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REDUCTION OF REACTIVITY FLUCTUATION WITH THE EULER-MACLAURIN METHOD

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ABSTRACT

This work presents the Euler-Maclaurin method with a first-order delay low-pass filter for reducing fluctuations in reactivity calculation. A Gaussian noise around the mean value of the measured neutron population density. This noise is simulated with different standard deviations, with a fixed seed to generate random numbers to reproduce the results. Different numerical experiments show that the proposed method offers high accuracy and low computational cost when compared to different methods reported in literature, especially when compared to the finite difference method and the FIR filter method for different forms of neutron population density.

Keywords: nuclear power plant, nuclear reactor, neutron population density, reactivity, numerical simulation.

INTRODUCTION

The production of electricity at a nuclear power plants is based on the physical principle of nuclear fission. This process, discovered and studied by Austrian physicist Lise Meitner and German radiochemists Otto Hahn and Fritz Strassman, describes the division experienced by a nucleus of a heavy excited atom due to the interaction with thermal neutrons, resulting in a self-sustaining chain reaction. In this fission process approximately 200 MeV of energy are released, which are distributed among fission products, electrons and neutrinos. This released energy is controlled in the nuclear reactor by means of control bars, which are made of neutron absorbent materials to keep the neutron population constant and therefore ensure a controlled chain reaction.

For the safety of a nuclear power plant, it is necessary to know the reactivity parameter; different methods have been developed to know its value more accurately; these methods are based on the discretization of neutron population density (Shimazu *et al.*, 1987; Hoogenboom *et al.*, 1988; Binney and Bakir, 1989; Ansari, 1991; Suescún *et al.,* 2008; Suescún *et al.*, 2013; Hessam and Vosoughi, 2013). Several of these works have not considered fluctuations in the reactor core (Stacey, 2018) and can affect real-time measurements, as well as propagating errors in reactivity calculation.

This work uses the Euler-Maclaurin method (Suescún *et al.*, 2013) with a first-order delay low-pass filter (Shimazu *et al.*, 1987) to reduce fluctuations in reactivity calculation, which is considered noise with Gaussian distribution around the mean density of neutrons.

MATERIALS AND METHODS

The inverse point kinetic equation

Point kinetic equations are a set of seven equations: six equations for the concentration of delayed neutron precursors, and one for neutron population density. (Duderstadt and Hamilton, 1976):

$$\frac{dP(t)}{dt} = \left[\frac{\rho(t)-\beta}{\Lambda}\right]P(t) + \sum_{i=1}^{6}\lambda_i C_i(t) \quad (1)$$

$$\frac{dC_i\ (t)}{dt} + \lambda_i C_i = \frac{\beta_i}{\Lambda}P(t) \quad (2)$$

Where C_i is the concentration of the *i-th* group of delayed neutron precursors, P(t) is the neuron population density, $\rho(t)$ is reactivity, Λ is the neutron generation time, β_i is the *i-th* fraction of delayed neutrons, β is the effective total fraction of delayed neutrons, and λ_i is the decay constant of the *i-th* group of delayed neutron precursors.

Equations (1) and (2) are subject to the following initial conditions that indicate the criticality of a reactor:

$$P(t=0) = P_0 \quad (3)$$

$$C_i(t=0) = \frac{\beta_i}{\Lambda}P_0 \quad (4)$$

To calculate reactivity, it is first of all necessary to solve equation (2) and solve $\rho(t)$ of equation (1), and the following can be obtained:

$$\rho(t) = \beta + \frac{\Lambda}{P(t)}\frac{dP(t)}{dt} - \frac{\langle P_0\rangle}{P(t)}\sum_{i=1}^{6}\beta_i\, e^{-\lambda_i t} - \frac{1}{P(t)}\sum_{i=1}^{6}\int_0^t \lambda_i\beta_i e^{-\lambda_i(t-t')}P(t')\,dt' \quad (5)$$

Equation (5) represents an expression for reactivity. It is used in the different methods which have been proposed, since it is the basis for the construction of digital reactivity meters, however there are difficulties in the implementation of this expression in real time, and it is therefore necessary to discretize the integral term known as neutron population density history.

