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EFFECT OF JOULE PARAMETER ON MHD MIXED CONVECTION IN AN OPEN CHANNEL WITH SEMI-CIRCULAR HEATER ON THE BOTTOM WALL

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ABSTRACT

A computational study has been done to investigate the heat and fluid flow in an open channel with a semi-circular heater on the bottom wall under the effect of magnetic field. The walls of the channel are adiabatic while the semi-circular heater in the bottom wall is kept at a constant temperature. The inlet and outlet are fixed at the left and right side of the channel. The governing equations are solved by using Galerkin weighted residual finite element technique. In this investigation the involved parameters are Reynolds number, Prandtl number, Hartmann number and joule heating parameter. The effect of Reynolds number, Prandtl number and joule heating parameter for different Rayleigh numbers are investigated while the magnetic parameter Ha (Hartmann number) is kept fixed at 10. The results show that at higher Rayleigh number, joule effect parameter can be utilized to control heat and fluid flow fields. In addition, the effect of Reynolds number on the heat and flow fields becomes insignificant at higher values of Rayleigh number. Finally, Prandtl number is found to have a positive effect on heat transfer rate.

Keywords: channel flow, finite element method, joule parameter, magnetohydrodynamics, semi-circular heater.

1. INTRODUCTION

Many investigators have studied the effect of magnetic field on the fluid flow and heat transfer in conductive fluids problems. Ozoe [1, 2] studied the ability of magnetic field to manipulate heat and fluid flow in metallurgical industries. Reynolds and prandtl numbers effects on MHD mixed convection in a lid-driven cavity along with joule heating and a centered heat conducting circular block investigated by Rahman *et al*. [3]. Oztop *et al*. [4] studied buoyancy induced flow in magneto hydrodynamic-fluid filled enclosure with sinusoidally varying temperature boundary conditions. They found that increasing Hartmann number decreases heat transfer rate within the enclosure. Salam *et al.* [5] numerically studied steady laminar natural convection of air flow in a sinusoidal corrugated enclosure containing a tilted hot thin plate located symmetrically at its centreline. Rahman *et al*. [6] studied the effects of magnetic field on mixed convection flow in a horizontal channel with a heated square cavity at the bottom wall numerically. Their investigation was carried out using Galerkin weighted residual finite element technique. It is found that magnetic field is mostly effective inside the cavity. Moreover, a numerical study on conduction-combined forced and natural convection heat transfer flow of air in a differentially heated lid-driven parallelogram-shaped enclosure divided by a solid partition has been conducted extensively by Chamkha *et al*. [7]. Rahman *et al*. [8, 9] also numerically studied mixed convection flow inside a ventilated cavity with different heat conducting solid bodies. They found that the characteristics of the heat transfer and fluid flow are influenced by the interaction between natural and forced convection in the cavity as well as the cavity aspect ratio. Azad *et al.* [10] also investigated the combined convection in an open cavity under constant heat flux boundary conditions in presence of magnetic field using finite element method. They found that heat transfer rate increase with increase of aspect ratio and decrease with the increase of Hartmann number.

The main objective of the present study is to understand the effect of joule heating parameter field in a long channel with a semi-circular heated cavity. These kinds of geometries can be found in literature for different applications of cavities and electronic devices studied by Kimura *et al*. [11], Oztop [12], Chandra and Chhabra [13], Basak *et al.* [14] and Ozalp [15]. The ultimate motivation is to extend the Billah *et al*. [16] study. They investigated magnetic effects and Rayleigh number effects for the mixed convection having a circular heater. In this study, the authors investigate Rayleigh number, Reynolds number, Prandtl number, and joule parameter effects on flow and heat transfer considering a semi-circular heater at the bottom wall.

In this investigation the paper is organised as flows: Section 2 gives the problem formulation consisting subsection Physical model and mathematical modelling. Section 3 discusses in short numerical methods to solve this problem divided in two subsections, one is numerical procedure and other one is grid sensitivity test with code validation to check the accuracy of the problem. In Section 4 elaborate the results and discussion while section 5 concludes the discussions.

