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ON MODELING THE MOTION OF A STRONG SHOCK WAVE ALONG A PERFORATED PLANE SURFACE

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ABSTRACT

We consider a gasmotion behind the front of a strong flat shock wave propagating along a flat surface, which, starting from a certain point, becomes perforated. The solution of Euler system of equations is constructed by a small parameter method. The characteristic ratio of gas densities at the shock front is chosen as a small parameter. An approximate analytical solution of the problem is constructed taking into account the terms of the first approximation. It is assumed that the gas flow through the permeable boundary is proportional to the pressure drop across it, which allows replacing the solution of the problem with the solution of a shock wave diffraction problem at an angle greater than π. the structure of the flow in the perturbed region behind the diffracted shock wave is analyzed. The shape of the wave front is constructed for different values of determining flow parameters.

Keywords: shockwave diffraction, perforated surface, small parameter method.

INTRODUCTION

To construct approximate analytical solutions in problems with strong shock waves, the most often used method is a thin shock layer method (the “boundary layer” method) [1-3]. This method was proposed in the late 50-ies of the last century to solve two-dimensional problems (plane steady and unsteady one-dimensional). It is now successfully used to construct approximate analytical solutions for plane and spatial unsteady gas dynamics problems with strong shock waves. [4-6].

The method of a thin shock layer is based on the assumption, natural for such problems, that the gas density ratio ahead of the front of an intense shock wave and immediately behind it is small compared with the unit. When constructing the gas-dynamic problem solution, the flow parameters in the perturbed region near the front of a strong shock wave are represented as special series in powers of this parameter. The solution of the problem begins with finding the so-called limiting flow, which is the exact solution (it is sometimes called the “Newtonian” solution) of the gas dynamics equations, when the ratio of gas densities at the shock front tends to zero. The greater the contribution of the limiting solution of the problem, the fewer expansion terms need to be sought to achieve the desired accuracy of the desired approximate solution.

The good agreement between the results of calculations obtained using this method and the results of numerical calculations and experimental data allows us to conclude that this method can be applied for the approximate solution of gas dynamics problems with intense shock waves [7-9].

In this paper, we consider the motion of a gas behind the front of a strong flat shock wave propagating along a flat surface, which, starting from a certain point becomes perforated. In the framework of the thin shock layer method, an approximate analytical solution of the problem is constructed taking into account the terms of the first approximation. It is assumed that the gas flow through the permeable boundary is proportional to the pressure drop across it, which allows replacing the solution of the problem with the solution of the problem of diffraction of a shock wave at an angle greater thanπ.

FORMULATIONOFTHEPROBLEM

Consider the infinite rigid wall EON (Figure-1), which consists of an absolutely impenetrable beam EO and a permeable portion of infinite length ON. The permeability of the wall will be assumed low.

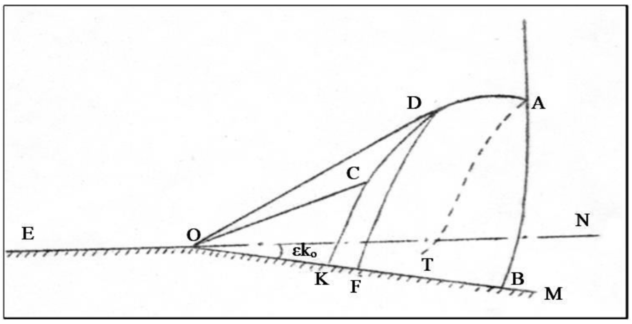

Figure-1. Flow pattern.

Let a strong plane shock wave, for which the characteristic ratio of gas densities at the front is much less than unity, propagates along a rectilinear rigid and impenetrable wall EO with velocity N_0. On the undisturbed front of the shock wave, the conditions of dynamic compatibility are fulfilled, which for strong shock waves can be approximately written as:

$$\frac{v_1}{N_0} = 1 - \varepsilon, \quad \frac{p_1}{\rho_0 N_0} = 1 - \varepsilon, \tag{1}$$
$$k = 1 + \frac{2\varepsilon}{1-\varepsilon}, \quad \frac{a_1^2}{N_0^2} = \varepsilon(1+\varepsilon)$$

At the time $t = 0$, the shock wave passes through point O and begins to move along the permeable portion of the rigid wall. ON.

