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MATRIX ANALYSIS OF PLANE TRUSSES BY SUBSTRUCTURING

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ABSTRACT

A typical case of a plane truss [1] is analyzed using a substructure analysis through the stiffness method proposed by Przemieniecki with the purpose of showing its advantages in engineering calculation when this strategy is used [2]. The results obtained are compared with the Ansys simulation software, in order to make important conclusions about the complexity reduction of the problem, and reduction of the cost when using computational techniques, since it is a topic of interest to structural designers in the research area. The substructure analysis using a finite elements software, such as Ansys, prove to have significant advantages on calculation used in research, since it allows to determine large structures using software that come with less features (such as the student version), in which structural division is possible, with the purpose of solving simultaneously several models from each divided part; hence, saving time and money is possible, since there is not a necessity for having full versions that come with more features for calculation.

Keywords: substructure, super element, przemieniecki, Ansys.

1. INTRODUCTION

The substructure analysis using the stiffness matrix method includes an initial analysis of each substructure separately, in which the movement of the nodes that share a common boundary with an adjacent substructure is fixed; these boundaries are then relaxed simultaneously and their actual boundary displacements are determined from the equation of equilibrium of forces, and each substructure is analyzed separately, taking into account the specified displacements and loading [2]. The size of the matrixes found through the substructure analysis for each part, are usually of lower order than the entire structure matrix; hence, the determination of the displacements of the common nodes implies a less amount of unknown variables [1]-[10].

The large structure analysis with substructure techniques using the stiffness method has great computational advantages, as a result, it has gained popularity in the area of current advanced software used for structural analysis and finite elements, in fact, the substructure analysis performed by Ansys® is a method that groups a set of finite elements inside a superelement; hence, the required calculation time and the solution of large complex problems is reduced. The performance of the superelements from the substructure is then defined with a small number of degrees of freedom called “master nodes”, and the solution of the model of the entire system is obtained through the use of an equilibrium and compatibility practice along the boundaries of the superelements. This reduction of the Ansys® model provides an efficient solution for large assemblies without losing accuracy.

The indicated methodology for substructure analysis is reliable when it is compared with finite element formulations. The mathematical model proposed was checked using Ansys®, in order to verify the accuracy of the method. Ansys® allows the user to define an adapted substructure analysis.

2. METHODS

2.1 Substructure analysis using the stiffness method proposed by Przemieniecki [1], [8]

The matrix equation of the displacements using the stiffness method is shown in equation (1).

$$\{F\} = [K]\{u\} \tag{1}$$

Where $[K]$ is the stiffness matrix of the system and $\{u\}$ is the displacement vector, and $\{F\}$ is the applied forces. The equation (1) can be generally described as $\{\bar{F}\} = [K]\{u\}$ where $\{\bar{F}\}$ is the effective forces vector.

When the structure is subdivided into substructures, and internal boundaries are added, the equation system (1) can be rewritten as shown in equation (2).

$$\begin{Bmatrix} \bar{F}_f \\ \bar{F}_i \end{Bmatrix} = \begin{bmatrix} k_{ff} & k_{fi} \\ k_{if} & k_{ii} \end{bmatrix} \begin{Bmatrix} \{u_f\} \\ \{u_i\} \end{Bmatrix} \tag{2}$$

Where $\{u_f\}$ is the displacement vector of the common boundaries between substructures and $\{u_i\}$ is the vector of uncommon internal displacements; hence, the displacements take place on the internal nodes of each substructure separately. The effective forces vectors are $\{\bar{F}_f\}$ and $\{\bar{F}_i\}$.