2. PROBLEM FORMULATION

2.1 Physical model

The physical model is a two-dimensional open channel of length L with a semi-circular heater of diameter 0.5L located at the center of the lower wall as seen in Figure-1. The walls are adiabatic except for the semi-circular part which is treated as an isothermal boundary condition with temperature T_h. It is assumed that the height of the channel w = 0.25L, the incoming flow is at a uniform velocity, u_i and ambient temperature, T_i. The magnetic field B_0 is applied horizontally. Gravity acts in vertical direction.

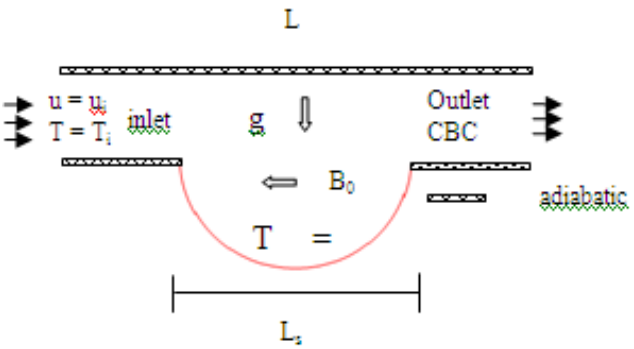

Figure-1. Physical model of the problem.

2.2 Mathematical modeling

The governing equations for the problem under consideration are based on the conservation laws of mass, momentum and thermal energy in two dimensions with Boussinesq approximation. The non-dimensional form of the governing equations become as the following (Rahman *et al*., [6]):

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{1}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{1}{Re}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \tag{2}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{1}{Re}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+\frac{Ra\,\theta}{Re^2 Pr}-\frac{Ha^2}{Re}V \tag{3}$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{Re\,Pr}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right)+JV^2 \tag{4}$$

where the dimensionless variables are defined as:

$$X=\frac{x}{L},Y=\frac{Y}{L},U=\frac{u}{u_i},V=\frac{v}{u_i},P=\frac{(p+\rho gy)L^2}{\rho u_i^2}\,and\,\theta=\frac{T-T_i}{T_h-T_i} \tag{5}$$

Those variables follow the common definition in thermo-fluid literature and are defined in the nomenclature. It can be seen from the Eqs. (2) - (4), five parameters that preside over this problem are the Reynolds number (*Re*), Prandtl number (*Pr*), Rayleigh number (*Ra*), Hartmann number (*Ha*) and joule heating parameter (*J*), which are defined respectively as

$$Re=\frac{u_i L}{\nu},\ Pr=\frac{\nu}{\alpha},\ Ra=\frac{g\beta(T_h-T_i)L^3}{\nu\alpha},$$
$$Ha=B_0 L\sqrt{\sigma/\mu}\ and\ J=\frac{\sigma B_0^2 L u_i}{\rho c_p(T_h-T_i)} \tag{6}$$

The boundary conditions are,

at inlet: $U=1,\ V=0,\ \theta=0$

at outlet: $\frac{\partial U}{\partial X}=0, V=0, \frac{\partial \theta}{\partial X}=0$

at all solid boundaries other than the semi-circle: $U=0, V=0, \frac{\partial \theta}{\partial N}=0$

on semi-circle: $U=V=0, \theta=1$

where *N* represents the normal direction to the solid wall.

The average Nusselt number evaluated along the heated surface of the channel, based on the dimensionless quantities, is expressed as

$$Nu=-\frac{1}{L_s}\int_0^{L_s/L}\frac{\partial \theta}{\partial Y}dX \tag{7}$$

where L_s is the length of the semi-circle.

The non-dimensional stream function is defined as

$$U=\frac{\partial \psi}{\partial Y},\ V=-\frac{\partial \psi}{\partial X} \tag{8}$$

3. NUMERICAL METHOD

3.1 Numerical procedure

Galerkin weighted residual method has been deployed to carry out the numerical solution. Entire domain of interest is discretized into several grids of triangular elements. Governing equations (Eqs. (1) - (4)) are transformed into set of algebraic equations using the finite element method. Bossinique approximation iteration technique has been used to get the numerical solution. The iteration has been done till the solution becomes convergent $|\gamma^{m+1}-\gamma^m|\le 10^{-5}$, where n is number of iteration and γ is the general dependent variable.

