



PERFORMANCE ANALYSIS OF ZERO-FORCING PROCESSING FOR MIMO MULTI-RELAY COMMUNICATION NETWORKS

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ABSTRACT

The problem of zero-forcing processing is studied for multi-input multi-output multi-relay communication system in which MIMO source-destination pairs communicate simultaneously. It is assumed that due to severe shadowing effects which communication links can be established only with the aid of relay node. The aim is to design the relay amplification matrix to maximize the achievable communication sum-rate through the relay, which in general amplifying-and-forward relaying mechanisms are considered. Zero-forcing (ZF) processing are employed at destination to maintain low-complexity. The performance analysis of the proposed algorithm is demonstrated through numerical simulations which are significantly improved.

Index Terms: multiple-input multiple-output(MIMO), multi-relay networks, zero-forcing (ZF) equalizer, amplify- and-forward(AF).

1. INTRODUCTION

In order to provide a reliable wireless transmission, one needs to compensate for the effects of signal fading due to multi-path propagation and strong shadowing. One way to address these issues is to transmit the signal through one or more relays [1]-[2], which can be accomplished via a wireless network consisting of geographically separated nodes. And then the basic motivation behind the use of cooperative communications lies in the exploitation of spatial diversity provided by the network nodes [5] and [6], as well as the efficient use of power resources [5]-[8] which can be achieved by a scheme that simply receives and forwards a given information, yet designed under certain optimality criterion.

Relay schemes can be broadly categorized into three general groups: amplify-and-forward (AF), decode-and-forward (DF), and compress-and-forward (CF). In the AF scheme, the relay nodes amplify the received signal and rebroadcast the amplified signals toward the destination node [1]-[2]. In the DF scheme, the relay nodes first decode the received signals and then forward the re-encoded signals toward the destination node [3]. In the CF method, the relay nodes compress the received signals by exploiting the statistical dependencies between the signals at the nodes [6]. In this paper we consider the AF strategy which is easier to implement compared with the other two approaches.

When nodes in the relay system are installed with multiple antennas, we call such system multiple-input multiple-output (MIMO) relay and multi-relay communication system. Recently, MIMO relay and multi-relay communication systems have attracted much research interest and provided significant improvement in terms of both spatial efficiency and link reliability [1], [2], and [6]. In this paper, we investigated the performance analysis of zero-forcing (ZF) algorithm in a MIMO multi-relay network in terms of bit error rate (BER). Note that the ZF algorithm has already been studied with single-hop MIMO [4] and MIMO relay channels [5] - [8]. In this

paper, we study the ZF algorithm in MIMO multi-relay channel. Our results show that algorithms in MIMO multi-relay system have significant performance improvement.

The rest of the paper is organized as follows: the system model is described in Section II; in Section III we study the detection techniques at the receiver by zero-forcing equalizer; Section IV shows the simulation results with ZF equalizer algorithms under various system scenarios and the conclusion is given in Section V.

2. SYSTEM MODEL FOR MIMO MULTI-RELAY

Fig. 1 illustrates a two-hop MIMO relay communication system consisting of one source node, K parallel relay nodes, and one destination node. We assume that the source and destination nodes have N_s and N_d antennas, respectively, and each relay node has N_r antennas. The generalization to the system with different number of antennas at each relay node is straightforward. To efficiently exploit the system hardware, each relay node uses the same antennas to transmit and receive signals. Due to its merit of simplicity, we consider the amplify- and-forward relaying scheme at each relay.

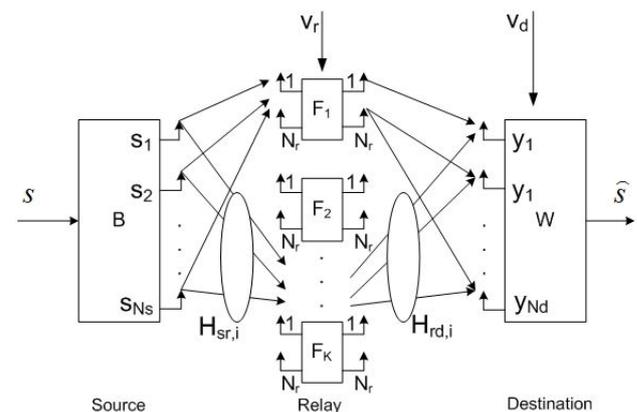


Figure-1. System model for MIMO Multi-Relay Channel.



The communication process between the source and destination nodes is completed in two time slots. In the first time slot, the $N_b \times 1$ modulated symbol vector s is linearly precoded as

$$x = Bs \tag{1}$$

where B is an $N_s \times N_b$ source precoding matrix. We assume that $E[ss^H] = I_{N_s}$, where $(\cdot)^H$ denotes matrix (vector) Hermitian transpose, $E[\cdot]$ stands for statistical expectation, and I_n is an $n \times n$ identity matrix. The precoded vector x is transmitted to the relay nodes and the received signal at the i th relay node can be written as

$$y_{r,i} = H_{sr,i} x + v_{r,i}, \quad i = 1, \dots, K \tag{2}$$

where $H_{sr,i}$ is the $N_r \times N_s$ MIMO channel matrix between the source and the i th relay node, $y_{r,i}$ and $v_{r,i}$ are the received signal and the additive Gaussian noise vectors at the i th relay node, respectively.