The problem will be solved in the polar coordinate system. By virtue of the self-similarity of the problem (there is no characteristic linear dimension), the system of equations describing the flow of gas in the perturbed region can be written in the vector form:

$$
\begin{aligned}
&(\bar{v}-\bar{r})\frac{d\bar{v}}{d\bar{r}} = -\varepsilon\tau\, grad\, p\\
&(\bar{v}-\bar{r})grad\,\tau = \tau\, div\,\bar{v}\\
&(\bar{v}-\bar{r})grad\, i = \varepsilon\tau(\bar{v}-\bar{r})\, grad\, p \qquad (2)\\
&\varepsilon\tau = \frac{k-1}{k}\frac{i}{p}.
\end{aligned}
$$

Here $\bar{v}(v_r, v_\theta), p, \rho = \frac{1}{\varepsilon\tau}, i$ – is the velocity vector of the gas, pressure, density and enthalpy, referred to N_0 , $\rho_0 N_0^2$, $\rho_0 u N_0^2$ respectively;$\bar{r}$isradius vector in the plane of dimensionless self-similar variables;$v_r u v_\theta$are the radial and transverse components of the velocity vector; $k$is a ratio of the specific heats of gas, $\varepsilon = \frac{k-1}{k+1}$.

The boundary conditions for system (2) are set at the ODA boundary with a uniform flow, at the perturbed front of the shock wave AB and at the permeable wall section ON.

At the ODA boundary, by virtue of (1) we have:

$$
\begin{aligned}
&\tau = 1\,,\quad p = 1-\varepsilon\,,\quad i = \tfrac{1}{2}(1-\varepsilon^2)\,, \qquad (3)\\
&v_r = (1-\varepsilon)\cos\theta\,,\qquad v_\theta = -(1-\varepsilon)\sin\theta\,.
\end{aligned}
$$

On OD equality is true

$$
sin^2\theta = \frac{\varepsilon(1+\varepsilon)}{(1-\varepsilon)^2}\,,\quad cos^2\theta = \frac{1-3\varepsilon}{(1-\varepsilon)^2}.
$$

Let us suppose that the gas flow through the permeable boundary is proportional to the pressure drop across the wall. Then the boundary condition on the permeable wall can be represented as:

$$
v_\theta = -\,\varepsilon\, k_0. \qquad (4)
$$

where k_0 is a dimensionless coefficient characterizing the degree of permeability of the boundaryON.

Let us set the front equation of the perturbed shock wave in the form$r = r(\theta)$. Then the dynamic compatibility conditions will take the form:

$$
\begin{aligned}
&v_r = r(1-\varepsilon\tau)cos^2\delta,\quad v_\theta = -r(1-\varepsilon\tau)\sin\delta\cos\delta, \quad (5)\\
&p = r^2(1-\varepsilon\tau)cos^2\delta,\qquad i = \frac{1}{2}r^2(1-\varepsilon^2\tau^2)cos^2\delta,
\end{aligned}
$$

where $tg\delta = \frac{1}{r}\frac{d}{d\theta}r(\theta)$.

Conditions (5) are written up to terms of order a_0^2/N_0^2 ,wherea_0 is the speed of sound in a gas at rest ahead of the shock wave front [10].

To determine the flow in the perturbed region, one should find a solution to the system of equations (2) that satisfies the boundary conditions (3) - (5). The boundary of the considered area (with the exception of the ON section) must be determined in the process of constructing the problem solution.

The problem (2) - (5) is equivalent to the problem of the diffraction of a strong shock wave near the angle $\pi + \varepsilon k_0$with the vertex at the point O. We place the beginning of the polar coordinate system at point O, and direct the polar axis along the fictitious rigid and already impenetrable wall OM (Figure-1). Then the gas parameters along the horizontal permeable wall will be equal to the values of the corresponding values in the problem of the diffraction on the beam$\theta = \varepsilon k_0$. Note that the curved shock wave should be perpendicular to the fictitious wall at B.