3.2 Grid sensitivity test and code validation

Governing equations are discretized by the triangular element and found that results vary with the variation of the element number. A grid independent test is performed considering Re = 100, Ra = 10^5, Ha = 10, Pr = 0.7 and J = 1 to check the accuracy of the numerical procedure and shown in Figure-2. It is evident from the figure that Nusselt number does not change significantly for the element number 4926, 6332 and 9752. Element number 4926 is taken as the standard independent grid size to get better results.

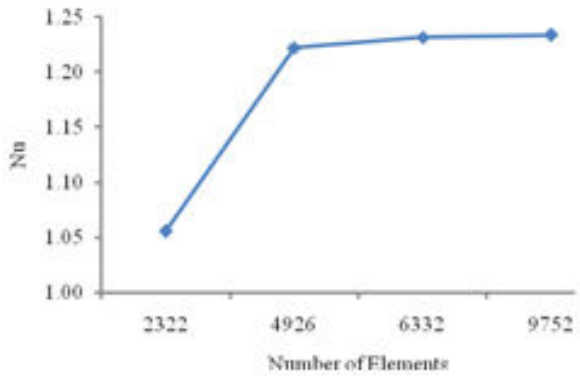

Figure-2. Grid independency study for different grid elements, while Re = 100, Ra = 10^5, Ha = 10, Pr = 0.7 and J = 1.

This numerical code is verified against the published work of Chamkha [17] in the light of average Nusselt number. The numerical solutions reported by Chamkha [17] is based on finite volume scheme using Hartmann number from 0.0 to 50.0 for Gr = 100, Pr = 0.71, and Re = 1000. The comparison of the present work with those of Ref. [17] is shown in Table 1 with respect to average Nusselt number at the hot wall. In the table is clear that present study has a good agreement with the Chamkha [17] solution.

Table-1. Comparison of results for validation at Gr = 100, Pr = 0.71, and Re = 1000.

Parameter Ha	Present Study Nu	Chamkha[17] Nu.	Error (%)
0.0	2.2158	2.2692	2.3532
10.0	2.1174	2.1050	0.5891
20.0	1.7052	1.6472	3.4013
50.0	1.0029	0.9164	8.6249

4. RESULTS AND DISCUSSIONS

The control parameters in the present study are the Rayleigh number, Reynolds number, Prandtl number and joule parameter while Hartmann number is fixed at 10. The effects of these parameters are shown by streamlines, isotherms, average Nusselt numbers, fluid temperature at the outlet of the channel, average pressure and temperature gradients in the domain.

Effects of joule heating parameter on streamlines and isotherms are presented in Figure-3. In these figures, results are presented for Pr = 0.7, Re = 100 and Ra = 10^5 at selected values of joule heating parameter J from 0 to 5. An ellipsoidal cell is formed inside the circular cavity and it rotates in clockwise direction with ψ_{min} = -0.007.

