HYDRODYNAMIC EFFECTS OF SECANT SLIDER BEARINGS LUBRICATED WITH SECOND - ORDER FLUIDS

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ABSTRACT
The constitutive equations governing the flow of secant slider bearings is analysed in this study. The bearing is lubricated with second order fluid. An attempt has been made to solve the equations governing the model and the characteristics of secant slider bearings is presented. An expression for the fluid film pressure is derived. The results reveal that second order fluids enhances the performance characteristics of lubrication indicating that second order fluids are better than Newtonian fluids.

Keywords: fluid film pressure, Newtonian fluids, second order fluids, slider bearings.

INTRODUCTION

In this paper an effort has been made to study the hydrodynamic effects of secant slider bearings lubricated with second order fluids. The equations’ governing the flow is highly nonlinear and an approximate method is employed to solve them.

MATHEMATICAL FORMULATION
The geometry constitutes a secant slider bearing lubricated with second order fluid. The lower surface of the bearing has a pure tangential sliding motion relative to the upper surface. The fluids constitutive equation for incompressible homogeneous of the second-order, based on the postulate proposed by Coleman and Noll [1] and is given by

\[ \mathbf{T}_{ij} = \alpha_0 A(1)_{ij} + \alpha_1 A(2)_{ij} + \alpha_2 A(1)_{ik} A(1)_{kj} - p\delta_{ij} \]

where \( \mathbf{T}_{ij} \) is the stress tensor, \( p \) is the pressure and \( \alpha_0, \alpha_1 \) and \( \alpha_2 \) are material constants.
The film thickness is,

\[ H_s = h_2 \sec \left( \frac{\pi (L-x)}{2L} \right); 0 < x \leq L \]  

where \( L \) is the length of the bearing.

The momentum and continuity equations are,

\[ \frac{\partial}{\partial x} \left( T_{xx} \right) = -\frac{\partial}{\partial y} \left( T_{xy} \right) \]  

\[ \frac{\partial}{\partial x} \left( T_{xy} \right) = -\frac{\partial}{\partial y} \left( T_{yy} \right) \]  

\[ \frac{\partial u}{\partial x} = -\frac{\partial u}{\partial y} \]  

Assume that the non-dimensional quantities:

\[ u^* = \frac{u}{U}, v^* = \frac{v}{V}, x^* = \frac{x}{L}, y^* = \frac{y}{e} \in L, \]

\[ p^* = \left( \frac{L}{h_o V^*} \right), H_s^* = \frac{H_s}{h_1}, \epsilon = \frac{h_1}{L} \]  

where \( V \) is the characteristic velocity and \( e \) is a very small quantity.

Hence,

\[ \frac{\partial p}{\partial x} = \frac{\partial u}{\partial x} = \frac{\partial \dot{u}}{\partial x} + \alpha_1 \frac{\partial \dot{u}}{\partial y} + \alpha_2 \frac{\partial \dot{u}}{\partial y} \]  

\[ + \alpha_1 \frac{\partial \dot{v}}{\partial y} + 4\alpha_2 \frac{\partial \dot{u}}{\partial y} + 2\alpha_2 \frac{\partial \dot{u}}{\partial y} \]  

\[ \frac{\partial p}{\partial y} = \left( 4\alpha_1 + 2\alpha_2 \right) \frac{\partial \dot{u}}{\partial y} \]  

The above equations are solved using the boundary conditions,

\[ u = U, v = 0 \text{ at } y = 0 \]  

\[ u = 0, v = 0 \text{ at } y = H_s \]

Assuming,

\[ u(x, y) = m(x) y^2 + n(x) y + u_0 \]  

From (3) and (7.a) and (7.b) we get,

\[ u(x, y) = m(x) y^2 + n(x) y + U \]  

\[ v(x, y) = -\frac{dm}{dy} \frac{y^3}{3} - \frac{dn}{dy} \frac{y^2}{2} \]  

where \( m(x) = \frac{3UH_s + c}{H_s}, n(x) = \frac{-4UH_s + c}{H_s} \)  

when \( c \) is an arbitrary constant.

