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SOLVING THE STOCHASTIC POINT-KINETIC EQUATIONS USING MILSTEIN'S METHOD FROM THE ITÔ LEMMA

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ABSTRACT

In this work we deduce Milstein's iterative scheme of Itô-Taylor expansion from the Itô Lemma. This scheme allows solving numerically stochastic differential equations. Particularly, we apply this scheme to the stochastic point kinetics model, which is more adequate to describe the stochastic behavior of the neutron population inside a nuclear reactor. The results are obtained using numerical experiments, considering one and six groups of neutron precursors with constant reactivities, different time steps, initial conditions and Wiener processes. These results are compared with values reported on the literature and with the deterministic point kinetic model. With the comparison we show that the Milstein model is efficient in the numerical solution to the stochastic point kinetic equation.

Keywords: Itô-Taylor expansion, milstein scheme, stochastic point kinetics, nuclear reactors, neutron population.

INTRODUCTION

The stochastic nature inside a nuclear reactor is very significative when there is the need to start or shut down the reactor. In the subcritical state, the movement of the control rods and the coolant force the stochastic effects to be taken into account [1].

Different methods have solved the stochastic point kinetic equations numerically, among which we can name: Monte Carlo and Stochastic Piecewise Constant Approximation [2], Euler-Maruyama and Taylor 1.5 [3], Fractional Stochastic Point-Kinetics [4], Simplificated Stocachastic Point Kinetics Equations [5], Analytical Exponential Model [6], Double Diagonalization-Descomposition Method [7], Efficient Stochastic Model [8], Implicit Euler Maruyama [9].

In this work we propose to implement Milstein's scheme, which will be deduced in the next section from the Itô-Taylor expansion starting from Itô's lemma. We seek to improve the mean value approximation of the neutron density and the precursor concentration in the stochastic point kinetics, making use of an approximation of the variance matrix.

THEORETICAL CONSIDERATIONS

Itô-Taylor expansion.

The differential for an arbitrary function with a random variable is defined by the Itô lemma [10] of the following form:

$$df(x_t) = \left[a\frac{\partial}{\partial x}f(x_t) + \frac{1}{2}b^2\frac{\partial^2}{\partial x^2}f(x_t)\right]dt + b\frac{\partial}{\partial x}f(x_t)d\omega_t \quad (1)$$

Where $a = a(x_t)$ is the drift coefficient, $b = b(x_t)$ is the diffusion coefficient and ω_t is Wiener's process, characterized for being a stochastic process of continuous time with statistically independent and stationary increases that are not differentiable in time.

Equation (1) can be written in the integral form:

$$f(x_t) = f(x_{t_0}) + \int_{t_0}^{t} L^0 f(x_{\tau_1})\, d\tau_1 + \int_{t_0}^{t} L^1 f(x_{\tau_1})\, d\omega_{\tau_1} \quad (2)$$

Where the operators L^0 and L^1 are defined as:

$$L^0 = a\frac{\partial}{\partial x} + \frac{1}{2}b^2\frac{\partial^2}{\partial x^2}$$
$$L^1 = b\frac{\partial}{\partial x} \quad (3)$$

The second integral on equation (2) is not a conventional Riemann or Lebesgue integral, instead it is a stochastic Itô integral. For$f(x_t) = x_t$, then $L^0 f(x_{\tau_t}) = a$ and $L^1 f(x_{\tau_t}) = b$, it is possible to obtain x_t of the form:

$$x_t = x_{t_0} + \int_{t_0}^{t} a(\tau_1)\, d\tau_1 + \int_{t_0}^{t} b(\tau_1)\, d\omega_{\tau_1} \quad (4)$$