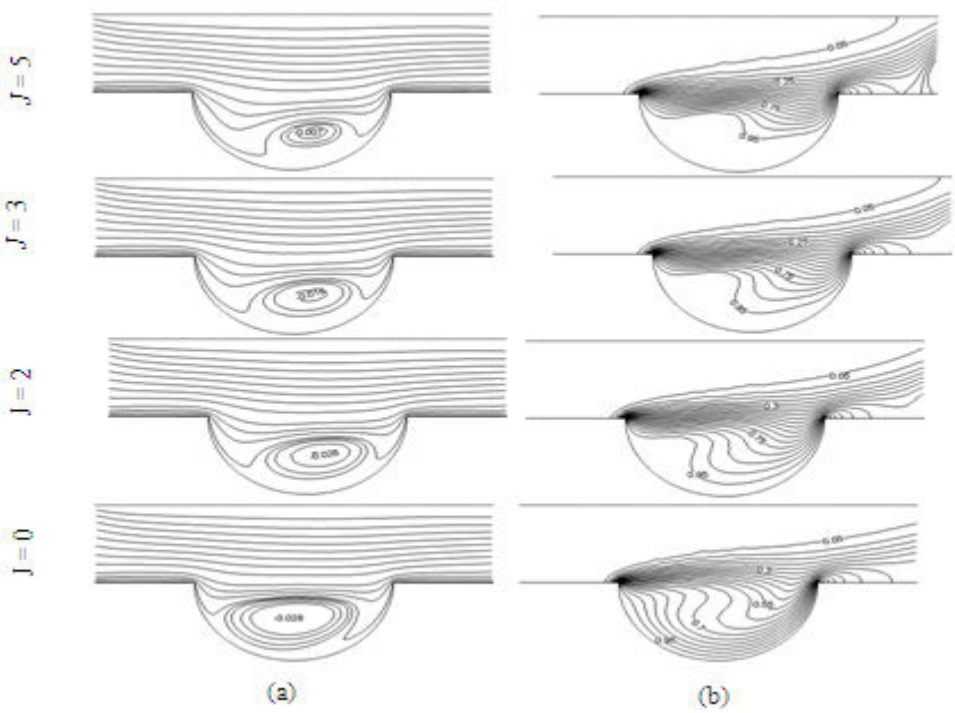

Figure-3. (a) Streamlines and (b) isotherms contours for Ha = 10, Pr = 0.7, Re = 100 and Ra = 10^5 at selected values of joule heating parameter J.

The center of this cell moves in the upstream direction with the decrease of joule parameter due to decreasing of inner heating. The size of the cell becomes larger with decreasing of joule heating. Flow strength also decreases with increasing of joule heating. Isotherms are wavy shaped near the heater. Temperatures are constant inside the channel due to lower conductivity value. For higher values of joule parameters, isotherms are stretched to the channel due to higher values of magnetic effect.

Effects of joule parameter on average Nusselt number, fluid temperature at the outlet of the channel, average pressure, and temperature gradient in the domain for different Ra at Re = 100, Pr = 0.7 are presented in Figure-4. As seen from the Figure-4(a) heat transfer rate decrease linearly in very short way for lower values of Rayleigh number. But it is roughly decrease at higher values of Rayleigh number with the increasing of joule heating parameter. In Figure-4(b) it is seen that for higher values of joule heating parameter fluid temperature at outlet of the channel increases linearly due to the magnetic effect while at higher value of Rayleigh number fluid temperature is increasing gradually. From the Figure-4(c) it is clear that joule heating parameter is not dominant upon the average pressure in the domain for lower values of Rayleigh number. But at higher value of Rayleigh number average pressure is negative and slightly dependent upon joule heating parameter. One the other hand temperature gradient is constant for variation of joule heating parameter at lower values of Rayleigh number shown in Figure-4(d). At higher value of Rayleigh number and different joule heating parameter temperature gradient in the domain makes a parabolic shape.

