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THEORETICAL MODELING OF FLOW FOR FRUSTUM - SHAPED CERAMIC WATER FILTERSVIA DISC-SHAPED CERAMIC WATER FILTERS

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ABSTRACT

In the past two decades, research on household technologies has been on the increase. Ceramic water filters are simple and appropriate technology proven to have capability to provide improved drinking water especially in rural and peri-urban communities. These filters can be fabricated into various shapes with careful consideration of their mechanical robustness. This paper gives an overview of various flow models in estimating flow rate for ceramic water filters, in particular frustum-shaped ceramic water filters. Analytical approach to estimate the quantity of water considering the geometrical shapes; frustum and cylindrical shapes of the Ceramic water filters are adopted. The analytical expressions deduced for the flows are applicable on the assumption that the materials used in the fabrication of the filters are the same, thus same material properties (chemical and thermal properties) and porosimetry characteristics. Also, the comparative flow equation is largely dependent on dimensions of the ceramic water filters. The scholarly articles on flow through ceramic water filters for frustum-shaped filter are theoretically explored.

Keywords: ceramic water filters, flow rate, hydraulic conductivity, disk-shaped filters; frustum-shaped filters.

1. INTRODUCTION

About 2.1 billion people do not have access to potable water and of this number; 844 million do not have even a basic drinking water service. This is alarming to the extent that a little over one quarter a billion people have to spend approximately half an hour per trip collecting water from sources outside the home. In addition, 159 million people still drink contaminated water from surface water sources, such as streams or lakes (Organization and UNICEF 2017). These statistics highlight the importance of household water treatment technologies.

The main household water treatment technologies enumerated are disinfection [this method includes boiling, solar pasteurization, chlorination, UV irradiation with lamps and solar disinfection (SODIS)], adsorption (the adsorption materials used include activated alumina/clay and activated carbon) and filtration (slow sand filter technologies, ceramic filter, cloth fiber, and bio-sand) (Yakub, Plappally *et al*. 2012) (Annan, Mustapha *et al.* 2014), Ceramic water filters (CWFs) are usually made from local materials such as rice husk, clay, starch/flour, water, and sawdust. The required shape of the filter is formed using a hydraulic press. The freshly made filter 'green filter' is then air-dried to remove inter water molecules, which duration is dependent on the climatic condition and fired in a kiln to a temperature of about 700 ^{o}C - 850 ^{o}C. At temperatures between 300 ^{o}C - 400 ^{o}C, the combustible materials are burnt out becoming porous.
The fired filter could be soaked in silver nanoparticles or silver nitrate solution to enhance their antimicrobial efficiency. The life-span of a filter is manly dependent on the type of water it filters. However, a typical ceramic water filter lifetime is 2-3 years. It is advised that regular cleaning of filters be undertaken using appropriate method. Details of processing of materials and the fabrication of CWFs is well described in Annan *et al*, 2014 (Annan, Mustapha *et al*. 2014).

The World Health Organization (WHO) recommends at least 2 liters of safe drinking water per day (Supply and Programme 2014). This implies that water filtration systems should be capable of providing enough water for the members of households. This can be theoretically estimated by developing a mathematical expression from Darcy's equation (if using a membrane or particle filtration system). The deductions are also to be noted to be shape dependent. For instance, work done by Schweitzer *et al*. 2013 (Schweitzer, Cunningham *et al.* 2013), Yakub *et al.* 2013 (Yakub, Plappally *et al.* 2012), van Halem 2006(Van Halem 2006), and Fahlin 200 Fahlin (2003) are mainly on frustum shaped CWF's.
With regard toparabolic-shaped filters, authors cited only recent works by Schweitzer *et al*. 2013 (Schweitzer, Cunningham *et al*. 2013), and Miller 2010 (Miller 2010). Most of the equations are fundamentally developed from Darcy's equation (Whitaker 1986). Table-1 below gives the historical perspective of published work on modelling associated with ceramic water filters. Table-2 gives the relevant parameters for each model for ceramic water filter flow. The main goal of this paper is to review flow through ceramic water filters in frustum - or disc shape and thereby model through frustum shaped CWFs through disks CWFs as surrogate.

