improvement method) functions individually and forecasts whether a specific observation refers to a specific ground improvement method. Therefore, it is necessary to adopt a probabilistic model to identify the most likely ground improvement method will be utilized given a set of observations. In this regard, logistic regression serves to assign probabilities to the model outcomes and eventually presents a multi-class classification model. After the probabilities of each class are defined, it is now possible to categorize the likelihoods of each observation. The logistic regression forecasts a logit transformation of the likelihood of a dichotomous variable on a linear basis. To elaborate more, assume that for a specific class:

Y= 0: not being chosen; 1: being chosen.

In this case, Y is a binary outcome. Therefore, if  $p(Y=1|X)$  is the probability of a specific class Y to be chosen given an array of independent variables X, the logistic regression of Y is described in equation (2):

$$\text{Logit } (p(Y=1|X)) = \log \frac{P(Y=1|X)}{1-P(Y=1|X)} = \alpha + \beta X. \quad (2)$$

( $\alpha$ : slope;  $\beta$ : intercept)

The term  $\frac{P(Y=1|X)}{1-P(Y=1|X)}$  is the odds ratio between the probability of a “Y” ground improvement method and the probability of “Y” not being selected. Solving Eq. (2) for p; the probabilities will be evaluated based on equation (3):

$$p = \frac{1}{1+e^{-(\alpha+\beta X)}} \quad (3)$$

To utilize Eq. (3) for the current problem; the generated GEP models will be replaced by X in Eq. (3).

## 6. SUMMARY AND CONCLUSIONS

This research offers an innovative approach for predicting the most suitable ground improvement technique using GEP. The data used to build the GEP models was collected from 83 ground improvement projects in the UAE. Data collected from each project includes the following parameters: fine content (%),

groundwater level (m), depth of improvement (m), distance to close by structures (m), and the used ground improvement method. The models have been trained and then tested to make proper predictions.

One GEP model is developed for each technique; the user will be able to input the parameters as mentioned above in each model and select the technique with higher accuracy.

79.31% of data (65 data points) were used to train the model, while the remaining data is used to validate and test the model. The developed GEP models have an accuracy of 100% for the training dataset. The accuracy is ranging from 88.9% to 94.4% for the validation dataset.

R<sup>2</sup> values are ranging from 0.72 to 0.95 for the training dataset and 0.26 to 0.83 for the validation dataset. Therefore, model results show that GEP can perform accurately.

Results can be presented in different forms such as ETs, K-expression, and Matlab code.

In conclusion, GEP can serve as a proper method to solve engineering problems with complex mechanisms, such as the selection of ground improvement technique. The developed GEP model can be employed for selecting the appropriate ground improvement technique.

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## APPENDIX A

Table-1. A: Model Output in Karva Notation.

State (Ground Improvement Method)	KARVA NOTATION		
VC	<div>Gene 1</div> <div>c0 = -6.71404412123277 c1 = 7.86288033692435 c2 = 6.71636312788796 c3 = 5.23798333689383 c4 = -22.8934539914727 c5 = -5.47227393414106 c6 = -1.5792488861568 c7 = -7.84059663686026 c8 = -9.08333299863064 c9 = 6.52577288125248</div> <div>Gene 2</div> <div>c0 = 9.53672902615436 c1 = -5.25498214667196E-02 c2 = -7.85027619251076 c3 = -80.6232970061342 c4 = -6.33001145594958 c5 = 179.271261424909 c6 = 7.28080080568865 c7 = -6.12964262825404 c8 = 1.28506881923887 c9 = -9.41035984374523</div> <div>Gene 3</div> <div>c0 = -1.21372112186041 c1 = 7.56285687760552 c2 = -2.43629261146886 c3 = 1.2564470351268 c4 = -1.52623065889462 c5 = -2.85849231070589 c6 = 9.53000542405469 c7 = -9.63295085216834 c8 = -12.401397177156 c9 = -2.31571728584728</div> <div>X2.+LT2A.GOE2A.NOT.*.LT2C.Avg2.d0.c0.c2.d1.d0.c4.d3.d1.d3 + GOE2C.*.LT2C.LT2E.LT2C.LT2E.+Ln.d1.d0.d2.d1.d1.d3.d3.d3 + X2.GOE2G.GOE2G.+Sqrt.*.LT2E.d1.c6.c5.d1.c9.c8.d0.d1.c5</div>		
RIC	<div>Gene 1</div> <div>c0 = -4.54031643208106 c1 = 1.99575792718284 c2 = -5.94286935026093 c3 = -9.32554094058046 c4 = -1.88860506302072 c5 = 6.96205633716849 c6 = -11.0589631433581 c7 = 4.62909421063875 c8 = 3.71202822679157 c9 = -4.15387432477798</div> <div>Gene 2</div> <div>c0 = -1.9815668202765 c1 = -9.8040711691641 c2 = 9.75824882076479 c3 = 7.98638874477371 c4 = 2.86715166648366 c5 = 4.42426831873531 c6 = 8.32393871883297 c7 = 5.07751701406903 c8 = -5.42222357860042 c9 = 0.372285808893094</div> <div>Gene 3</div> <div>c0 = 9.47839681451036 c1 = 8.06329538865322 c2 = -1.26377147740104 c3 = -4.71236304818873 c4 = -9.55809198278756 c5 = -5.65477462080752 c6 = -1.3243507187109 c7 = -2.16345713675344 c8 = 4.4564296395764 c9 = -6.73197046651411</div> <div>GOE2E.GOE2E.Logi.3Rt.Atan.LT2C.-.3Rt.d2.c4.c7.c8.d1.d0.d3.d3.c6 + GOE2B.LT2G.LT2C.*.+.-+.GOE2B.d0.c4.d3.d0.c4.d3.d1.c6.d2 + GOE2B./Atan.Sqrt.+LT2A.*.LT2C.d1.d2.d1.c0.d0.c9.d0.d0.d3</div>		
SC	<div>Gene 1</div> <div>c0 = -5.34149754325999 c1 = -8.81591235084078 c2 = 8.89846542642524 c3 = 0.752229377117222 c4 = 3.15490327631928 c5 = 5.37705618457595 c6 = 6.24073000274667 c7 = -9.33408612323374 c8 = 9.09761403009763 c9 = -2.85622730185858</div> <div>Gene 2</div> <div>c0 = -6.98903686605426 c1 = 7.91779140598773 c2 = 4.5616016113773 c3 = -3.20398571733757 c4 = 8.00434457007181 c5 = 2.89111301004059 c6 = 10.0025971251564 c7 = 8.78470134172387 c8 = -6.00826837977233 c9 = 8.73958555864132</div> <div>Gene 3</div> <div>c0 = 4.64873195593127 c1 = 6.99906494499116 c2 = -7.73186437574389 c3 = 6.67348246711631 c4 = 0.315571459089938 c5 = 6.04663228247932 c6 = 2.69875179296243 c7 = 6.9997378460036 c8 = 4.62202826013977 c9 = -7.64996709939543E-02</div> <div>LT2G.+.*.GOE2C.GOE2A.GOE2B.LT2C.LT2A.c2.d3.d1.d1.d2.d1.d0.c8.d0 + -LT2G.+GOE2C.GOE2G.Sqrt.LT2A.GOE2G.d2.d0.d2.d3.d1.c7.d1.c4.d1 + -LT2C.GOE2G.c1.GOE2G.-LT2A.LT2A.d3.d0.d1.d1.d3.d2.c7.c3.d2</div>		