To approximate the result of the integrals, we apply Itô's lemma according to equation (1) for $f(x_t) = a(x_t)$ and $f(x_t) = b(x_t)$. The results obtained are replaced on equation (4) such that we obtain the following expression:

$$x_t = x_{t_0} + a(t_0)\int_{t_0}^{t} d\tau_1 + b(t_o)\int_{t_0}^{t} d\omega_{\tau_1} + R \quad (5)$$

where R are the remaining terms:

$$R = \int_{t_0}^{t}\int_{t_0}^{\tau_2} L^0 a\, d\tau_1 d\tau_2 + \int_{t_0}^{t}\int_{t_0}^{\tau_2} L^1 a\, d\tau_1 d\omega_{\tau_2} + \int_{t_0}^{t}\int_{t_0}^{\tau_2} L^0 b\, d\omega_{\tau_1} d\tau_2 + \int_{t_0}^{t}\int_{t_0}^{\tau_2} L^1 b\, d\omega_{\tau_1} d\omega_{\tau_2} \quad (6)$$

Similarly, for $f(x_t) = L^1 b(x_t)$, we obtain:

$$x_t = x_{t_0} + a(t_0)\int_{t_0}^{t} d\tau_1 + b(t_o)\int_{t_0}^{t} d\omega_{\tau_1} + L^1 b(t_o)\int_{t_0}^{t}\int_{t_0}^{\tau_2} d\omega_{\tau_1} d\omega_{\tau_2} + \tilde{R} \quad (7)$$

Where the new remaining terms are:

$$\widetilde{R} = \int_{t_0}^{t}\int_{t_0}^{\tau_2} L^0 a\, d\tau_1 d\tau_2 + \int_{t_0}^{t}\int_{t_0}^{\tau_2} L^1 a\, d\tau_1 d\omega_{\tau_2} + \int_{t_0}^{t}\int_{t_0}^{\tau_2} L^0 b\, d\omega_{\tau_1} d\tau_2 + \int_{t_0}^{t}\int_{t_0}^{\tau_2}\int_{t_0}^{\tau_3} L^0 L^1 b\, d\omega_{\tau_1} d\omega_{\tau_2} d\tau_3 + \int_{t_0}^{t}\int_{t_0}^{\tau_2}\int_{t_0}^{\tau_3} L^1 L^1 b\, d\omega_{\tau_1} d\omega_{\tau_2} d\omega_{\tau_3} \quad (8)$$

Equation (7) can be rewritten in terms of Itô's integrals:

$$x_t = x_{t_0} + a(t_0) I_{(0)} + b(t_0) I_{(1)} + L^1 b(t_0) I_{(1,1)} + \widetilde{R} \quad (9)$$

A more extensive expression for the Itô-Taylor expansion as a function of the Itô integrals can be obtain [10]. The Itô-Taylor expansion is the base for the different iterative schemes that give numerical solution to the stochastic differential equations, which is not possible to achieve analytically, similarly to the Taylor expansion.

Truncating the expansion given by equation (9) in its fourth term, the Milstein scheme is obtained, which has a convergence order of 1.0. However, to reach to this point, it is critical to solve the $I_{(0)}$, $I_{(1)}$, $I_{(1,1)}$ integrals. Integrals $I_{(0)}$ *and* $I_{(1)}$ are intuitive, but $I_{(1,1)}$ requires some work, but it has the following solution:

$$I_{(1,1)} = \frac{1}{2}\left[\left(w_t - w_{t_0}\right)^2 - (t - t_0)\right] \quad (10)$$

Therefore, Milstein's scheme written in its discrete form is simply:

$$x_{n+1} = x_n + a_n h + b_n \Delta\omega + \frac{1}{2} b_n \frac{\partial}{\partial x_n} b_n [(\Delta\omega)^2 - h] \quad (11)$$

Where

$$h = t_{n+1} - t_n \quad (12)$$

$$\Delta\omega = \omega_{n+1} - \omega_n \quad (13)$$

Equation (13) represents a Wiener process, with the characteristic that: $\omega_{t=0} = 0$ with probability 1 and $\omega_t - \omega_s \sim \aleph(0, t-s)$ for $0 \le s \le t$, where $\aleph(\mu, \sigma^2)$ denotes the normal distribution with expected value μ and variance σ^2[11].

In the following section we present a stochastic model for the point kinetics and we show a numerical solution for it, using the Milstein scheme given by equation (11).