PROPOSED METHOD

To discretize the neutron population density history of equation (5), the Euler-Maclaurin formula is used, which permits to change from continuous space to discrete space:

$$\int_0^n F(k)dk = \sum_{x=1}^{n-1} F[x] + \frac{1}{2}[F[0] + F[n]] - \sum_{m=1}^{\infty} \frac{B_m}{(2m)!}[F^{(2m-1)}[n] - F^{(2m-1)}[0]] \quad (6)$$

Term B_m refers to the Bernoulli numbers.

By replacing the value of m by the unit, it is possible to rewrite equation (6) considering only the first Bernoulli number

$$\int_0^n F(k)dk = \sum_{x=1}^{n-1} F[x] + \frac{1}{2}[F[0] + F[n]] - \frac{B_1}{2!}[F^{(1)}[n] - F^{(1)}[0]] \quad (7)$$

where $B_1 = 1/6$.

The continuous response of the system to a unit impulse function (Suescún *et al.*, 2008) is defined by:

$$h_i(t - t') = \lambda_i \beta_i e^{-\lambda_i(t-t')} \quad (8)$$

By replacing equation (8) in the integrand of equation (5), we can express:

$$F(t') = h_i(t - t')P(t') \quad (9)$$

A discrete version of equation (9) will be:

$$F[x] = h_i[n - x]P[x] \quad (10)$$

Since the final term of equation (7) indicates the first derivative, it is necessary to derive equation (10) once, this is:

$$F^{(1)}[x] = h_i^{(1)}[n - x]P[x] + h_i[n - x]P^{(1)}[x] \quad (11)$$

Equation (10) is evaluated in $x = n$ and $x = 0$ respectively. Obtaining:

$$F[n] = h_i[0]P[n] \quad (12)$$

$$F[0] = h_i[n]P[0] \quad (13)$$

By replacing equations (12) and (13) in equation (11) we have:

$$F^{(1)}[n] = h_i^{(1)}[0]P[n] + h_i[0]P^{(1)}[n] \quad (14)$$

$$F^{(1)}[0] = h_i^{(1)}[n]P[0] + h_i[n]P^{(1)}[0] \quad (15)$$

By subtracting (14) and (15) the final term of equation (7) can be rewritten in the following way:

$$F^{(1)}[n] - F^{(1)}[0] = \left[h_i^{(1)}[0]P[n] + h_i[0]P^{(1)}[n]\right] - \left[h_i^{(1)}[n]P[0] + h_i[n]P^{(1)}[0]\right] \quad (16)$$

Replacing the value of B_1 in equation (7) we have:

$$\int_0^n F(k)dk = \sum_{x=1}^{n-1} F[x] + \frac{1}{2}[F[0] + F[n]] - \frac{1}{12}[F^{(1)}[n] - F^{(1)}[0]] \quad (17)$$

Now, replacing equations (9-10) in equation (17) the following is obtained:

$$\int_0^n h_i(t - t')P(t')dt' = \sum_{x=1}^{n-1} h_i[n - x]P[x] + \frac{1}{2}[F[0] + F[n]] - \frac{1}{12}[F^{(1)}[n] - F^{(1)}[0]] \quad (18)$$

Replacing equations (12-13) along with equation (16) in equation (18) the following is obtained:

$$\int_0^n h_i(t - t')P(t')dt' = \sum_{x=1}^{n} h_i[n - x]P[x] - \frac{1}{2}[h_i[n]P[0] + h_i[0]P[n]]] - \frac{1}{12}\left[h_i^{(1)}[0]P[n] + h_i[0]P^{(1)}[n] - h_i^{(1)}[n]P[0] - h_i[n]P^{(1)}[0]\right] \quad (19)$$

It is important to note that there is a relationship of equivalence between continuous time and discrete time, given by $t = nT$, where T is the time step in the reactivity calculation.