DR	<p>Gene 1</p> <p>c0 = -8.60537215432763 c1 = -0.56633198034608 c2 = 6.60756248664815 c3 = 0.881405072176275 c4 = 5.20061952574236 c5 = -4.88789712899258 c6 = -4.78831268959624 c7 = 5.56477191943452 c8 = -9.45100461590058 c9 = 3.54655598620563</p> <p>Gene 2</p> <p>c0 = -0.957726444957145 c1 = -1.98028647724845 c2 = 0.266956638560137 c3 = 2.63161107211524 c4 = 0.902432325205237 c5 = -7.43055048295542 c6 = -8.24499603579438 c7 = -11.4995605334635 c8 = 5.75988175603503 c9 = 4.47864293191251</p> <p>Gene 3</p> <p>c0 = 0.74422367015595 c1 = -8.08943647106521 c2 = -3.63936277352214 c3 = -7.68695108810645 c4 = 9.01398349172109 c5 = -8.90342417676321 c6 = -2.34717856379894 c7 = 6.65639210180975 c8 = 7.87530137028108 c9 = -6.81020538956877</p> <p>GOE2C.LT2C.*.GOE2E.*.-.LT2E.c7.c6.d0.d0.d1.c8.d3.d3.c0 + *.LT2G.LT2E+.LT2E.-.+*.c0.d1.d2.c6.d0.d2.c5.c9.d2 + LT2E.LT2G.d3.*.LT2E.+.-.+d2.c9.d1.c1.c4.d0.c3.c4.d1</p>
DC	<p>Gene 1</p> <p>c0 = -3.54268013550218 c1 = -8.0657368694113 c2 = -2.59864803003021 c3 = 5.6167666768395 c4 = 9.95178075502792 c5 = -1.66173284096805 c6 = -8.98416364024781 c7 = -6.29566331980346 c8 = -4.27228614154485 c9 = 2.20923269173254</p> <p>Gene 2</p> <p>c0 = -3.03942098842086 c1 = 8.73409222693564 c2 = -3.83817712668233 c3 = 7.86117130039369 c4 = 3.955504013184 c5 = 8.94589068269906 c6 = -3.01428333719901 c7 = -6.47999511703848 c8 = 7.61528366954558 c9 = 7.62959803972081</p> <p>Gene 3</p> <p>c0 = 6.56142765587329 c1 = -0.718710898159734 c2 = -6.77342548905911 c3 = 1.45359660634175 c4 = 3.09940407016205 c5 = -10.8157846017461 c6 = 4.08172310369366 c7 = 8.38373973815119 c8 = 7.23324610126041 c9 = 7.08230536820582</p> <p>Logi.LT2C.*.X2.Avg2.Avg2.+.-.d1.d3.d1.d0.c9.c5.c2.c5.d2 + 3Rt.LT2B.X2.LT2G.LT2G.LT2C.LT2C.LT2C.c9.d0.d1.c0.d0.d3.d1.d3.d1 + LT2G.LT2B.X4.LT2G.GOE2A.X2.+LT2G.d2.d1.c9.c5.d0.d2.c4.d1.c0</p>