SOLUTION

We will solve the problem using the thin shock layer method. A distinctive feature of the problem under consideration is the presence of regions in which the asymptotic expansions in the parameter ε of dependent and independent variables have significantly different nature.

a) Consider first the region where $1 - r = O(1)$, i.e. $r \ll 1$. The effect of the permeable wall ONon the flow behind the shock-wave front downstream will be limited by the straight-line characteristic of the first family OD. To the right of the OD characteristic, the flow pattern should be similar to the rarefaction flow that occurs when a larger angle flows aroundπ. If we assume that for all the main flow parameters, the dependence on the angle θ is more significant than on the distance to the origin of coordinates in the plane of self-similar variables, then in the first approximation from (2) we obtain a system of ordinary differential equations. Integrating and defining the arbitrary constants from conditions (3) we obtain

$$
v_r = 1 - \frac{3}{2}\varepsilon + O(\varepsilon)\,,\ v_\theta = -\sqrt{\varepsilon}\left(1 + \frac{1}{2}\varepsilon + O(\varepsilon)\right). \quad (6)
$$

In the area between the OC and the OB wall, there is a uniform flow with parameters:

$$
\begin{aligned}
&v_r = (1-\varepsilon)\cos\theta,\quad v_\theta = -(1-\varepsilon)\sin\theta\\
&\mathrm{p} = 1 - \sqrt{\varepsilon}\mathrm{k}_0 + \mathrm{O}(\varepsilon)\,,\ \tau = 1 + \sqrt{\varepsilon}\mathrm{k}_0 + \mathrm{O}(\varepsilon)\,.
\end{aligned}
$$

The areas of these currents are bounded to the right by the characteristic of the second family DK.

System of equations (2) to the right of the DK characteristic is hyperbolic down to the parabolicity line DF, whose equations are determined by the relation $(v_r - r)^2 + v_\theta^2 = a^2$,where $a$is the local speed of sound. It is easy to verify that in the approximation under consideration, the gas flow in the DFK region can be described by the formulas (6), (7).

b) In the intermediate zone, in the region of ellipticityof system (2), to the right of the DF parabolicity line, we move on to the new independent variables$\sigma$and$\varphi$by the formulas:

$$
r = 1 + \sqrt{\varepsilon}\,\sigma\ ,\ \theta = \sqrt{\varepsilon}\,\varphi\,. \qquad (8)
$$

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Given the orders of the desired functions, the solution of the problem in this area will be sought in the form:

$$v_r = 1 + \varepsilon u_1, \quad v_\theta = -\sqrt{\varepsilon}\,\varphi + \varepsilon v_1\,, \qquad (9)$$
$$p = 1 + \sqrt{\varepsilon} p_1\,,\ \ \tau = 1 + \sqrt{\varepsilon}\tau_1\,,\ \ i = \frac{1}{2} + \varepsilon i_1.$$

As a result, we obtain the system of differential equations in partial derivatives, which, putting$\sigma = R\cos\lambda$,$\varphi = R\sin\lambda$, we reduce to a second order equation for the functionp_1. Note that in the considered approximation, we can assume that$R = 1$ on DF . Integrating the resulting equation, we obtain an expression for the first approximation correctionp_1. Using the solution obtained from the remaining equations, it is easy to determine the velocity field in the considered region, which is characterized by the fact that the radius vector differs from unity by an amount of the order of$\sqrt{\varepsilon}$ $(1 - r = O(\sqrt{\varepsilon}))$.

c) We proceed to consider the flow in the region where$1 - r = O(\varepsilon)$. In this region immediately adjacent to the disturbed shock front AB, we set:

$$r = 1 + \varepsilon\,\zeta\ \ ,\ \ \theta = \sqrt{\varepsilon}\,\varphi\,. \qquad (10)$$

Then, taking into account the orders of the desired functions, the solution of the problem in this area will be sought in the form:

$$v_r = 1 + \varepsilon u_2 + \cdots, \quad v_\theta = \sqrt{\varepsilon} v_2 + \cdots,$$
$$p = 1 + \varepsilon p_2 + \cdots\ ,\ \ \tau = 1 + \varepsilon\tau_2 + \cdots,$$
$$i = \tfrac{1}{2} + \varepsilon i_2 + \cdots. \qquad (11)$$