Replacing equation (19) in equation (5) we obtain:

$$\rho[n] = \beta + \frac{\Lambda}{P[n]}P^{(1)}[n] - \frac{\langle P_0 \rangle}{P[n]}\sum_{i=1}^{6} \beta_i e^{-\lambda_i nT} - \frac{T}{P[n]}\sum_{i=1}^{6}\left[\sum_{x=1}^{n} h_i[n - x]P[x] - \frac{1}{2}[h_i[n]P[0] + h_i[0]P[n]]]\right] + \frac{T^2}{12P[n]}\sum_{i=1}^{6}\left[h_i^{(1)}[0]P[n] + h_i[0]P^{(1)}[n] - h_i^{(1)}[n]P[0] - h_i[n]P^{(1)}[0]\right] \quad (20)$$

In equation (20) the first four terms represent a FIR filter with trapezoidal correction (Suescún *et al.*, 2008). Equation (20) was obtained by (Suescún *et al.*, 2013), but the noise present in neutron density was not considered when making real-time measurements in a nuclear reactor (Stacey, 2018).

This study analyses the accuracy of the method for calculating reactivity when the first four terms of equation (20) are combined, representing a FIR filter, with the correction of the first Bernoulli number in the presence of noise in neutron density.

In order, the method can be applied when the neutron population density measurement presents random noise; the fluctuations in the input signal of the neutron population density are considered to have Gaussian noise around the mean value (Kitano *et al.*, 2000):

$$\overline{P_i} = \frac{1}{N}\sum_{j=1}^{N} P_j \quad (21)$$

To reduce fluctuation in the neutron population density signal, a digital filter called first-order delay low-pass filter is applied (Shimazu *et al.*, 1987), being used to filter noise in low-frequency signals, and is given by the expression:

$$P_i = P_{i-1} + \frac{T}{T+\tau}(\overline{P_i} - P_{i-1}) \quad (22)$$

Where τ is named the filtration constant, and for this case τ = 0.1 *s* and τ = 1.5 *s* is considered.

RESULTS AND DISCUSSIONS

The results of the numerical simulations performed for different forms of the neutron population density are shown below, where the term $\frac{\Lambda}{P[n]}P^{(1)}[n]$ in equation (20) is not taken into account, taking different time step in the calculation of the reactivity. The standard deviation is varied, for all the examples, between σ = 0.01 and σ = 0.001, the low pass filter is applied with 10 numbers of samples in all the numerical experiments and a filtering constant of τ = 0.1 s in the first two examples. Then, the filtering constant is increased for τ = 1.5 s. To generate noise with Gaussian distribution, a seed generating random numbers of $2^{31} - 1$ is used. The physical constants considered to develop this research are presented in Table-1. The reference method is obtained by solving analytically the equation (5). All simulations were performed with a time step of T = 0.1 s.

Table-1. Typical precursor coefficients for ^{235}U.

Group	1	2	3	4	5	6
λ_i [s^{-1}]	0.0127	0.0317	0.115	0.311	1.4	3.87
β_i	0.000266	0.001491	0.001316	0.002849	0.000896	0.000182
Λ=2× 10^{-5} [s]						
β=7 × 10^{-3}						

Case I

Tables 2-3 show the results obtained by comparing the Euler-Maclaurin method with the finite difference method by applying low-pass filter, with filtering constant τ=0.1 and a sample number of 10; the population density of neutrons is of the form $P(t) = a + bt^3$ with $b = 3.67\text{x}10^{-11}$ and a standard deviation of σ = 0.001 and σ =0.01 respectively.

For the numerical experiments under the same conditions presented in the finite difference method (Suescún and Senra, 2010), it is evident that the proposed Euler-Maclaurin with low-pass filter method gives significant reduction in neutron population density fluctuations without causing attenuation.

Table-2. Maximum difference in pcm for a neutron population density of the form $P(t) = a + bt^3$ with $b = 3.67x10^{-11}$ with σ =*0.001*.

$P(t) = a + bt^3$, $t_f = 10000$ s, σ=0.001, τ=0.1		Maximum Differences in reactivity [pcm]	
		Finite Diff + Low-Pass Filter	Euler-Maclaurin + Low-Pass Filter
$a = 1$	b = $3.67\text{x}10^{-11}$	0.76 in t=5374 s	0.60 in t=1697.1 s

Table-3. Maximum difference in pcm for a neutron population density of the form $P(t) = a + bt^3$ with $b = 3.67x10^{-11}$with σ =*0.01*.