The total displacements of the structure are determined by the superposition of two vectors that satisfy the equation (3).

$$\{u\} = \{u^\alpha\} + \{u^\beta\} \tag{3}$$

As shown in equation (3), $\{u^\alpha\}$ is the displacement vector related to $\{\bar{F}_i\}$ when $\{u_f\} = 0$, and $\{u^\beta\}$ is the displacement adjustment $\{u^\alpha\}$, in order to allow boundary displacements $\{u_f\}$ when $\{\bar{F}_i\} = 0$. Hence,

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the equation (3) can be rewritten as shown in equation (4), where $\{u_f^\alpha\} = 0$ by definition.

$$\{u\} = \begin{Bmatrix} u_f \\ u_i \end{Bmatrix} = \underbrace{\begin{Bmatrix} u_f^\alpha \\ u_i^\alpha \end{Bmatrix}}_{\substack{restricted \\ boundaries}} + \underbrace{\begin{Bmatrix} u_f^\beta \\ u_i^\alpha \end{Bmatrix}}_{\substack{correction\ released \\ boundaries}} \tag{4}$$

An equal analysis is done for effective force vectors as shown in equation (5), where $\{\overline{F_i^\alpha}\} = \{\overline{F_i}\}$, and $\{\overline{F_i^\beta}\} = 0$ by definition.

$$\{\overline{\mathrm{F}}\} = \{\overline{\mathrm{F}}^\alpha\} + \{\overline{\mathrm{F}}^\beta\} \tag{5}$$

$$\{\overline{\mathrm{F}}\} = \begin{Bmatrix} \overline{\mathrm{F}}_\mathrm{f} \\ \overline{\mathrm{F}}_\mathrm{i} \end{Bmatrix} = \underbrace{\begin{Bmatrix} \overline{\mathrm{F}}_\mathrm{f}^\alpha \\ \overline{\mathrm{F}}_\mathrm{i}^\alpha \end{Bmatrix}}_{\substack{\text{fixed} \\ \text{boundaries}}} + \underbrace{\begin{Bmatrix} \overline{\mathrm{F}}_\mathrm{f}^\beta \\ \overline{\mathrm{F}}_\mathrm{i}^\beta \end{Bmatrix}}_{\substack{\text{adjustment of fixed} \\ \text{boundaries}}}$$

(6)

When the boundaries of the superstructure are fixed, the vectors $\{u_i^\alpha\}$ and $\{\overline{F_f^\alpha}\}$, proposed by Przemieniecki [8], are determined as shown in (7) and (8).

$$\{\mathrm{u}_\mathrm{i}^\alpha\} = [\mathrm{k}_\mathrm{ii}]^{-1}\{\overline{\mathrm{F}}_\mathrm{i}\} \tag{7}$$

$$\{\overline{\mathrm{F}}_\mathrm{f}^\alpha\} = [\mathrm{k}_{\mathrm{f}_\mathrm{i}}][\mathrm{k}_\mathrm{ii}]^{-1}\{\overline{\mathrm{F}}_\mathrm{i}\} = \{\overline{\mathrm{R}}_\mathrm{f}\} \tag{8}$$

As shown in equation (8), $\{\overline{F_f^\alpha}\}$ is the reaction forces or boundary forces required to remain $\{u_f\} = 0$, when interior forces $\{\overline{F_i}\}$ are applied. When the boundaries of the substructure are relaxed, the displacements $\{u^\beta\}$are determined using equation (3) and the displacements $\{u_i^\beta\}$ and $\{u_f^\beta\}$ are calculated using equations (9) and (10).

$$\{\mathrm{u}_\mathrm{i}^\beta\} = -[\mathrm{k}_\mathrm{ii}]^{-1}[\mathrm{k}_\mathrm{if}]\{\mathrm{u}_\mathrm{f}^\beta\} \tag{9}$$

$$\{\mathrm{u}_\mathrm{f}^\beta\} = [\mathrm{K}_\mathrm{f}]\{\overline{\mathrm{F}}_\mathrm{f}^\beta\} \tag{10}$$

The stiffness matrix of the boundary$[K_f]$ is obtained using the equation (11), which represents the static concentration of equation (3).

$$[\mathrm{K}_\mathrm{f}] = [\mathrm{k}_\mathrm{ff}] - [\mathrm{k}_\mathrm{fi}][\mathrm{k}_\mathrm{ii}]^{-1}[\mathrm{k}_\mathrm{if}] \tag{11}$$

The vector $\{\overline{F}\}$ from equation (12) is obtained using the equations (6) and (8).