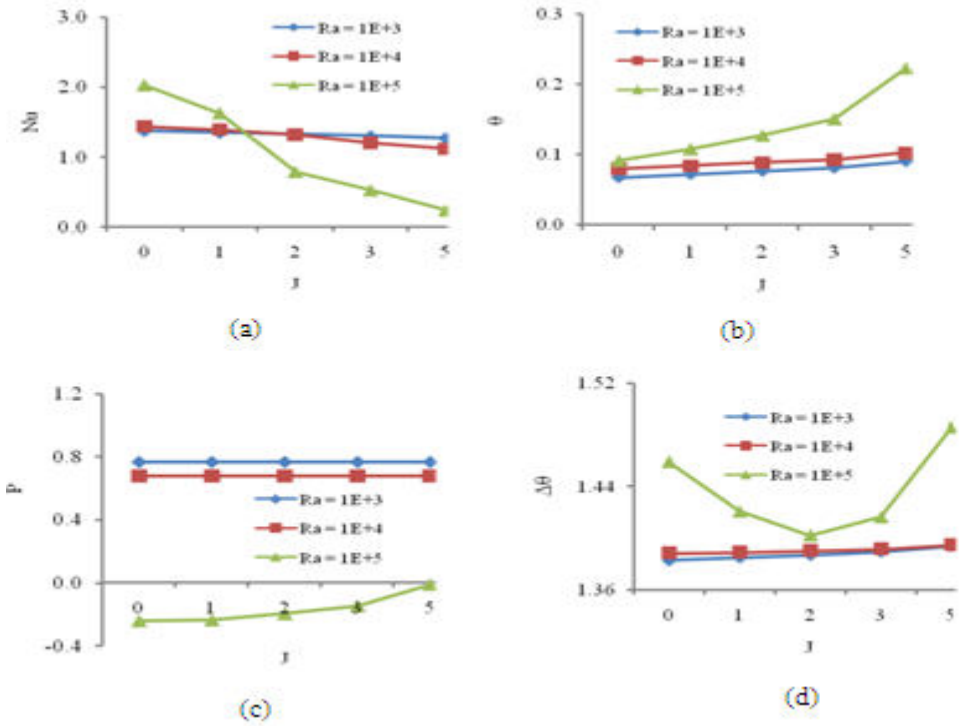

Figure-4. (a) Average Nusselt number, (b) fluid temperature at the outlet of the channel (c) average pressure in the domain, and (d) temperature gradient in the domain versus joule heating parameter *J* for different Ra and Re = 100, Ha = 10, Pr = 0.7.

Streamlines and isotherms for J = 1, Re = 100 and different values of Pr are presented in Figure-5. The figure shows that a circular cell is formed for Pr = 1 with ψ_{min} = -0.018 while higher values of Prandtl number is insignificant over flow field due to increasing of thermal diffusivity. For this case, thermal boundary layer becomes higher in comparison with the case of higher Prandtl numbers and the convection becomes stronger inside the circular cavity. At the downstream part of the semi-circular cavity, thinner thermal boundary layer is formed with increasing of Prandtl number.

Effects of Prandtl number on average Nusselt number, fluid temperature at the outlet of the channel, average pressure in the domain, and temperature gradient in the domain are shown in Figure-6 at Re = 100, J = 1 and different Ra. In Figure-6 (a), average Nusselt number gradually increases with increasing of Prandtl number due to increasing of kinetic energy of the fluid. This increment is smoother for Pr = 1 and 10. As expected, higher values of average Nusselt number is found at higher Rayleigh numbers. However, fluid temperature at the outlet of the channel decreases with increasing of Prandtl number as given in Figure-6 (b). Due to domination of conduction mode of heat transfer values are very close to each other at lower values of Rayleigh number. It brings constant average pressure in the domain for variation of Prandtl

number and pressure increases from P = -0.4 to P = 0.7 at Ra = 10^5 as shown in Figure-6(c). It is interesting to observe that for the higher value of Prandtl number meaning more energy into system and Rayleigh number becomes insignificant. Figure-6 (d) shows temperature gradient changes with Prandtl number. As seen from the figure, temperature gradients increase linearly with Prandtl number and the highest value is obtained for highest value of Rayleigh number.