$P(t) = a + bt^3$, $t_f = 10000$ s, σ=0.01, τ=0.1		Maximum Differences in reactivity [pcm]	
		Finite Diff + Low-Pass Filter	Euler-Maclaurin + Low-Pass Filter
$a = 1$	b = $3.67\text{x}10^{-11}$	6.61 in *t=2932 s*	5.79 in t=1697.1 s

Case II

Tables 4-7 show a comparison of mean errors and maximum difference in reactivity for the Euler-Maclaurin method and the FIR filter method, applying the low-pass filter, for an exponential neutron population density with different values of ω and different time step in reactivity calculation. In these experiments, it is shown that applying the Euler-Maclaurin method with first-order delay low-pass filter, a reduction in the reactivity fluctuation is obtained. Although the first four terms of equation (13) determine the FIR filter, the correction of the first

Bernoulli number improves the results when comparing with the FIR filter.

Table-4. Absolute mean error for a neutron population density of the form *P(t)=exp(ωt)* with different values of ω with σ *=0.001*.

P(t)=exp(wt) σ=0.001, τ=1.5		Mean absolute error [pcm]	
		Trapezoidal FIR + Low-Pass Filter	**Euler-Maclaurin + Low-Pass Filter**
$t_f = 1000$	$\omega = 0.00243$	0.43	0.09
$t_f = 800$	$\omega = 0.01046$	0.55	0.28
$t_f = 600$	$\omega = 0.02817$	0.86	0.69

Table-5. Maximum difference in pcm for a neutron population density of the form *P(t)=exp(ωt)* with different values of ω with σ *=0.001*.

P(t)=exp(wt) σ=0.001, τ=1.5		Maximum Differences in reactivity [pcm]	
		Trapezoidal FIR + Low-Pass Filter	**Euler-Maclaurin + Low-Pass Filter**
$t_f = 1000$	$\omega = 0.00243$	2.18 in *t=4 s*	1.79 in *t=4 s*
$t_f = 800$	$\omega = 0.01046$	7.42 in *t=3 s*	7.04 in *t=3 s*
$t_f = 600$	$\omega = 0.02817$	18.52 in *t=3 s*	18.13 in *t=3 s*

Table-6. Absolute mean error for a neutron population density of the form *P(t)=exp(ωt)* with different values of ω with σ *=0.01*.

P(t)=exp(wt) σ=0.01, τ=1.5		Mean absolute error [pcm]	
		Trapezoidal FIR + Low-Pass Filter	**Euler-Maclaurin + Low-Pass Filter**
$t_f = 1000$	$\omega = 0.00243$	0.74	0.65
$t_f = 800$	$\omega = 0.01046$	0.82	0.72
$t_f = 600$	$\omega = 0.02817$	1.06	0.95

Table-7. Maximum difference in pcm for a neutron population density of the form *P(t)=exp(ωt)* with different values of ω with σ *=0.01*.

P(t)=exp(wt) σ=0.01, τ=1.5		Maximum Differences in reactivity [pcm]	
		Trapezoidal FIR + Low-Pass Filter	**Euler-Maclaurin + Low-Pass Filter**
$t_f = 1000$	$\omega = 0.00243$	4.05 in *t=5 s*	3.66 in *t=5 s*
$t_f = 800$	$\omega = 0.01046$	8.96 in *t=4 s*	8.59 in *t=4 s*
$t_f = 600$	$\omega = 0.02817$	19.28 in *t=4 s*	18.91 in *t=4 s*

Figures 1-2 shows the reactivity curve when carrying out simulations without applying the low pass filter, and when the low pass filter is applied, respectively. Gaussian noise is generated with a standard deviation $\sigma = 0.01$. It can be seen that considering the Euler-Maclaurin method only is not satisfactory for reactivity calculation because it produces errors of approximately 35% of its real value, and when combined with the first-order delay low-pass filter, there is a reduction in fluctuation, obtaining a maximum difference of 8.59 pcm in *t=4 s*, close to 11% of the real value, but with a very low absolute mean error of 0.72 pcm, which is shown in Figure-3, for $\omega = 0.01046$.