Figure-1 gives a schematic diagram of the geometrical shapes of the ceramic water filters. It is important to deduce the rate at which the level or height of the water changes with time, thus dh/dt (where h, is height or level of water and t is time). In here, we have deduced the dh/dt for the three models under consideration:

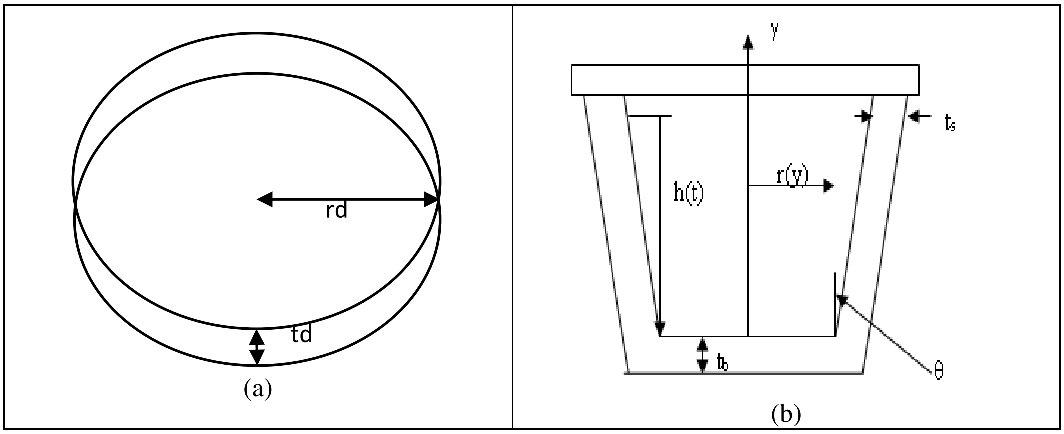

Figure-1. Schematic diagram of the geometrical shapes of the ceramic water filters (a) Disk - shaped and (b) Frustum-shaped.

Table-1. A comparative table for different flow models of ceramic filter and their assumptions.

Assumption Category	**Eriksen (2001)**	**Lantagne (2001)**	**Fahlin (2003)**	**van Halem (2006)**	**Miller (2010)**	**Schweitzer (2013)**	**Yakub (2013)**
Geometry	Cylindrical	Cylindrical	Frustum	Frustum	Parabolic	Parabolic & Frustum	Frustum
Conductivity	Bottom and side flow conductivity equal	Bottom and side flow conductivity equal	Bottom and side conductivities different due to different forces during pressing	Bottom and side conductivity different	Bottom and side conductivity different	Bottom and side conductivity different	Bottom and side conductivity different
Head	Same head across bottom and sides	Same head across bottom and sides	Head on bottom and sides is different	Head on bottom and sides is different	Head on bottom and sides is different	Head on bottom and sides is different	Head on bottom and sides is different
Overall flow	Batch process (fully filled then emptied)	Batch process (fully filled then emptied)	Continuous Flow	Continuous flow, constant head on bottom	Continuous flow, constant head on bottom	Instantaneous h(t) solved to find Q(t)	Instantaneous h(t) solved to find Q(t)
Sidewall flow	Flowthrough sides by integrating Darcy's Law	flow through sides by integrating Darcy's Law	No integration; continuous flow	Integrating flow from 0 ->h_w	As a function of slant height, dl	Included as a function of radius	function of slant height and radius

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Table-2. Relevant parameters for each model for ceramic water filter flow.