STOCHASTIC POINT KINETICS

Stochastic point kinetics was presented [2] and consists in a set of $m+1$ stochastic differential equations of Itô which are non-linear and have strong variable coupling. This system represents the temporal evolution of the neutron and precursor population for a given reactivity. The matrix expression of this system is:

$$d|P(t)\rangle = \left[\widehat{A}|P(t)\rangle + |Q(t)\rangle\right]dt + \widehat{B}^{1/2}\, d|\omega(t)\rangle \quad (14)$$

Where

$$|P(t)\rangle = \begin{bmatrix} n(t) \\ C_1(t) \\ C_2(t) \\ \vdots \\ C_m(t) \end{bmatrix} \quad (15)$$

$$\widehat{A} = \begin{bmatrix} \frac{\rho(t)-\beta}{\Lambda} & \lambda_1 & \lambda_2 & \cdots & \lambda_m \\ \frac{\beta_1}{\Lambda} & -\lambda_1 & 0 & \cdots & 0 \\ \frac{\beta_2}{\Lambda} & 0 & -\lambda_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\beta_m}{\Lambda} & 0 & 0 & \cdots & -\lambda_m \end{bmatrix} \quad (16)$$

$$|Q(t)\rangle = \begin{bmatrix} q(t) \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (17)$$

$$\widehat{B}^{1/2} = \begin{bmatrix} \sqrt{\xi} & 0 & 0 & \ldots & 0 \\ 0 & \sqrt{r_1} & 0 & \ldots & 0 \\ 0 & 0 & \sqrt{r_2} & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & \sqrt{r_m} \end{bmatrix} \quad (18)$$

$$|\omega(t)\rangle = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \vdots \\ \omega_{m+1} \end{bmatrix} \quad (19)$$

Where n is the neutron density, C_m is the precursor concentration of the $m-th$ group, ρ is the reactivity which gives information about the neutron production, β is the total fraction of the delayed neutron precursors, Λ is the median time of neutron generation, λ_m is the decay constant for the $m-th$ class of the delayed neutron precursors, q is the magnitude of the external source of neutrons, ω_m are the Wiener processes that are characterized for being stochastic processes of continuous time and independent stationary increments.

The elements of the $\hat{B}^{1/2}$ matrix are described by:

$$\xi = \gamma\, n(t) + \sum_{i=1}^{m} \lambda_i C_i(t) + q(t) \quad (20)$$

$$\gamma = \frac{-1-\rho+\upsilon(1-\beta)^2+2\beta}{\Lambda} \quad (21)$$

$$r_m = \frac{\nu\, \beta_m^2}{\Lambda} n(t) + \lambda_m C_m(t) \quad (22)$$

Where υ is the average number of generated neutrons by each fission event.

For simplicity we choose to work with the square root of the variance matrix on this work, as presented by [5] given by equation (18), since this simplifies the implementation of the Milstein scheme.

If we use the iterative Milstein scheme, given by equation (11), to the stochastic point kinetics, we obtain the system of equations:

$$\begin{aligned} n_{k+1} &= n_k + \left[\frac{\rho_k-\beta}{\Lambda} n_k + \sum_{i=1}^{m} \lambda_i C_{i\,k} + q_k\right] h + \sqrt{\xi_k}\,\Delta\omega_1 + \frac{1}{4}\gamma[(\Delta\omega_1)^2 - h] \\ C_{m\,k+1} &= C_{m\,k} + \left[\frac{\beta_m}{\Lambda} n_k - \lambda_m C_{m\,k}\right] h + \sqrt{r_{m\,k}}\Delta\omega_{m+1} + \frac{1}{4}\lambda_m[(\Delta\omega_{m+1})^2 - h] \end{aligned} \quad (23)$$

Equation system (23) provides an approximated numerical solution to the stochastic point kinetics. However, equation (23) can be re-written in the matrix form:

$$P_{k+1} = P_k + [A_k P_k + Q_k - C\,]\Delta + \left[B_k^{1/2} + C^* W_k\right]\Delta\omega_k \quad (24)$$