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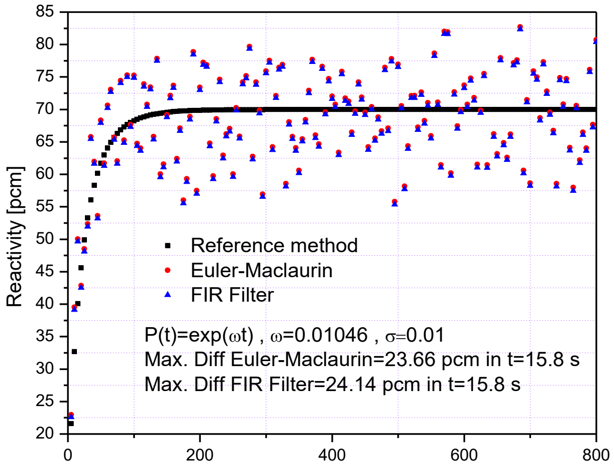

Figure-1. Comparison of reactivity for neutron population density of the form *P(t)=exp(*ωt*)* with $\omega = 0.01046$ without applying low-pass filter with $\sigma = 0.01$.

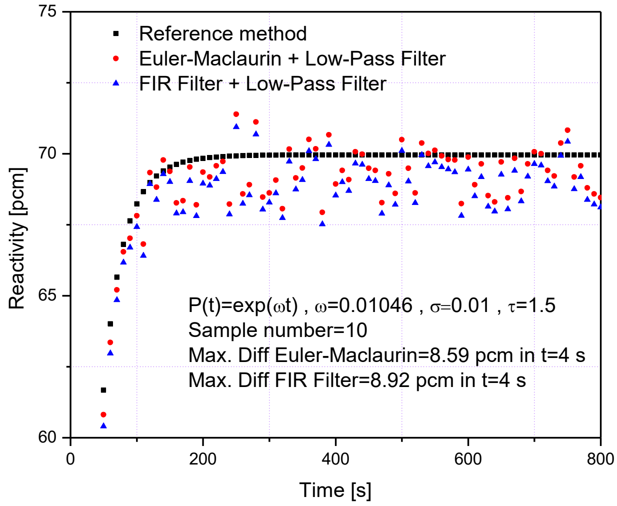

Figure-2. Comparison of reactivity for neutron population density of the form *P(t)=exp(*ωt*)* with $\omega = 0.01046$ applying low-pass filter with $\sigma = 0.01$.

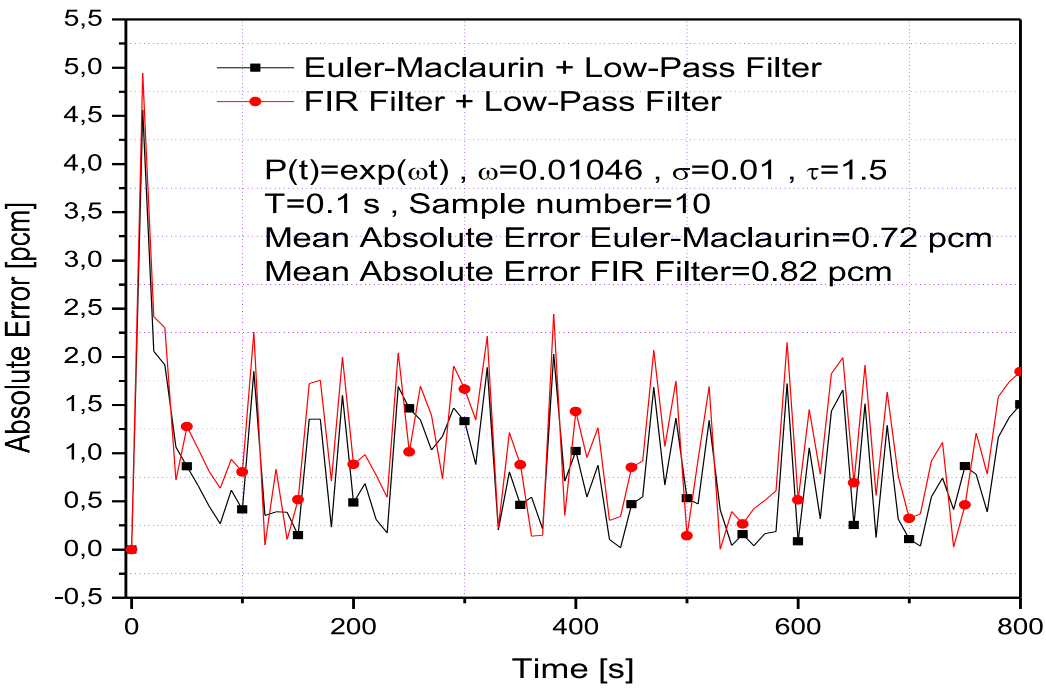

Figure-3. Comparison of absolute mean error for a neutron population density of the form *P(t)=exp(*ωt*)* with $\omega = 0.01046$ using low-pass filter with con $\sigma = 0.01$.