Substituting (10), (11) into the original system of equations (2), we obtain the system of nonlinear equations describing the flow in the considered domain:

$$(u_2 - \zeta)\frac{\partial u_2}{\partial\zeta} + v_2\frac{\partial u_2}{\partial\varphi} - v_2^2 = -\tau\,\frac{\partial p_2}{\partial\zeta},$$
$$(u_2 - \zeta)\tfrac{\partial v_2}{\partial\zeta} + v_2\tfrac{\partial v_2}{\partial\varphi} - v_2 = 0\,, \qquad (12)$$
$$\frac{\partial u_2}{\partial\zeta} + \frac{\partial v_2}{\partial\varphi} + 1 = 0$$
$$(u_2 - \zeta)\frac{\partial i_2}{\partial\zeta} + v_2\frac{\partial i_2}{\partial\varphi} = 0$$

The third equation of system (12) allows us to enter a function $F(\varphi, \zeta)$ such, that:

$$F_\varphi = -u_2\,,\ \ F_\zeta = v_2 + \varphi\,. \qquad (13)$$

Then, using the second equation of system (12), we obtain the nonlinear second-order equation in partial derivatives for determining the function $F(\varphi, \zeta)$:

$$(\zeta + F_\varphi)F_{\zeta\zeta} + (\varphi - F_\zeta)F_{\zeta\varphi} = 0\,, \qquad (14)$$

which, using the Euler-Ampere transform,

$$F = \Phi(q, \varphi) + q\zeta, \qquad (15)$$

where $\Phi(q, \varphi)$is a new desired function, $q$is a parameter, and, $q = v_2 + \varphi = F_\zeta$, is reduced to:

$$[\Phi_\varphi - \Phi_q + (\varphi - q)\Phi_{\varphi q}]F_{\zeta\zeta} = 0$$

From here we have two cases. In the first case, we have the Euler-Darboux equation, whose solution with two arbitrary functions allows us to obtain a parametric representation of the velocity field in the considered region:

$$v_2 = q - \varphi, \qquad (16)$$
$$u_2 = -[L_2(\varphi) + H_2(q) - (\varphi - q)L_2'(\varphi)]\ ,$$
$$\zeta = -[L_2(\varphi) + H_2(q) + (\varphi - q)H_2'(\varphi)]\ ,$$

where $H_2(q) = L_1'(q)$, $L_2(\varphi) = H_1'(\varphi)$ are arbitrary functions.

In the second case, we have:

$$F(\varphi, \zeta) = \zeta f_1(\varphi) + f_2(\varphi)\,, \qquad (17)$$

or on based (13):

$$v_2 = f_1(\varphi) - \varphi,\ \ u_2 = -\zeta f_1'(\varphi) - f_2'\,. \qquad (18)$$

From the second equation of system (12) it follows that, in addition to the solution of (16), the system of equations describing the velocity field in the considered region has a special solution of the form:

$$v_2 = -\varphi,\ \ u_2 = f(\varphi)\,, \qquad (19)$$

or$v_2 = $ c, $u_2 = f(\varphi) - \zeta$, where $f_2'(\varphi) = f(\varphi)$isan arbitrary function, a$c = const$. Note that these solutions are not contained in the general solution (16) under any functions. $H_1(q)$и$H_2(\varphi)$.

The general solution of the form (16) describes the vortex flow of gas, and the special solution (19) is valid for the isentropic flow. Solutions of this type can be conjugated along an entropy characteristic AT.

d) We will look for the shape of the front of a perturbed shock wave AB in the form $\zeta = \zeta_2(\varphi)$, then from (5) we have:

$$v_2 = -\zeta_2'(\varphi),\ \ u_2 = \zeta_2(\varphi) - {\zeta_2'}^2(\varphi) - 1. \qquad (20)$$

Substituting (20) into (16) we arrive at a system of three equations for determining the functions $H_2(q), \zeta_2(\varphi)$, $q_2(\varphi)$, where$q_2(\varphi)$is the equation of the wave front AB on the plane of variables (q, φ).