In the second time slot, the source node is silent, while each relay node transmits the amplified signal vector to the destination node as

$$x_{r,i} = F_i y_{r,i}, \quad i = 1, \dots, K \tag{3}$$

where F_i is the $N_r \times N_r$ amplifying matrix at the i th relay node. Thus the received signal vector at the

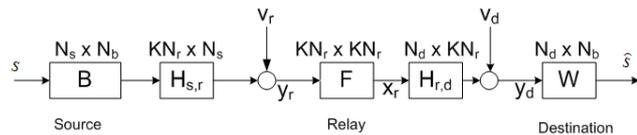


Figure-2. Equivalent MIMO Relay Channel.

destination node can be written as

$$y_d = \sum_{i=1}^K H_{rd,i} F_i x_{r,i} + v_d \tag{4}$$

where $H_{rd,i}$ is the $N_d \times N_r$ MIMO channel matrix between the i th relay and the destination node, y_d and v_d are the total received signal and the additive Gaussian noise vectors at the destination node, respectively.

Substituting (1)-(3) into (4), we obtain

$$y_d = \sum_{i=1}^K (H_{rd,i} F_i H_{sr,i} B s + H_{rd,i} F_i v_{r,i}) + v_d = H_{rd} F H_{sr} B s + H_{rd} F v_r + v_d \tag{5}$$

where we define

$$H_{sr} = [(H_{sr,1})^T, (H_{sr,2})^T, \dots, (H_{sr,K})^T]^T$$

$$H_{rd} = [H_{rd,1}, H_{rd,2}, \dots, H_{rd,K}]$$

$$F = \text{bd}[F_1, F_2, \dots, F_K]$$

$$v_r = (v_{r,1})^T, (v_{r,2})^T, \dots, (v_{r,K})^T \text{ }^T$$

Here $(\cdot)^T$ denotes the matrix (vector) transpose, $\text{bd}(\cdot)$ stands for a block-diagonal matrix, H_{sr} is a $K N_r \times N_s$ channel matrix between the source node and all relay nodes, H_{rd} is an $N_d \times K N_r$ channel matrix between all relay nodes and the destination node, v_r is obtained by stacking the noise vectors at all the relays and F is the equivalent $K N_r \times K N_r$ block diagonal relay matrix. The diagram of the equivalent MIMO relay system described by (5) is shown in Fig. 2.

By introducing

$$\bar{F} = H_{rd} F \tag{6}$$

the received signal vector at the destination can be equivalently written as

$$y_d = \bar{F} H_{sr} B s + \bar{F} v_r + v_d = H_s + v$$

where we define $H = \bar{F} H_{sr} B$ as the effective MIMO channel matrix of the source-relay-destination link, and v as the equivalent noise with $v = \bar{F} v_r + v_d$.

In this paper, we try to improve the system BER performance by using zero-forcing (ZF) equalizer. A simple approach to design the relay is to treat it as an all-pass AF unit, which we construct as $F = \alpha I_{N_r}$, where α is the amplifying factor of the relay and I_{N_r} is an identity matrix of dimension N_r . We can find α from

$$P_r = \alpha^2 \text{tr} \{ P_s / N_s H_{s,r} H_{s,r}^H + I_{N_r} \}$$

Here $P_s > 0$ and $P_r > 0$ are the transmit power available at the source and the relay nodes respectively, $(\cdot)^H$ denotes matrix Hermitian and $\text{tr}\{\cdot\}$ indicates trace of a matrix.

3. DETECTION FOR MIMO MULTI-RELAY SYSTEM

We study the following detection algorithms for MIMO multi-relay systems such as the ZF equalizer. If we consider the received signal vector at the destination in (5) then our proposed MIMO multi-relay channel (Fig. 1) reduces to a MIMO relay channel (Fig. 2) into MIMO channel with the equivalent channel matrix of $H_s + v$ where $H = \bar{F} H_{sr} B$, the signals vector of s and the equivalent noise vector of $v = \bar{F} v_r + v_d$.

Now we can analyze the signal detection at the receiver with the zero-forcing equivalent MIMO channel. For simplicity, we consider here a 2×2 MIMO transmission which essentially means both the transmitter



and the receiver are equipped with two antennas. Thus, the received signal on the first receive antenna is

$$y_{d1} = h_{1,1} s_1 + h_{1,2} s_2 + v_1$$

$$= \begin{bmatrix} h_{1,1} & h_{1,2} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + v_1 \quad (7)$$

and that on the second receive antenna is

$$y_{d2} = h_{2,1} s_1 + h_{2,2} s_2 + v_2$$

$$= \begin{bmatrix} h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + v_2 \quad (8)$$

where y_{d1} and y_{d2} are the received symbols on the first and the second antennas respectively, $h_{j,i}$ is the channel from i th transmit antenna to j th receive antenna, s_1 and s_2 are the transmitted symbols, and v_1 and v_2 are the noises on first and second receive antenna respectively. For convenience, the above two equations can be combined in matrix notation as

$$\begin{bmatrix} y_{d1} \\ y_{d2} \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad (9)$$