Where

$$P_k = \begin{bmatrix} n_k \\ C_{1\,k} \\ C_{2\,k} \\ \vdots \\ C_{m\,k} \end{bmatrix} \quad (25)$$

$$A_k = \begin{bmatrix} \frac{\rho_k-\beta}{\Lambda} & \lambda_1 & \lambda_2 & \cdots & \lambda_m \\ \frac{\beta_1}{\Lambda} & -\lambda_1 & 0 & \cdots & 0 \\ \frac{\beta_2}{\Lambda} & 0 & -\lambda_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\beta_m}{\Lambda} & 0 & 0 & \cdots & -\lambda_m \end{bmatrix} \quad (26)$$

$$Q_k = \begin{bmatrix} q_k \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (27)$$

$$C = \frac{1}{4}\begin{bmatrix} \gamma \\ \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_m \end{bmatrix} \quad (28)$$

$$C^* = \frac{1}{4}\begin{bmatrix} \gamma & 0 & 0 & \ldots & 0 \\ 0 & \lambda_1 & 0 & \ldots & 0 \\ 0 & 0 & \lambda_2 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & \lambda_m \end{bmatrix} \quad (29)$$

$$W_k = \begin{bmatrix} \Delta\omega_1 & 0 & 0 & \ldots & 0 \\ 0 & \Delta\omega_2 & 0 & \ldots & 0 \\ 0 & 0 & \Delta\omega_3 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & \Delta\omega_m \end{bmatrix} \quad (30)$$

$$B_k^{1/2} = \begin{bmatrix} \sqrt{\xi_k} & 0 & 0 & \ldots & 0 \\ 0 & \sqrt{r_{1\,k}} & 0 & \ldots & 0 \\ 0 & 0 & \sqrt{r_{2\,k}} & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & \sqrt{r_{m\,k}} \end{bmatrix} \quad (31)$$

Equation (24) is the numerical solution to the stochastic point kinetics that is developed on this work. This scheme will be tested using constant reactivities and diverse initial conditions, as well as one and six precursor groups. This is $m = 1$ and $m = 6$ respectively. The results are presented in the following section.

RESULTS

In this section we present four simulations with constant reactivities, from which two are realized for a precursor group and two are for six precursor groups. The numerical simulations were realized, we implemented a variety of initial conditions, time steps, precursor groups and Wiener processes. The reproduction of the results of these stochastics processes is achieved using $2^{31} - 1$ as seed. The results obtained on this work are compared on median values and standard deviation with the schemes reported on the literature, such as: SPCA (Stochastic piecewise Constant Approximation) y MC (Monte Carlo) [2], EM (Euler Maruyama) and T 1.5 (Taylor 1.5) [3], FSNPK (Fractional stochastic point kinetic equations) [4], SSPK (Simplificated Stocachastic Point Kinetics Equations) [5], AEM (Analytical Exponential Model) [6], Double DDM (Double Diagonalization-Descomposition Method) [7], ESM (Efficient Stochastic Model) [8], IEM (Implicit Euler Maruyama) [9]. The values reported on the literature have been written with four significative figures. In cases where the number of significative figures is less, we have added zeros. On the other hand, the results obtained through the stochastic model are compared with the median values with the deterministic formulation, because the last does not have values for the standard deviation in the tables, said information will be stated as *non-applicable* (NA). The values of the deterministic model have been calculated using the Euler method in its implicit version. The time steps for this scheme are the same as for the proposed scheme.

The first two simulations were executed for one precursor group and reactivities of $\rho = -{}^{1}/_{3}\,pcm$ and $\rho = 300\,pcm$. In the first we used the parameters: neutron generation time $\Lambda = {}^{2}/_{3}\,(s)$, decay constant $\lambda_1 = 0.1(s^{-1})$, delayed neutron total precursor fraction $\beta = 0.05$, average of generated neutrons for each fission