Substituting (9) and (10) into (6) we get,

\[ \frac{\partial p}{\partial x} = 2\alpha_1 \left( \frac{3UH_s + c}{H_s^3} \right) + 4y^2 \left( 2\alpha_1 + \alpha_2 \right) \frac{d \left( \frac{3UH_s + c}{H_s^3} \right)^2}{dx} \]  

\[ + 4y \left( 2\alpha_1 + \alpha_2 \right) \frac{d \left( \frac{-4UH_s + c}{H_s^3} \right)^2}{dx} \]  

\[ + \frac{3\alpha_1 + 2\alpha_2}{2} \frac{d \left( \frac{-4UH_s + c}{H_s^3} \right)^2}{dx} + 2U\alpha_1 \left( \frac{3UH_s + c}{H_s^3} \right) \]  

\[ \frac{\partial p}{\partial y} = 4 \left( 2\alpha_1 + \alpha_2 \right) \left( \frac{3UH_s + c}{H_s^3} \right) \frac{y}{2} \]  

\[ + \left( \frac{3UH_s + c}{H_s^3} \right) \left( \frac{-4UH_s + c}{H_s^3} \right) \left( \frac{-4UH_s + c}{H_s^3} \right) \]  

From (11),

\[ p(x, y) = 2\alpha_1 \left( \frac{3UH_s + c}{H_s^3} \right) dx + 4 \left( 2\alpha_1 + \alpha_2 \right) \left( \frac{3UH_s + c}{H_s^3} \right)^2 y^2 \]  

\[ + 4 \left( 2\alpha_1 + \alpha_2 \right) \left( \frac{3UH_s + c}{H_s^3} \right) \left( \frac{-4UH_s + c}{H_s^3} \right) y \]  

\[ + \left( \frac{3\alpha_1 + 2\alpha_2}{2} \right) \left( \frac{-4UH_s + c}{H_s^3} \right)^2 + 2U \left( \frac{3UH_s + c}{H_s^3} \right) \alpha_1 + d \]  

where \( d \) is a constant.

The average pressure distribution \( p \) across the film thickness is,
\[ p = \frac{1}{H_s} \int_0^H p \, dy \]
\[ = 2\alpha_0 \left[ \frac{3UH_s + c}{H_s^3} \right] dx + \frac{4}{3} \left( 2\alpha_0 + \alpha_1 \right) \left( \frac{3UH_s + c}{H_s^3} \right) H_s \]
\[ + 2 \left[ 2\alpha_0 \left( \frac{3UH_s + c}{H_s^3} \right) - \frac{4}{3} \left( \frac{4UH_s + c}{H_s^3} \right) \right] H_s \]
\[ = \phi_2 \left[ \frac{4}{3} \left( \frac{3UH_s + c}{H_s^3} \right) H_s \right] \]
\[ + 2 \left( \frac{3UH_s + c}{H_s^3} \right) \left( \frac{4UH_s + c}{H_s^3} \right) H_s \]
\[ + \left( \frac{4UH_s + c}{H_s^3} \right)^2 \alpha_1 + d = 0 \]
\[ m_2 = \sin 3 \left( \frac{\pi (L-x)}{2L} \right) + 9 \sin \left( \frac{\pi (L-x)}{2L} \right) \]

The average total stress is:
\[ \frac{1}{H_s} \int_0^H (p - \tau_{xx}) \, dy = 0 \quad \text{at} \quad x = 0 \quad \text{and} \quad x = L \]

Where \( \tau_{xx} = \frac{1}{H_s} \int_0^H \tau_{xx} \, dy \)

From (14),
\[ d = 2\alpha_0 M - \frac{8\alpha_1}{3} \left( \frac{3U}{h_s^2} + \frac{c}{H_s^3} \right) \]
\[ + 4\alpha_1 \left( \frac{3U}{h_s^2} + \frac{c}{H_s^3} \right) \left( \frac{4U}{h_s^2} + \frac{c}{H_s^3} \right) \frac{L_{cm}}{6\pi h_s^3} \]
\[ = \frac{\pi (L-x)}{L} + \sin \frac{\pi (L-x)}{L} \]

\[ \alpha_1 = \frac{L}{6\pi} \left[ \sin 3 \left( \frac{\pi (L-a)}{2L} \right) + 9 \sin \left( \frac{\pi (L-a)}{2L} \right) \right] \]

From (17) and (18),
\[ \frac{-c^2 k}{6h_s^2 U} \left( \frac{1}{a_2^2} - 1 \right) + \frac{2k\alpha_0}{\alpha U} - 10k \left( \frac{1}{a_2^2} - 1 \right) \]
\[ + \frac{U^2}{h_s^2} \left( \frac{6k}{U} \left( \frac{1}{a_2^2} - 1 \right) \right) + 2ak_0 \alpha_0 \frac{\alpha U h_s^3}{\alpha U} = 0 \]