A. THE ZF EQUALIZER

The first decoding technique to be described in this paper is ZF. The ZF linear detector meeting the constraint $W^H H = I_N$ is given by

$$W = H(H^H H)^{-1} \quad (10)$$

W is also known as the pseudo-inverse for a general $m \times n$ matrix and $(\cdot)^{-1}$ indicated simple matrix inversion. In order for a pseudo-inverse to exist, N_d must be greater than or equal to N_s . Then the estimate for the transmit vector will be

$$\hat{s} = W^H y_d \quad (11)$$

Using the ZF equalization approach described above, the receiver can obtain an estimate of the two transmitted symbols s_1 and s_2 as

$$\begin{bmatrix} \hat{s}_1 \\ \hat{s}_2 \end{bmatrix} = (H^H H)^{-1} H^H \begin{bmatrix} y_{d1} \\ y_{d2} \end{bmatrix} \quad (12)$$

4. SIMULATION RESULTS AND DISCUSSIONS

In the simulations, the transmission signaling is in spatial multiplexing mode (i.e., the source transmits independent data streams from different antennas) with total transmit power uniformly distributed among the transmit antennas. We study the performance of the proposed zero-forcing equalizer algorithm for MIMO multi-relay systems. All simulations are conducted in a flat Rayleigh fading environment using the BPSK constellation, and the noises are i.i.d. Gaussian random variables with zero mean and unit variance. The channel matrices have zero-mean entries with variances σ^2/N_s and σ^2/K for H_{SR} and H_{RD} , respectively. We vary the signal-to-noise ratio (SNR) in the relay-to-destination link SNR_r while fixing the SNR in the source-

to-relay link SNR_s to 30dB. We transmit 1000 randomly generated bits in each channel realization, and the bit-error-rate (BER) results are averaged through 200 channel realizations.

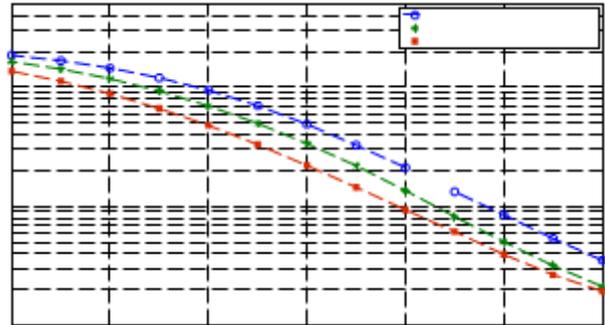


Figure 3- BER versus SNR_r . $N_s = N_r = N_d = 2$, and $SNR_s = 30$ dB for MIMO Multi-Relay channel.

In the first example, we study the effect of the number of relays ($K=2$) and compare the system BER performance of the zero-forcing equalizer algorithm with the naive amplify-and-forward (NAF) algorithm and PMF algorithm where both the source and relay matrices are scaled identity matrices. We choose $N_s = N_r = N_d = 2$. From Fig. 3, it can be seen that the NAF algorithm has the worst performance. The zero-forcing equalizer algorithm outperforms the other two approaches.

In the second example, we simulate a MIMO multi-relay system with different number of antennas at relay ($K = 2, K=4$ and $K=8$). We also simulate the system BER performance of ZF equalizer in MIMO Multi-Relay channel with varying SNR in the relay-to-destination SNR (SNR_r) keeping the source-to-relay link (SNR_s) at 30 dB. Fig. 4 shows the BER performance with $N_s = N_r = N_d = 3$. It can be seen that at $BER = 10^{-2}$, we achieve a 5 dB gain by increasing from $K = 2$ to $K = 8$.

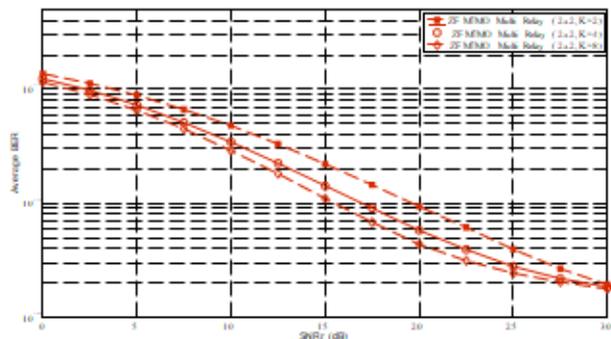


Figure 4- BER versus SNR_r . $N_s = N_r = N_d = 2$, and $SNR_s = 30$ dB for MIMO Multi-Relay channel.



5. CONCLUSIONS

In conclusion, we have demonstrated the advantage of using ZF processing algorithms in MIMO multi-relay network. We designed relays as all-pass amplify-and-forward (AF) units which are simpler to implement. Our results demonstrate that ZF processing algorithms outperform the NAF and PMF algorithms. Future works may include analysis the MMSE equalizer for MIMO multi-relay networks and optimizing the source and the relay matrices to allocate power efficiently in a cooperative MIMO relay network.

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