$$\{\overline{\mathrm{F}}_\mathrm{f}^\beta\} = \{\overline{\mathrm{F}}_\mathrm{f}\} - \{\overline{\mathrm{F}}_\mathrm{f}^\alpha\} = \{\overline{\mathrm{F}}_\mathrm{f}\} - [\mathrm{k}_\mathrm{ft}][\mathrm{k}_\mathrm{ii}]^{-1}\{\overline{\mathrm{F}}_\mathrm{i}\} = \{\overline{\mathrm{S}}_\mathrm{f}\} \tag{12}$$

Substructures are completely isolated when boundaries displacements are forced to be null ($\{u_f\} = 0$), thereby displacements are only generated on the substructure in which internal forces are applied. Therefore, when boundaries are fixed, the internal displacements$\{u_i^\alpha\}$are calculated independently for each substructure from equation (7). The boundary displacements $\{u_f^\beta\}$are calculated from equation (10), where the inverse of a stiffness matrix$[K_f]$is of lower order than the global stiffness matrix of the system$[K]$.

2.2 Boundaries fixing: displacements and forces of the substructure [1], [8]

The stiffness matrix of a substructure can be split as shown in equation (13).

$$[\mathrm{K}^\mathrm{n}] = \begin{bmatrix} [\mathrm{k}_\mathrm{ff}^\mathrm{n}] & [\mathrm{k}_\mathrm{fi}^\mathrm{n}] \\ [\mathrm{k}_\mathrm{if}^\mathrm{n}] & [\mathrm{k}_\mathrm{ii}^\mathrm{n}] \end{bmatrix}$$

(13)

Where n is the umpteenth structure and the subscripts i and $f$are the internal and boundary displacements respectively. The external forces and the displacements of the substructure are related such as $[K^n]\{u^n\} = [\overline{F}^n]$using equation (14).

$$\begin{Bmatrix} [k_{\mathrm{ff}}^n][k_{\mathrm{fi}}^n] \\ [k_{if}^n][k_{ii}^n] \end{Bmatrix} \begin{Bmatrix} u_f^n \\ u_i^n \end{Bmatrix} = \begin{Bmatrix} \overline{F}_f^n \\ \overline{F}_i^n \end{Bmatrix} \tag{14}$$

In equation (15) and (16) is shown the calculation of the internal displacements and reaction forces of the completely fixed boundaries in a nth substructure due to $\{\overline{F}\}$ when $\{u_f^n\} = 0$.

$$\{\mathrm{u}_\mathrm{i}^\mathrm{n}\}_{\substack{\text{fixed} \\ \text{boundaries}}} = [\mathrm{k}_\mathrm{ii}^\mathrm{n}]^{-1}\{\overline{\mathrm{F}}_\mathrm{i}^\mathrm{n}\} \tag{15}$$

$$\{\overline{\mathrm{F}}_\mathrm{f}^\mathrm{n}\} = [\mathrm{k}_\mathrm{fi}^\mathrm{n}][\mathrm{k}_\mathrm{ii}^\mathrm{n}]^{-1}\{\overline{\mathrm{F}}_\mathrm{i}^\mathrm{n}\} = \{\overline{\mathrm{R}}_\mathrm{f}^\mathrm{n}\} \tag{16}$$

Equation (17) is obtained from equation (11), which allows determining the stiffness matrix of the boundary of the nth substructure.

$$[\mathrm{K}_\mathrm{f}^\mathrm{n}] = [\mathrm{K}_\mathrm{ff}^\mathrm{n}] - [\mathrm{k}_\mathrm{fi}^\mathrm{n}][\mathrm{k}_\mathrm{ii}^\mathrm{n}]^{-1}[\mathrm{k}_\mathrm{if}^\mathrm{n}] \tag{17}$$

The stiffness matrix of the boundary of the entire structure, $[K_f]$, is obtained through the assembly of every matrix of the boundary of each substructure $[K_f^n]$, which are calculated using equation (17).

2.3 Substructure relaxation: a general solution for finding the boundary displacements [1], [8]

Determining the stiffness matrices $[K_f^n]$and the reaction forces of the boundary$\{\overline{R}\}$, every boundary is simultaneously relaxed; hence, the reaction forces of the boundary and any external load applied to it, are not in equilibrium, therefore, displacements of the boundary take place, which satisfy the equilibrium of each boundary node.