Hence, \( A_1 c^2 + A_2 c + A_3 = 0 \)
\[ A_1 = -k \frac{1}{6h_s^2 U} \left( \frac{1}{a_2^2} - 1 \right); \quad A_2 = \frac{2k\alpha_0}{\alpha U h_s^3} - 10k \left( \frac{1}{a_2^2} - 1 \right) ; \quad A_3 = 0 \]
\[ A_3 = \left[ \frac{6k}{U} \left( \frac{1}{a_2^2} - 1 \right) \right] ; A_4 = \frac{2a_4k\alpha_0}{\alpha_iU} \]

\[ A_1 = \frac{a_3}{h_2^2} ; A_2 = \frac{a_4}{h_2} ; A_3 = a_5 \]

where

\[ a_3 = \frac{k}{6U} \left( \frac{1}{a_2^2} - 1 \right) ; a_4 = \left[ \frac{2ka_0\alpha_0}{\alpha_iU} - 10k \left( \frac{1}{a_2^2} - 1 \right) \right] ; \]

\[ a_5 = \left[ \frac{6k}{U} \left( \frac{1}{a_2^2} - 1 \right) \right] \]

\[ c = \frac{-A_2 + \sqrt{A_2^2 - 4A_4A_3}}{2A_4} \]

\[ c = h_2A_i \text{ where } A_i = \frac{a_4 + \left( \frac{a_3}{h_2^2} \right)^2 + 4 \left( \frac{a_3}{h_2^2} \right) a_5}{2a_3} \]


Hence,

\[ p = -2a_3 \left[ \frac{3umL}{2\pi h_2^2} + \frac{Lcm_1}{6\pi h_2^2} \right] \alpha_0 \left( \frac{c}{H_i} \right)^2 \]

\[ + \frac{2U\alpha_0}{H_i} \left( \frac{3U}{H_i} + \frac{c}{H_i} \right) \frac{a_2}{3} \left[ \frac{4U}{H_i} + \frac{c}{H_i} \right]^2 \]

\[ + d - \frac{8a_3}{3} \left( \frac{3U}{h_2} + \frac{c}{h_2} \right)^2 + 4a_0 \left( \frac{3U}{h_2} + \frac{c}{h_2} \right) \frac{4U}{h_2} \]

\[ -2a_3 \left( \frac{4U}{h_2} + \frac{c}{h_2} \right) \frac{4U}{h_2} - 2Ua_0 \left( \frac{3U}{h_2} + \frac{c}{h_2} \right) \frac{4U}{h_2} \]  \hspace{1cm} (20) \]

The Non-dimensionless pressure is,

\[ p^* = \frac{ph_2^2}{\alpha_0UL} \]

The Non-dimensional pressure \( p^* \) is estimated for the second-order fluids A (Osteoarthritic fluid, \( \alpha_0 = 2.5 \text{ Nsm}^{-2}, \alpha_1 = 0.025 \text{ Ns}^2\text{m}^{-2}, \alpha_2 = 0.05 \text{ Ns}^2\text{m}^{-2} \)), B(Polyisobutylene (5.39%), \( \alpha_0 = 18.5 \text{ Nsm}^{-2}, \alpha_1 = 0.3 \text{ Ns}^2\text{m}^{-2} \)), and C(Polyisobutylene (5.4%), \( \alpha_0 = 18.5 \text{ Nsm}^{-2}, \alpha_1 = 0.2 \text{ Ns}^2\text{m}^{-2}, \alpha_2 = 1 \text{ Ns}^2\text{m}^{-2} \)) with material parameters \( \alpha_0, \alpha_1, \alpha_2 \) for the fluids A, B, C respectively. Figure-2 indicates the variation of Non-dimensional pressure \( p^* \) and \( x^* \) for the second order fluids A, B, C.

**CONCLUSIONS**

The performance characteristic of a secant slider bearing is analysed. It is evident that the Non-dimensionless pressure \( p^* \) is more for the second-order fluids when compared to Newtonian fluid. The results reveal that second order fluids enhances the lubrication effects of machinery when coupled to Newtonian fluid.

**NOMENCLATURE**

- \( \alpha_0, \alpha_1, \alpha_2 \) Material constants.
- \( L \) Length of the bearing.
- \( U \) Velocity of slider.
- \( h_1 \) Minimum film thickness.
- \( h_2 \) Maximum film thickness.
- \( h^* \) Film thickness ratio, \( h^*_2 = \frac{h_2}{h_1} \)
- \( p \) pressure in the film
- \( p^* \) Non-dimensional pressure.
- \( c \) coefficient of friction.

**REFERENCES**