The system, thus obtained, is reduced to one equation for the function $\zeta_2(\varphi)$:

$$\zeta_2''({\zeta_2'}^2 - 1) + {\zeta_2'}^2 L_2''(\varphi) = -1$$

After determining $L''_2(\varphi)$ from the condition of splicing solution (19) with a solution in the intermediate region to determine the shape of the shock wave front AB, we obtain the equation:

$$Z_2' = 1 + \frac{1}{\pi\sqrt{\varepsilon}} \frac{Z_2^2}{1-Z_2^2} \frac{\sqrt{1-\varphi^2}}{\varphi^4}\, ln\left(1 + 2k_0\sqrt{\varepsilon}\frac{\varphi^2}{1-\varphi^2}\right), \qquad (21)$$

where$Z_2(\varphi) = \zeta_2'(\varphi)$.

From the conditions for the passage of a shock wave through point $A\left(\zeta_A = \frac{1}{2},\ \varphi_A = 1\right)$ and the impenetrability of the tangent vector to the front at this point, we have:

$$\zeta_2(1) = \frac{1}{2}, \quad \zeta_2'(1) = 1.$$

At point B, the condition of perpendicularity of the shock wave to the wall gives$\zeta_2'(0) = 0$. This condition cannot be satisfied for all values of the parameter k_0, since for $k_0 > \frac{\pi}{8}$, equation (21) has no solutions passing through the point$(0, 0)$. Restricting ourselves to the case of $k_0 \leq \frac{\pi}{8}$, for equation (21) we have two boundary conditions:

$$Z_2(0) = 0, \quad Z_2(1) = 1.$$

However, it is easy to show that equation (21) does not have a continuous solution that satisfies both boundary conditions, despite the fact that points (0, 0) and (1, 1) are special points of this equation. In such a situation, it is necessary to assume the presence of a break in the front of a diffracted shock wave AB at a certain point. Thus, in the vicinity of the break point, an irregular (Mach) configuration of shock waves with a reflected wave, a tangential discontinuity, and a Mach wave must occur.

The shape of the front of the diffracted shock wave was determined by numerical integration of equation (21). Two solutions of this equation were constructed with initial conditions $Z_2(1) = 1$, $Z_4(0) = 0$respectively. In this case, the departure from the singular points was carried out with the help of approximate analytical solutions.

$$Z_2(\varphi) = 1 + \frac{2}{\sqrt{3\pi\sqrt{2\varepsilon}}}(1-\varphi)^{3/4}\sqrt{-\ln(1-\varphi)},$$

$$Z_4(\varphi) = A\varphi, \quad A = \frac{\pi}{4k_0}\left(1 - \sqrt{1 - \frac{8k_0}{\pi}}\right),$$

where $Z_4(0) = \zeta_4'(0) = 0, \zeta(\varphi)$ is the Mach wave equation QB, and the position of the breakpoint Q is determined from the relation $(Z_2 \cdot Z_4)_Q = 1$.

The figures below show the calculation results. They are grouped in such a way that it is easy to evaluate the influence of the main parameters of the problem on the change in the shape of the shock wave.

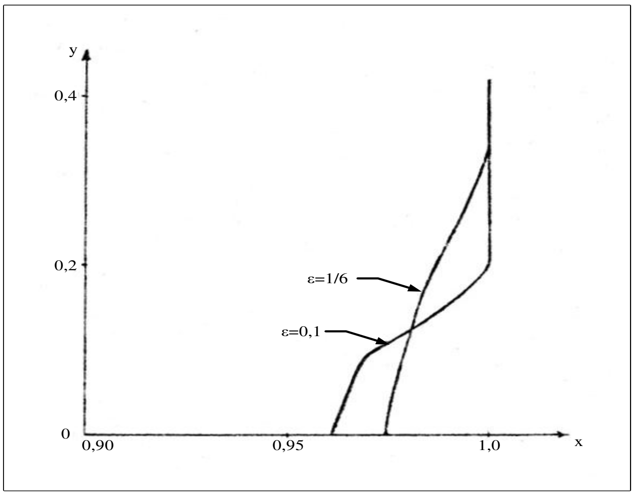

Figure-2. The shape of thedisturbed shock wave front. $(k_0 = 2{,}0)$.