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In order to calculate the displacements of the boundary, the entire structure is assumed as an assembly of substructures subjected to external loading (Przemieniecki), as shown in equation (18).

$$\left\{\overline{S}_f^n\right\} = -\sum_n \left\{\overline{R}_f^n\right\} + \left\{\overline{\overline{F}}_f\right\} \tag{18}$$

As shown in equation (18), the variables are the sum of the reaction forces of the fixed boundaries, and $\{\overline{F_f}\}$ is the external force vector applied to the boundaries. The negative sign converts the force reactions into externally applied forces.

Afterwards, the equilibrium equations are rewritten as seen in equation (18-a), which is a function of the displacements of the boundaries of the entire structure.

$$[K_f]\{u_f\} = \left\{\overline{\overline{S}}_f\right\} \tag{18-a}$$

$[K_f]$ is calculated from the assembly of the matrices $[K_f^n]$; therefore, the boundary displacements are determined as shown in equation (19).

$$\{u_f\} = [K_f]^{-1}\left\{\overline{\overline{S}}_f\right\} \tag{19}$$

2.4 Mathematical model for substructure analysis

The calculation algorithm is used to solve the substructure problem, proposed by Przemieniecki, and it was developed using Excel. The summarized steps are listed below:

(a) Define the stiffness matrices of both substructures using assembly rules and following the usual conventions used for giving direction to bars.

(b) Propose a system of equations for each substructure, characterizing each node of the boundaries and interior nodes.

$$\begin{Bmatrix} \overline{F}_f \\ \overline{F}_i \end{Bmatrix} = \begin{bmatrix} [K_{ff}] & [K_{fi}] \\ [K_{if}] & [K_{ii}] \end{bmatrix} \begin{Bmatrix} \{u_f\} \\ \{u_i\} \end{Bmatrix}.$$

(c) Determine the stiffness matrices of the boundary for each substructure, considering that:

$[K_f^A] = [K_{ff}^A] - [K_{fi}^A][k_{ii}^A]^{-1}[k_{if}^A]$ when there are interior nodes.

$[K_f^B] = [K^B]$ when there are not interior nodes.

(d) Calculate the stiffness matrix of the boundary of the system, $[K_f] = [K_f^A] + [K_f^B]$.

(e) Define two force vectors, the first performs directly on the boundary, the second performs on the internal degrees of freedom, $\{\overline{F_f}\}$ and $\{\overline{F_i}\}$ respectively.

(f) Calculate the resulting boundary: $\{\overline{S_f}\} = \{\overline{F_f}\} - \{\overline{R_f}\}$. (Choosing the substructure with internal nodes):

$\{\overline{S_f}\} = \{\overline{F_f}\} - \{\overline{R_f^A}\} = \{\overline{F_f}\} - [K_{fi}^A][k_{ii}^A]^{-1}\{\overline{F_i^A}\} = \{\overline{S}\}.$

(g) Determine the boundary displacement $\{u_f\} = [K_f]^{-1}\{\overline{S}\}$.

(h) Calculate the internal displacements of each substructure from its own equation system (or calculate the substructure that has interior nodes)

$[k_{if}^A]\{u_f^A\} + [k_{ii}^A]\{u_i^A\} = \{f_i^A\}.$

$\{u_i^A\} = [k_{ii}^A]^{-1}\{\overline{F}\} - [k_{ii}^A]^{-1}[k_{if}^A]\{u_f^A\}.$

(i) Determine the total displacements of the structure.

$$\{u\} = \begin{Bmatrix} \{u_i^A\} \\ \{u_f^A\} \\ \{u_i^B\} \end{Bmatrix}.$$

A model of substructure analysis was developed using Ansys®, a finite element software, and a calculation system was developed in Excel using the proposal by Przemieniecki [8] in order to solve the problem proposed by Tena [1]. A plane truss referenced in Tena [1] is shown in Figure-1, with the purpose of obtaining the displacements of each node using Ansys® and the substructure technique proposed by Przemieniecki.