Variable	Eriksen (2001)	Lantagne (2001)	Fahlin (2003) low t	Fahlin (2003) high t	van Halem (2006)	Miller (2010)	Schweitzer (2013)	Yakub (2013)
Tortuosity	1	2	4	19	--	--	--	11 - 60
t_{silver} or c	0.1 mm	4 mm	2.5 mm	10 mm	--	--	--	--
t_b	10 mm	10 mm	14.5 mm	14.5 mm	15 +/- 3 mm	2 cm	1.92 cm (paraboloid), 1.42 cm (frustum)	15 mm
T_{min}	25 min	25 min	25 min	25 min	--	--	--	--
max height, H	24 cm	24 cm	20.34 cm	20.34cm	--	19.5 cm	19.5 cm	24 cm
flow time, T	1 hour	8 hours	--	--	--	--	--	--
inner filter diameter, D	0.2 m	0.2 m	0.2 m	0.2 m	0.2 m +/- 0.2 m (at top)	0.119 m (at top)	0.23 m (parabolic), 0.195 (frustum)	0.183 m
k_{max}	0.00001 m/h	0.0004 m/h	0.00171 m/hr	0.00325 m/hr	--	--	--	--
k_{actual}	0.03 m/h	0.004 m/h	0.00104 m/hr	0.00287 m/hr	0.00047 to 0.00016 m/hr	0.00234 m/hr	0.000281 m/hr	from K = 1 - 50 mD; 0.035 to 1.7 m/hr
k_{actual} / k_{max}	3000	10	0.61	0.89	--	--	--	--

In this paper, we focus our studies on three main models: Schweitzer *et al*. 2013 (Schweitzer, Cunningham *et al*. 2013), Yakub *et al*. 2013 (Yakub, Plappally *et al.* 2012) and van Halem 2006 (Van Halem 2006).

2. FLOW MODELS FOR FRUSTUM-SHAPED FILTERS

The main determinant to the flow rate in a filtering element is the pore size and pore distribution. In ceramic water filters the pore sizes are determined by the combustible materials in the mix prior firing. It is significant to reiterate that there are three main types of pores: dead-end pores, isolated pores and interconnected pores (Figure-1). The inter-connected pores are the ones that mainly contribute to flow through ceramic water filters. Table-1 gives some physical characterization form the three locations.

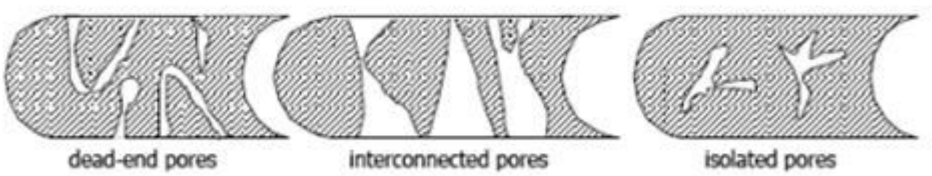

Figure .1 Types of pores [Xiaolong, 2005]

2.1 Flow Model And Van Halem 2006 (Van Halem 2006)

Van Halem considered the flow characteristics of frustum-shaped ceramic water filters from three different locations: Ghana, Cambodia and Nicaragua. Table-3 shows the values for porosity via Direct Method and Mercury Intrusion Porosimetry. It is clear from Table-2 that for all filters from all locations, mercury Intrusion porosimeter gives higher porosity values compared to that of the direct testing method. Also, the ceramic water filters impregnated with colloidal silver was found to have lower porosity compared with ceramic water filters with 'no colloidal silver'. It appears therefore that the 'silver particles' in the colloid attached itself to the surfaces of the filter element. Another interesting research may be the need to measure the force of adhesion between the agents involved: Clay-silver nano-particles; silver nano-particles - Bacteria; and Clay - Bacteria.

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Table-3. Porosity and Density measurement for filters (Van Halem).

Country of Origin	Direct Method Porosity (%)	Direct Method Density(g/mL)	Mercury Intrusion Porosimetry Porosity (%)	Mercury Intrusion Porosimetry Density (g/ml)
Cambodia	36	1.28	43	1.35
Ghana	38	1.22	39	1.27
Nicaragua	30	1.34	34	1.30
Nicaragua (no silver)	35	1.30	44	1.30