event $\upsilon = 2.5$, and neutron external source $q(t) = 200$. The initial conditions where $n(0) = 400$ and $C(0) = 300$. This simulation was realized with 40 iterations in a time interval of $[0,2]$ seconds, making use of 5000 Wiener processes. However, in the second simulation we modeled a TRIGA reactor with the parameters: neutron generation time $\Lambda = 0.0001\ (s)$, decay constant $\lambda_1 = 0.077(seg^{-1})$, delayed neutron total precursor fraction $\beta = 0.0079$, average of generated neutrons for each fission event $\upsilon = 2.432$, neutron external source $q(t) = 10000$, and initial conditions $n(0) = 0$, $C(0) = 0$. This simulation is realized with 100 iterations for a time interval of $[0,0.1]$ seconds and using 10000 Wiener processes. For this last simulation we have increased the time step by a factor of 10^4 with respect to the reported value in [5]. The results obtained with the proposed scheme for the first simulation are shown on Table-1. These results are satisfactory, because in the median value of the neutron density, it has a good approximation to the deterministic value, making it better than most of the schemes reported in the literature, and only being beaten by *Monte Carlo* and *Implicit Euler-Maruyama**. The standard deviation values agree with the reported values. It is worth noting that the *Efficient stochastic model* and *Implicit Euler-Maruyama** schemes have modifications to the square root of the variance matrix and the median value matrix respectively, which allows them to reduce the variance in the random variables. The results for the TRIGA reactor are presented on Table-2 and show how the Milstein scheme provides good approximations in both the median values as in the standard deviation respect to the reported values and the deterministic model. On Figure-1 we show the neutron density for the deterministic model and the stochastic model. The values of the stochastic model fluctuate around the values of the deterministic model, which is a behavior also observable on Figure-2. The small fluctuations make the stochastic model for the point kinetics able to describe in a more real way the behavior of the neutron population inside a nuclear reactor.

Table-1. Result comparison of the proposed scheme (M), the schemes reported in the literature and the deterministic model (DM).

Method	$\boldsymbol{E[n(2s)]}$	$\boldsymbol{\sigma[n(2s)]}$	$\boldsymbol{E[C(2s)]}$	$\boldsymbol{\sigma[C(2s)]}$
SPCA	395.3200	29.4110	300.6700	8.3564
MC	400.0300	27.3110	300.0000	7.8073
EM	412.2300	34.3910	315.9600	8.2656
T 1.5	412.1000	34.5190	315.9300	8.3158
FSNPK	412.2300	34.3918	315.9690	8.2656
AEM	396.2800	31.2120	300.4200	7.9576
Double DDM	402.3500	28.6100	305.8400	7.9240
ESM	396.6200	0.9199	300.3900	0.0016
IEM	399.7100	31.4310	299.7700	7.9411
IEM*	399.9874	0.5439	299.8730	6.8405
M	400.1790	31.8185	299.9753	7.9038
DM	400.0000	NA	300.0000	NA