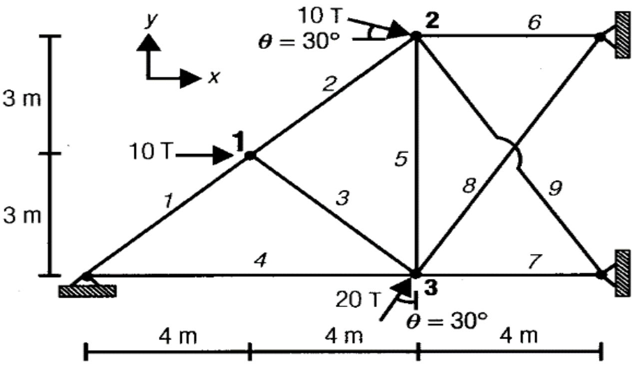

Figure-1. Plane truss[1].

In the substructure process, the substructure A and B are used as shown in Figure-2. The stiffness of each bar expressed in Ton/mmarek_1=k_5=k_7=1, k_2=k_3=k_6=2, k_4=2.5 yk_8=k_9=3.

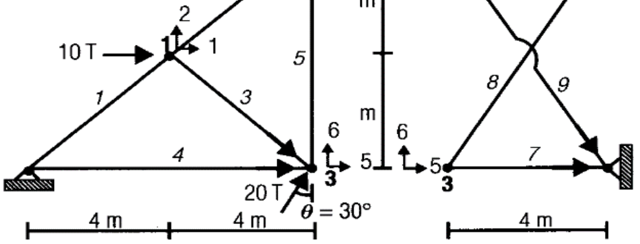

Figure-2. Substructure analysis[1].

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After applying the substructure technique proposed by Przemieniecki [8] using Excel, the results are the stiffness matrices of the boundary $[K_f^A]$and $[K_f^B]$, and the force vector of the boundary $\{\overline{F_f}\}$, the stiffness matrix of boundary $[K_f]$, the intern force vector $\{\overline{F_i}\}$, the resulting forces of the boundary $\{\overline{S_f}\}$, the boundary displacement $\{u_f\}$, the intern displacements and total displacements of the structure as shown below:

Substructure A

$$[k_{ii}^A]^{-1} = \begin{bmatrix} 0.3255 & -0.0868 \\ -0.0868 & 0.5787 \end{bmatrix}$$

$$[k_{fi}^A][k_{ii}^A]^{-1} = \begin{bmatrix} -0.3333 & -0.4444 \\ -0.25 & -0.3333 \\ -0.5 & 0.6667 \\ 0.375 & -0.5 \end{bmatrix}$$

$$[k_{fi}^A][k_{ii}^A]^{-1}[k_{if}^A] = \begin{bmatrix} 0.8533 & 0.64 & 1E\text{-}16 & 0 \\ 0.64 & 0.48 & 0 & 8E\text{-}17 \\ 1E\text{-}16 & 0 & 1.28 & -0.96 \\ 6E\text{-}17 & 1E\text{-}16 & -0.96 & 0.72 \end{bmatrix}$$

Kf Substructure A

$$[K_f^A] = [k_{ff}^A] - [k_{fi}^A][k_{ii}^A]^{-1}[k_{if}^A] = \begin{bmatrix} 0.43 & 0.32 & 0.00 & 0.00 \\ 0.32 & 1.24 & 0.00 & -1.00 \\ 0.00 & 0.00 & 2.50 & 0.00 \\ 0.00 & -1.00 & 0.00 & 1.00 \end{bmatrix}$$

Substructure B

Kf Substructure B

$$[K_f^B] = [K^B] = \begin{bmatrix} 2.92 & -1.38 & 0.00 & 0.00 \\ -1.38 & 2.08 & 0.00 & 0.00 \\ 0.00 & 0.00 & 1.92 & 1.38 \\ 0.00 & 0.00 & 1.38 & 2.08 \end{bmatrix}$$

K_f Boundaries

$$[K_f] = [K_f^A] + [K_f^B] = \begin{bmatrix} 3.35 & -1.06 & -0 & -6E\text{-}17 \\ -1.06 & 3.32 & -0 & -1 \\ -0 & -0 & 4.42 & 1.385 \\ -0 & -1 & 1.38 & 3.077 \end{bmatrix}$$