The flow equation for Van Halem, 2006 was deduced from Darcy's equation(Miller 2010). When developing the model, filters from the following countries were considered; Cambodia, Nicaragua and Ghana. The flow rate for base of filter element (Q_b) and side element (Q_s) is given respectively as:

$$Q_b = k\frac{\pi r_2{}^2 h_w}{t_b} \tag{1}$$

Whereas the flow through the filter side or wall, denoted as Q_s is given by

$$Q_s = 2\pi\frac{k}{t_s}\left[\frac{(r_1-r_2)}{6L}h_w{}^3 + \frac{1}{2}r_2 h_w{}^2\right] \tag{2}$$

Total flow rate equation equals Qw + Qb

$$Q(m^3/s) = \frac{\pi k h_w}{t_s}\left[\frac{(r_1-r_2)}{3L}h_w{}^2 + r_2 h_w\right] + \frac{\pi k}{t_b}[r_2{}^2 h_w] \tag{3}$$

Where the symbols, t_s- filter wall thickness, t_b- filter bottom thickness, r_1- radius at the filter's top, r_2- radius at the filter bottom, L is slanght height, and$k$ hydraulic conductivity (m/s). The symbols have the same meaning through the document except where redefined.

Therefore according to Van Halem (Van Halem 2006), the total flow rate though a frustum shaped filter is given as:

$$\mathrm{Q(t)} = \frac{\pi k h}{t_s}\left\{\frac{(r_1-r_o)h^2}{3L} + r_o h\right\} + \frac{k\pi r_o{}^2 h}{t_b} \tag{4}$$

The expression below depicts the variation in volume with time and is equal to the volumetric:

$$\frac{dV(t)}{dt} = Q(t) \tag{5}$$

$$\frac{dV(t)}{dt} = \pi[r_z(t)]^2\frac{dh}{dt} \tag{6}$$

The variation in radius as the water level decreases is depicted in the Van Halem's model as:

$$r_z(t) = \frac{(r_1-r_o)z}{L} + r_o \tag{7}$$

Combining equations (6) into (7) gives

$$\frac{dV(t)}{dt} = \pi\left[\frac{(r_1-r_o)z}{L} + r_o\right]^2\frac{dh}{dt} \tag{8}$$

Simplifying equation (8) and re-arranging gives

$$\frac{dh}{dt} = \frac{Q(t)}{\pi\left[\frac{(r_1-r_o)^2 z^2}{L^2} + \frac{2r_o(r_1-r_o)z}{L} + r_o{}^2\right]} \tag{9}$$

Equation (14) which represents the rate of change of height is then obtained by combining equations (8) and (9)

$$\frac{dh}{dt} = kh\frac{\left[\frac{(r_1-r_o)h^2}{3t_s L} + \frac{r_o h}{t_s} + \frac{r_o{}^2}{t_b}\right]}{\left[\frac{(r_1-r_o)^2 h^2}{L^2} + \frac{2r_o(r_1-r_o)h}{L} + r_o{}^2\right]} \tag{10}$$

Since equation (10) is a differential equation, it can be solved using Euler's method or numerically. The following boundary conditions are required in the numerical solution:

$h(0) = h_{max}$
$h(\infty) = 0$

Where the maximum height, h_{max} is given by

$$h\frac{3t_s L}{2(r_1-r_0)}\left[-\frac{r_0}{t_s} + \sqrt{\left(\frac{r_0}{t_s}\right)^2 - \frac{4}{3}\cdot\left(\frac{r_1-r_0}{t_s L}\right)\cdot\frac{r_0{}^2}{t_b}}\right]_{max}$$

2.2 Flow modeling: Yakub *et al*. 2012(Yakub, Plappally *et al.* 2012) and Schweitzer *et al*. 2013

This model considers flow through frustum-shaped CWF. The major difference between Yakub *et al* and Schweitzer *et al* has to do with the uniformity of thickness (that is $t=t_b=t_w$). The flow rate expression for the bottom is the same and given in equation (4). Equations (5), and (6) are flow rate through filter walls and total flow rate of the frustum CWFs ceramic water filters respectively, as deduced in Yakub *et al*, 2012 Yakub, Plappally *et al.* (2012) model (Modified by Annan *et al*, 2014 (Annan, Mustapha *et al*. 2014)).