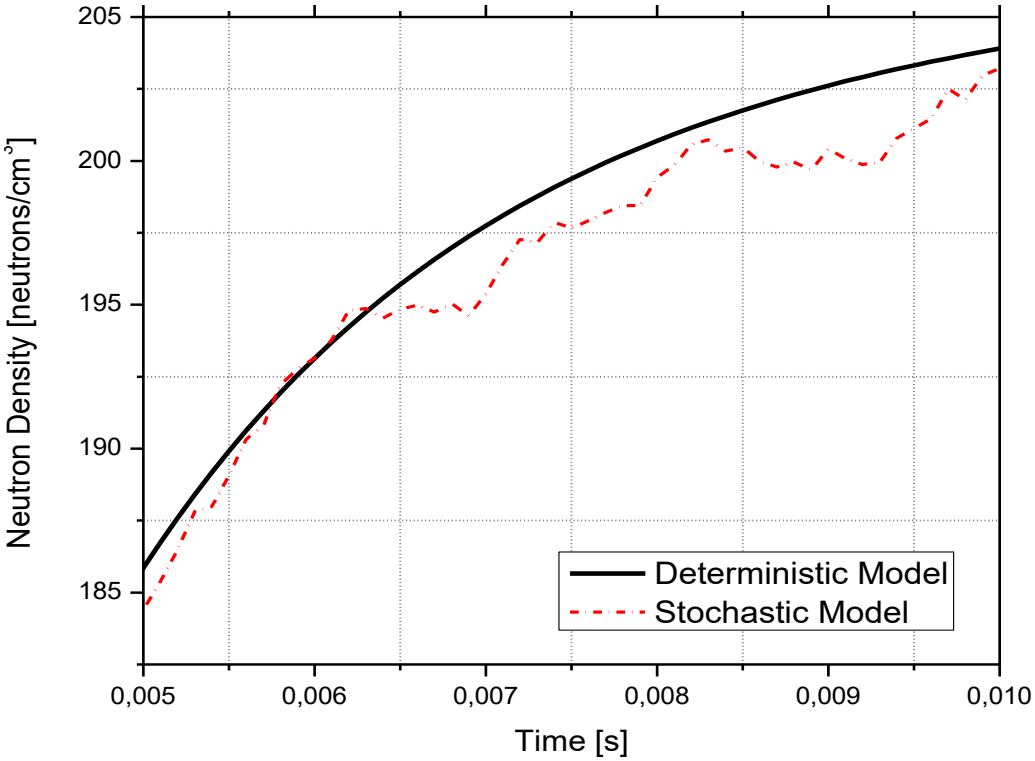

Figure-1. Neutron density by the stochastic model and the deterministic model (DM) for a TRIGA reactor with reactivity $\rho = 300\ pcm$.

Table-2. Result comparison of the proposed scheme (M), the schemes reported in the literature and the deterministic model (DM) for a TRIGA reactor.

Method	$E[n(0.1s)]$	$\sigma[n(0.1s)]$	$E[C(0.1s)]$	$\sigma[C(0.1s)]$
SPCA	204.5200	174.0300	1.2940×10^3	620.6800
MC	199.1500	152.6300	1.2545×10^3	613.9400
SSPK	208.1400	174.3000	1.2932×10^3	622.1200
M	203.2214	166.8899	1.2757×10^3	613.2044
DM	203.9027	NA	1.2861×10^3	NA

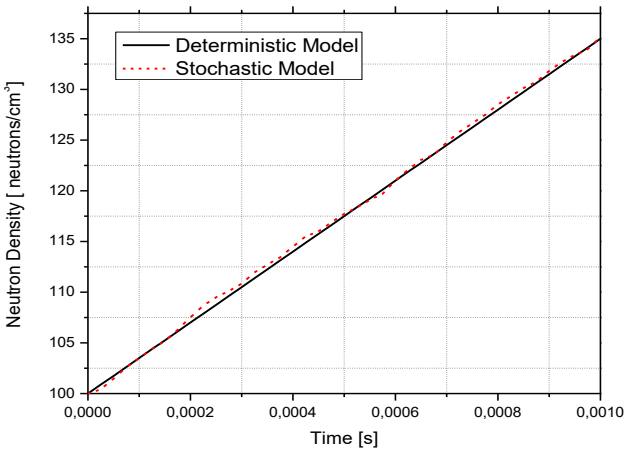

Figure-2. Neutron density by the stochastic model and the deterministic model (DM) for a reactivity $\rho = 700\ pcm$.

The last two simulations are executed for six precursor groups with subcritical and critical reactivities of$\rho = 300\ pcm$ and $\rho = 700\ pcm$ respectively, and the following parameters: neutron generation time $\Lambda = 0.00002(s)$, decay constants $\lambda_i = [0.0127, 0.0317, 0.1150, 0.3110\ 1.4000, 3.8700](s^{-1})$, delayed neutron total precursor fraction $\beta = 0.007$, delayed neutron precursor fraction of the i-th group $\beta_i = [0.000266, 0.001491, 0.001316, 0.002849, 0.000896, 0.000182]$, average of generated neutrons by each fission event $\upsilon = 2.5$, external neutron source $q(t) = 0$, and initial conditions $[n(0), C_1(t), C_2(t), \dots, C_6(t)]^T = 100\left[1, \frac{\beta_1}{\lambda_1 \Lambda}, \frac{\beta_2}{\lambda_2 \Lambda}, \dots, \frac{\beta_6}{\lambda_6 \Lambda}\right]^T$, employing 5000 Wiener processes and 40 iterations.