RESULT OF FORCE IN THE BOUNDARY

Substructure A

$$\{\overline{S_f}\} = \{\overline{F_f}\} - \{\overline{R_f}\} = \{\overline{F_f}\} - \{\overline{R_f^A}\} = \{\overline{F_f}\} - [k_{fi}^A][k_{ii}^A]^{-1}\{\overline{F_i^A}\}$$

$$\begin{bmatrix} 8.66 \\ -5.00 \\ 10.00 \\ 17.32 \end{bmatrix} - \begin{bmatrix} -3.3333 \\ -2.5 \\ -5 \\ 3.75 \end{bmatrix} = \begin{bmatrix} 11.99 \\ -2.50 \\ 15.00 \\ 13.57 \end{bmatrix} \begin{matrix} Fx2 \\ Fy2 \\ Fx3 \\ Fy3 \end{matrix}$$

BOUNDARY DISPLACEMENT

$$\{u_f\} = [K_f]^{-1}\{\overline{S_f}\}$$

$$\begin{bmatrix} 0.337 & 0.122 & -0.014 & 0.046 \\ 0.122 & 0.385 & -0.046 & 0.145 \\ -0.014 & -0.046 & 0.269 & -0.136 \\ 0.046 & 0.145 & -0.136 & 0.433 \end{bmatrix} * \begin{bmatrix} 11.99 \\ -2.50 \\ 15.00 \\ 13.57 \end{bmatrix} = \begin{bmatrix} 4.15113 \\ 1.7956 \\ 2.1277 \\ 4.03652 \end{bmatrix} \begin{matrix} u2 \\ v2 \\ u3 \\ v3 \end{matrix}$$

INTERNAL DISPLACEMENT ***Substructure A***

$$\{u_i^A\} = [k_{ii}^A]^{-1}\{\overline{F_i^A}\} - [k_{ii}^A]^{-1}[k_{if}^A]\{u_f^A\}$$

$$\begin{bmatrix} 0.3255 & -0.0868 \\ -0.0868 & 0.5787 \end{bmatrix} * \begin{bmatrix} 10 \\ 0 \end{bmatrix} = \begin{bmatrix} 3.26 \\ -0.87 \end{bmatrix}$$

$$\begin{bmatrix} -0.3333 & -0.25 & -0.5 & 0.375 \\ -0.4444 & -0.3333 & 0.6667 & -0.5 \end{bmatrix} * \begin{bmatrix} 4.15 \\ 1.8 \\ 2.13 \\ 4.04 \end{bmatrix} = \begin{bmatrix} -1.38 \\ -3.04 \end{bmatrix}$$

$$\begin{bmatrix} 3.2552 \\ -0.8681 \end{bmatrix} * \begin{bmatrix} -1.3828 \\ -3.0433 \end{bmatrix} = \begin{bmatrix} 4.638 \\ 2.1752 \end{bmatrix} \begin{matrix} u1 \\ v1 \end{matrix}$$

2.5 Computational modeling using Ansys®

The modeling of the problem of the plane truss using the software Ansys®, is developed with the finite element LINK1 and the superelement MATRIX for the substructure analysis.

LINK1 is a two-dimensional straight element subject to uniaxial compression or tension, and possess two degrees of freedom for each node: translations on X and Y.

Ansys®uses a method that groups finite elements into a superelement (MATRIX). The performance of the super element that was determined using the substructure analysis, is grouped with a less quantity of degrees of freedom called "master nodes" and the solution of the model of the entire system is obtained using the equilibrium and compatibility process, across the boundaries of the super element.

3. RESULTS AND DISCUSSIONS

Displacements of the nodes of the boundaries are called $u_2 and u_3$, and they were obtained through the substructure analysis technique, using Excel and Ansys®, these results (degrees of freedom of the super element) are shown in Table-1.

Table-1. Boundary DOF.