Case III

Tables 8-11 show the results of simulations for neutron population density of the form of $P(t) = a + bt^3$ where *a=1* is fixed and the value of *b* is varied, obtaining a constant value in absolute mean error when using the Euler-Maclaurin method with a σ =0.001 standard deviation. It is shown that Euler-Maclaurin methods with low-pass filter significantly reduce the fluctuations which are present in neutron population density.

Table-8. Absolute mean error for a neutron population density of the form $P(t) = a + bt^3$ with different values of b with σ =*0.001*.

$P(t) = a + bt^3, t_f = 10000\ s$ σ=0.001 , τ=1.5		Mean absolute error [pcm]	
		Trapezoidal FIR + Low-Pass Filter	**Euler-Maclaurin + Low-Pass Filter**
$a = 1$	b= $(0.0127)^5/9$	0.40	0.06
$a = 1$	b= $(0.0127)^4/40$	0.40	0.06
$a = 1$	b= $(0.0127)^4/4$	0.40	0.06

Table-9. Maximum difference in pcm for a neutron population density of the form $P(t) = a + bt^3$ with different values of b with σ =*0.001*.

$P(t) = a + bt^3, t_f = 10000\ s$ σ=*0.001*, τ=*1.5*		Maximum Differences in reactivity [pcm]	
		Trapezoidal FIR + Low-Pass Filter	**Euler-Maclaurin + Low-Pass Filter**
$a = 1$	b= $(0.0127)^5/9$	0.76 in *t=4503 s*	0.36 in *t=4503 s*
$a = 1$	b= $(0.0127)^4/40$	0.78 in *t=1697 s*	0.39 in *t=1697 s*
$a = 1$	b= $(0.0127)^4/4$	0.79 in *t=307 s*	0.39 in *t=307 s*

Table-10. Absolute mean error for a neutron population density of the form $P(t) = a + bt^3$ with different values of b with σ =*0.01*.

$P(t) = a + bt^3, t_f = 10000\ s$ *σ=0.01, τ=1.5*		Mean absolute error [pcm]	
		Trapezoidal FIR + Low-Pass Filter	**Euler-Maclaurin + Low-Pass Filter**
$a = 1$	b= $(0.0127)^5/9$	0.73	0.65
$a = 1$	b= $(0.0127)^4/40$	0.73	0.65
$a = 1$	b= $(0.0127)^4/4$	0.72	0.64

Table-11. Maximum difference in pcm for a neutron population density of the form $P(t) = a + bt^3$ with different values of b with σ =*0.01*.

$P(t) = a + bt^3, t_f = 10000\ s$ *σ=0.01,τ=1.5*		Maximum Differences in reactivity [pcm]	
		Trapezoidal FIR + Low-Pass Filter	**Euler-Maclaurin + Low-Pass Filter**
$a = 1$	b= $(0.0127)^5/9$	3.91 in *t=4503 s*	3.51 in *t=1697 s*
$a = 1$	b= $(0.0127)^4/40$	3.90 in *t=4503 s*	3.49 in *t=1697 s*
$a = 1$	b= $(0.0127)^4/4$	3.90 in *t=4503 s*	3.49 in *t=4502.9 s*

Figures 4-6 show a comparison between the FIR Filter method and the Euler-Maclaurin method by applying Gaussian noise with σ =0.001 standard deviation. When applying the low-pass filter in both methods, it can be seen that the Euler-Maclaurin method overlaps the reference method given by the analytical solution of equation (5), while the FIR filter with low-pass filter is below this method, obtaining an absolute mean error greater than that obtained using the Euler-Maclaurin method. This is, when using the proposed method, in other words, using the Euler - Maclaurin method with the first-order delay low-pass filter, there are no attenuations in reactivity calculation, distinct from what happens in the case of the FIR filter and the finite difference method (Suescún *et al*., 2010) with a first-order delay low-pass filter.