The time interval in which the nuclear reactor is simulated with subcritical and critical reactivity is $[0,0.1]$

and[0,0.001] seconds respectively. Tables 3 and 4 show the results obtained by the proposed scheme (M) and the reported in the literature along with the values for the deterministic model (DM). In these is clear that the Milstein scheme is excellent for the calculation of the expected values in comparison with the methods that do not have a reduction in the variance. It is to expect that when implementing a method with reduction of the variance to this scheme, it will be the one that approximates the most to the deterministic model values.

Table-3. Result comparison of the proposed scheme (M), the schemes reported in the literature and the deterministic model (DM) for subcritical reactivity.

Method	$E[n(0.1s)]$	$\sigma[n(0.1s)]$	$E[C(0.1s)]$	$\sigma[C(0.1s)]$
SPCA	186.3100	164.1600	$4.4910x10^5$	$1.9172x10^3$
MC	183.0400	168.7900	$4.4780x10^5$	$1.4957x10^3$
EM	208.6000	255.9500	$4.4980x10^5$	$1.2333x10^3$
T 1.5	199.4080	168.5470	$4.4970x10^5$	$1.2188x10^3$
FSNPK	208.5990	255.9540	$4.4981\ x10^5$	$1.2333\ x10^3$
SSPK	184.8000	186.9600	$4.4890x10^5$	$0.9826x10^3$
AEM	186.3000	164.1400	$4.4900x10^5$	$1.9119x10^3$
Double DDM	187.0500	167.8300	$4.4880x10^5$	$1.4756x10^3$
ESM	179.9300	10.5550	$4.4890x10^5$	$0.0947x10^3$
IEM	178.2700	165.1100	$4.4886x10^5$	$1.2536x10^3$
IEM*	179.9461	0.2178	$4.4888x10^5$	$0.0604x10^3$
M	180.5300	227.1684	$4.4881\ x10^5$	$2.0703\ x10^3$
DM	179.9485	NA	$4.4888x10^5$	NA

Table-4. Result comparison of the proposed scheme (M), the schemes reported in the literature and the deterministic model (DM) for critical reactivity.

Method	$E[n(0.001s)]$	$\sigma[n(0.001s)]$	$E[C(0.001s)]$	$\sigma[C(0.001s)]$
SPCA	134.5500	91.2420	$4.4640x10^5$	19.4440
MC	135.6700	93.3760	$4.4640x10^5$	16.2260
EM	139.5680	92.0420	$4.4630x10^5$	6.0710
T 1.5	139.5690	92.0470	$4.4630x10^5$	18.3370
AEM	134.5400	91.2340	$4.4640x10^5$	19.2350
Double DDM	135.8600	93.2100	$4.4630x10^5$	17.8450
ESM	134.9600	6.8527	$4.4640x10^5$	2.5290
IEM	134.0200	93.2730	$4.4636x10^5$	18.7760
IEM*	134.9218	5.9661	$4.4636x10^5$	6.0686
M	135.1546	92.3398	$4.4636\ x10^5$	18.0927
DM	135.0010	NA	$4.4636x10^5$	NA

In this section we presented simulations for one and six groups of precursors with different initial conditions, time steps and Wiener processes for constant reactivities. The results obtained by Milstein have been compared in expected value and standard deviation with other methods reported in the literature. Also, we compared with the expected value of the deterministic model for the point kinetics. This comparison determines the efficiency of the proposed scheme for the numerical solution of the stochastic point kinetics.

CONCLUSIONS

This work deduces the Itô-Taylor expansion successfully from the Itô lemma. The iterative Milstein scheme has been presented and then applied to the stochastic point kinetics using an approximated form of the square root of the variance matrix. The results obtained

by the proposed scheme have been analyzed and compared with schemes reported in the literature. This comparison showed that the Milstein scheme has a good approximation to the median values and the standard deviation for the neutron density and the precursor concentration, being possible to determinate the mean value of the neutron density.

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