$$Q_b = \frac{K}{\mu}\frac{\pi r_o{}^2 \rho g h(t)}{t_b} \tag{11}$$

$$Q_s = \frac{\pi K \rho g}{t_s \mu}\left[r_o(h(t))^2 + \frac{\tan\theta(h(t))^3}{3}\right] \tag{12}$$

$$Q = \frac{K}{\mu}\pi\rho g h(t)\left[\frac{r_o{}^2}{t_b} + \frac{r_o(h(t))}{t_s} + \frac{(h(t))^2}{3t_s}\tan\theta\right] \tag{13}$$

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Where, *h(t)* is water-head varying with time. Schweitzer *et al*. 2013 (Supply and Programme 2014) equation (with assumed uniform thickness) for flow rate of the frustum shaped ceramic water filter is as below:

$$Q=\frac{\pi k h(t)}{t}\left[r_o^2+r_o h(t)+\frac{1}{3}\tan\theta\,(h(t))^2\right] \quad (14)$$

Adopting similar deductions as in section 2.1, the rate at with the water level varies with time could be deduced as in equation (11).

$$\frac{dh(t)}{dt}=-\frac{kh}{d}\frac{{}^{1}\!/_{3}\tan\varphi h^2+r_o h+r_o^2}{\tan^2\varphi h^2+2r_o\tan\varphi h+r_o^2} \quad (15)$$

Solution could be obtained with the initial condition described in equation below

$$h(0)=h_{max}$$

Where

$$h\frac{3}{2_0}\tan(\theta)\left[-1+\sqrt{1-\frac{4}{3}\cdot\tan(\theta)}\right]_{max}$$

$$Q(t)=\pi kh\left[\frac{r_o^2}{t_b}+\frac{r_o h}{t_s}+\frac{h^2\tan\varphi}{3t_s}\right] \quad (16)$$

$$r_z=r_o+z\tan\varphi \quad (17)$$

Combining (9), (10), (16) and (17) gives

$$\frac{dh}{dt}=\frac{Q(t)}{\pi[r_z(t)]^2}=\frac{\pi kh\left[\frac{r_o^2}{t_b}+\frac{r_o h}{t_s}+\frac{h^2\tan\varphi}{3t_s}\right]}{\pi[r_o+h\tan\varphi]^2}$$

$$\frac{dh}{dt}=kh\frac{\left[\frac{r_o^2}{t_b}+\frac{r_o h}{t_s}+\frac{h^2\tan\varphi}{3t_s}\right]}{[r_o^2+2r_o h\tan\varphi+\tan^2\varphi h^2]} \quad (18)$$

3. MODELLING: FRUSTUM-AND DISK-SHAPED CWFS

3.1 Van Halem 2006 [9]

$$Q_f=\frac{\pi kh}{t_s}\left\{\frac{(r_1-r_o)h^2}{3L}+r_o h\right\}+\frac{k\pi r_o^2 h}{t_b} \quad (19)$$

In equation (19), the top and bottom radii of frustum-shaped CWFs are represented by r_1 and r_o, respectively. The thickness of the bottom of the frustum shaped CWF and the thickness of the disk-shaped CWF are represented with t_b and t_d, respectively.

The expression for the flowrate of the disk-shaped CWF is presented in equation (20) below:

$$Q_d=\frac{k_d\pi r_d^2 h}{t_d} \quad (20)$$

Where, thickness of disk, $t_d = t_s$, thickness of side, r_d is the radius of the disk, re-arranging equation (20)

$$\frac{Q_d}{r_d^2}=\frac{\pi k_d h}{t_d} \quad (21)$$

Re-arranging (20) and putting in equation (19)

$$Q_f=\frac{Q_d}{r_d^2}\left\{\frac{(r_1-r_o)h^2}{3L}+r_o h\right\}+\frac{Q_d r_o^2 t_d}{r_d^2 t_b} \quad (22)$$

$$\frac{Q_f}{Q_d}=\frac{1}{r_o^2}\left[\frac{(r_1-r_o)h^2}{3L}+r_o h+\frac{r_o^2 t_d}{t_b}\right] \quad (23)$$

Where equation (23) gives the ratio of the flow rate of the frustum-shaped filter to the disk filter.