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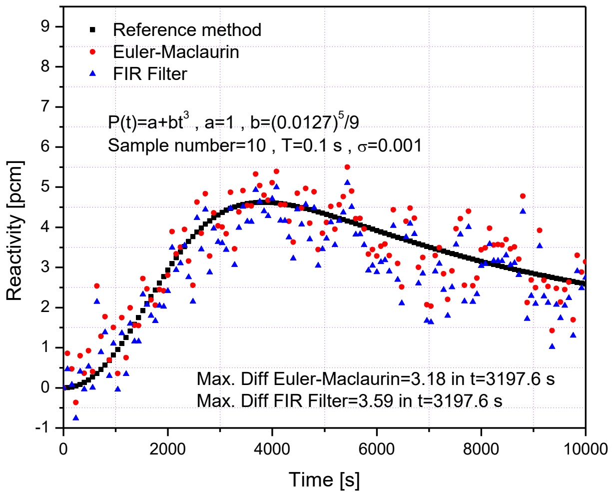

Figure-4. Comparison of reactivity for a neutron population density of the form $P(t) = a + bt^3$ with $b = (0.0127)^5/9$ without applying low-pass filter with $\sigma = 0.001$.

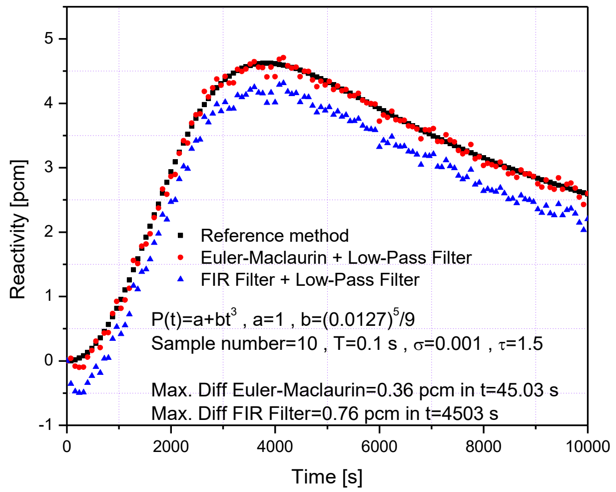

Figure-5. Comparison of reactivity for a neutron population density of the form $P(t) = a + bt^3$ with $b = (0.0127)^5/9$ applying low pass filter with $\sigma = 0.001$.

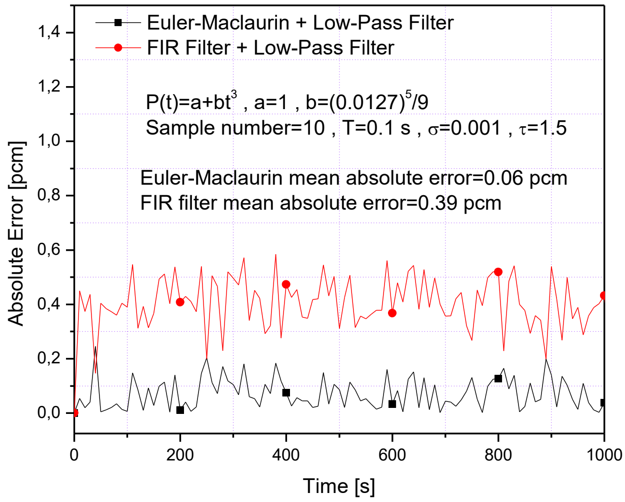

Figure-6. Comparison of absolute mean error for a neutron population density of the form $P(t) = a + bt^3$ with $b = (0.0127)^5/9$ using low-pass filter with $\sigma = 0.001$.

CONCLUSIONS

Numerical experiments were carried out to reduce fluctuations in the calculation of reactivity for different forms of neutron population density, generating random numbers with a seed of $2^{31} - 1$, to simulate noise with different standard deviations around a mean nuclear density value, ignoring the term $\frac{\Lambda}{P[n]} P^{(1)}[n]$ in equation (20). The filtering constant of the low-pass filter varied from τ=0.1 to τ=1.5, and the corresponding simulations were carried out, leaving the time step fixed at T=0.1 s. The numerical experiments show that when the proposed method is used, there is no attenuation, and the reduction of fluctuation in reactivity calculations is successfully achieved.

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