	ANSYS		USER	
NODE	**UX**	**UY**	**UX**	**UY**
2	4.1511	1.7956	4.1483	1.7966
3	2.1277	4.0366	2.1238	4.0428

Figure-3 and Figure-4 show the results of the displacements in the X and Y directions, obtained by the substructure analysis using Ansys®

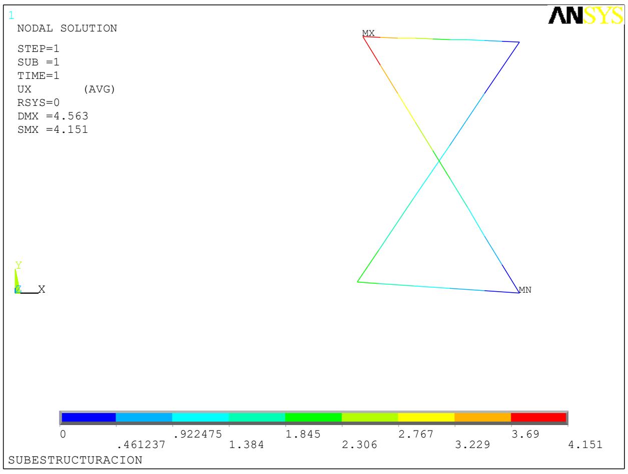

Figure-3. ANSYS®DOF (UX) - Substructure B.

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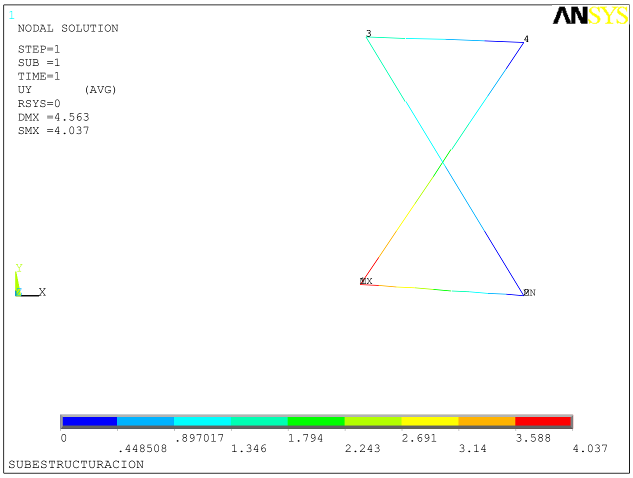

Figure-4. ANSYS®DOF (UY) - Substructure B.

4. CONCLUSIONS

- When stiffness matrices of each substructure $[K_f^n]$ are assembled into the boundary stiffness matrix of the entire structure $[K_f]$, its relative position depends on the sequence in which boundary displacements are chosen individually, according to the assembly rules.
- When substructures that are not connected physically to the linking matrices (boundary) exist, the matrix must be equalized to zero.
- When a reference enumeration for the substructure and the displacements is selected, it is recommended that the enumeration is done to order the submatrices that are part of the linking matrix $[K_f]$, across the principal diagonal, in order to create a band matrix; hence, large time savings are obtained, especially when solution algorithms are used for band matrices.
- A classic calculation methodology, of a systematic implementation based on the stiffness analysis, is able to equalize the results obtained from finite element software, proving to be a trustable choice to determine plane trusses.
- The efficiency and the accuracy of the developed model were verified, showing great advantages for the analysis of large plane trusses.
- The use of the calculation techniques offered by the actual resources and the progress of the methods used for mathematical calculation; allow to solve structural engineering problems, without the expensive use of a commercial finite element software.

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REFERENCES

[1] A. Tena-Colunga. 2007. Análisis de Estructuras con Métodos Matriciales. México: Gedisa.

[2] J. S. PRZEMIENIECKI. 1963. Matrix Structural Analysis of Substructures. AIAA J.

[3] R. H. Gallagher. 1975. Finite Element Analysis Fundamentals. Englewood Cliffs: Prentice-Hall.

[4] M. Petyt. 2003. Theory of matrix structural analysis. J. Sound Vib.

[5] W. McGuire, R. H. Gallagher, y H. Saunders. 1982. Matrix Structural Analysis. J. Mech. Des.

[6] J. S. Przemieniecki. 2012. Finite Element Structural Analysis: New Concepts.

[7] T. H. G. Megson. 2007. Matrix Methods of Analysis. en Structural and Stress Analysis.

[8] S. Przemieniecki J. 1969. Theory of matrix structural analysis.

[9] M. Paz, W. Leigh, M. Paz, y W. Leigh. 2011. Static Condensation and Substructuring. En Integrated Matrix Analysis of Structures.

[10] H. H. West. 1993. Fundamentals of Structural Analysis. Eur. J. Eng. Educ.