3.2 Flow and Schweitzer *et al*. 2013 [8]

The expression of flow rate in this model is as below:

$$Q_f=\frac{\pi kh}{t}\left[r_o^2+r_o^2 h+\frac{1}{3}h^2\tan\varphi\right] \quad (24)$$

Where the symbols are defined as, t_s the thickness of side (which is assumed to be same as thickness of bottom), hydraulic conductivity, $k_s = k_b = k$; r_o is the radius of the bottom of the frustum-shaped filter. Putting equation (24) into equation (28) and this gives us the expression for the ratio of flow rates, equation (25).

$$\frac{Q_f}{Q_d}=\frac{1}{r_d^2}\left[r_o^2+r_o h+\frac{1}{3}h^2\tan\varphi\right] \quad (25)$$

3.3 Flow and Yakub *et al*, 2012 (Yakub, Plappally *et al*. 2012) (Annan, Mustapha *et al*. 2014))

The comparative flow expression for disk- and frustum-ceramic water filtersisdeducedas:

$$\frac{Q_f}{Q_d}=\left(\frac{t_d}{r_d^2}\right)\left[\frac{r_o^2}{t_b}+\frac{r_o h}{t_s}+\frac{h^2\tan\varphi}{3t_s}\right] \quad (26)$$

The three selected models - van Halem, Schweitzer, and Yakub models which predicts the flow rate through frustum-shaped CWFs, the equations (23), (25) and (26) can be used to predict flow rate of corresponding disk-shaped CWF, respectively. The volume of water can be deduced from the flow equations. Van Halem's model deduced the volume expression as given in equation (27). Schweitzer *et al*, and Yakub *et al*, 2012 had same expression to compute the volume as given in equation (32).

$$V(t)=\frac{\pi(r_1-r_o)^2h^3}{3L^2}+\frac{\pi(r_1-r_o)h^2 r_o}{L}+\pi r_o^2 h \quad (27)$$

$$V(t)=\pi r_o^2[h_o-h(t)]+\pi r_o\tan\varphi\,[h_o-h(t)]^2+\frac{\pi}{3}\tan^2\varphi\,[h_o-h(t)]^3 \quad (28)$$

4. IMPLICATIONS

This research has significant implications to be considered. First of all, research output show that flow through a ceramic water filter can be inferred using different geometrical shape with the assumptions.

Environmental Engineers, Materials Engineers and researchers have a new framework to compute the amount of water for any geometrical shape of ceramic water filter with just the knowledge of fore option. Computational analysis could be developed to easily simulate the equations in this proposed model via Matlab or likewise softwares, thereby making it easier for supervisors and factory workers to produce reliable flow dependent ceramic water filters.

As a follow-up from the three models, the capture of experimental flow may not be the true total discharge trend of the filter and such further statistical reference is future work to perfect each model and ultimately the comparative expressions. The initial flow rates are easier to estimate but the initial flow rate data do not capture whole range to determine trends in the flow over the period of discharge. Statistics inform us that the range of flow values to capture true trend is to consider minimum of at least 20 consecutive readings. However, these require more detailed analyses of flow data that are probably well beyond the capabilities of most ceramic water filter factory engineers. Hence, simple software is needed to enable engineers to establish the variability in the effective permeabilities for efficient applications in quality control.

5. CONCLUSIONS

The quantity of water from a filtration system has to be enough for people in the household or community. An expedient approach to estimate the quantity via flow rate or permeability has been deduced and proposed in this paper. We have also given a platform for experimental start-up for various geometrical shape ceramic water filters. The authors are aware of the differences in clay and combustible material properties in various geographical locations and do we recommend consistence in such endeavor. Indeed, the implications of this result are many but primary with regard to providing guidance for the materials or Environmental Engineer in design, processing and fabrication. Researchers can explore the statistical flow relationship for any two geometrical shapes.

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