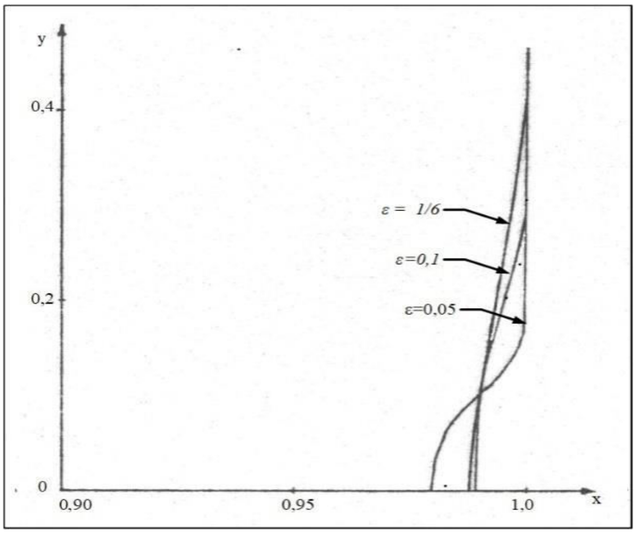

Figure-3. The shape of thedisturbed shock wave front. $(k_0 = 1{,}0)$.

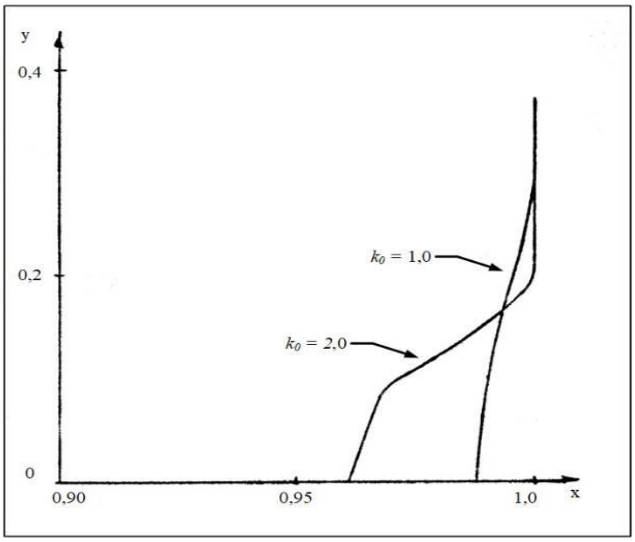

Figure-4. The shape of thedisturbed shock wave front . $(\varepsilon = 0{,}1)$.

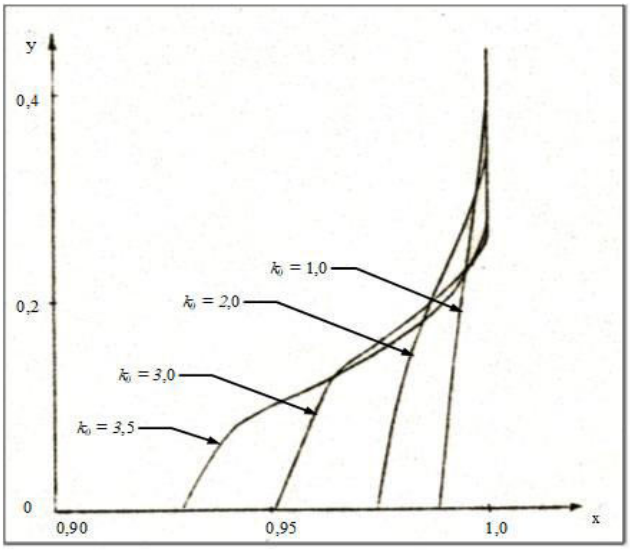

Figure-5. The shape of thedisturbed shock wave front. ($\varepsilon = 1/6$).

The calculations have shown that the position of the break point relative to the wall is very sensitive to small changes in the angle of deflection of the wall at point O, and hence, to the coefficient of permeability of the wallk_0.

CONCLUSIONS

Note that the presence of a triple point and a decrease in its distance from point B with an increase in the wall break angle is in qualitative agreement with experimental data on ashock wave diffraction [11].

The results obtained in this work can be useful in solving more complex non-stationary gas dynamics problems with intense shock waves, such as gas moving behind a strong shock wave front, taking into account dust, non-equilibrium, etc. The presented solution can be used as an intermediate asymptotic when building a solution that more adequately describes the gas flow (taking into account the finite Mach number) at considerable distances from the shock wave front, as well as at the initial design stage to identify specific points and areas of gas flow in the disturbed region.

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