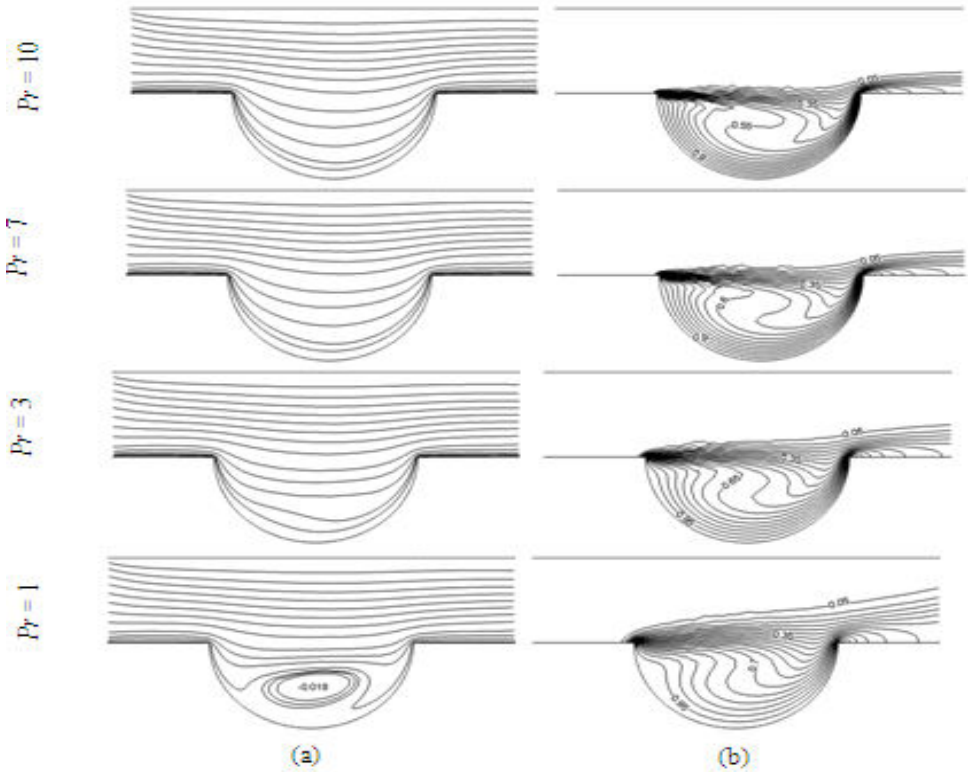

Figure-5. (a) Streamlines and (b) isotherms contours for J = 1, Re = 100, Ha = 10 and Ra = 10^5 at selected values of Prandtl number Pr.

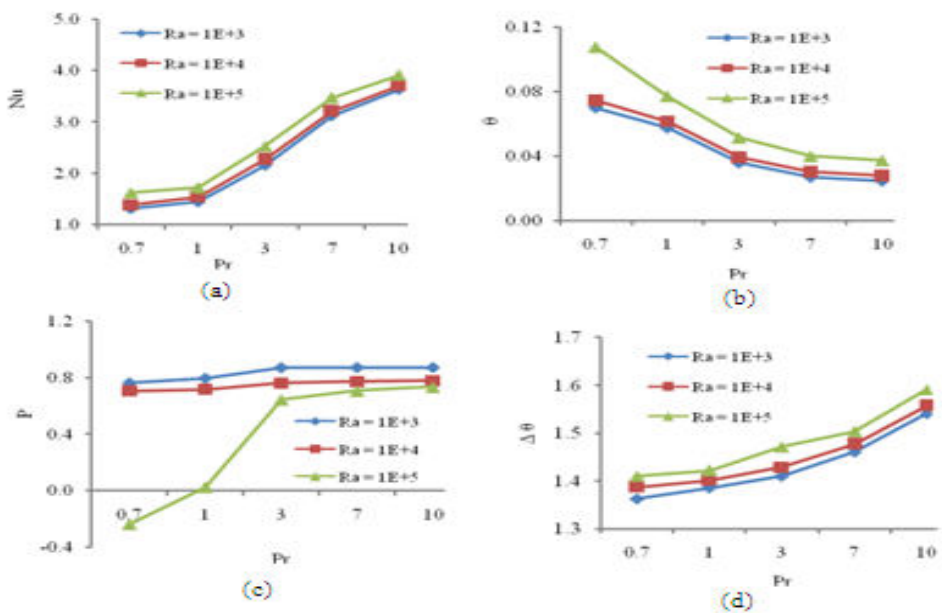

Figure-6. (a) Average Nusselt number, (b) fluid temperature at the outlet of the channel (c) average pressure in the domain, and (d) temperature gradient in the domain versus Prandtl number Pr for different Ra and Re = 100, Ha = 10, J = 1.

Effects of Reynolds number on streamlines and isotherms contours for $J = 1$, $Pr = 0.7$, 10 and Ra = 10^5 at selected values of Reynolds number *Re* is presented in Figure-7. From the figure it is clearly seen that the flow field is not affected by variation of Reynolds number. One the other hand thinner thermal boundary layer is formed for higher values of Reynolds number due to increasing of kinetic energy.

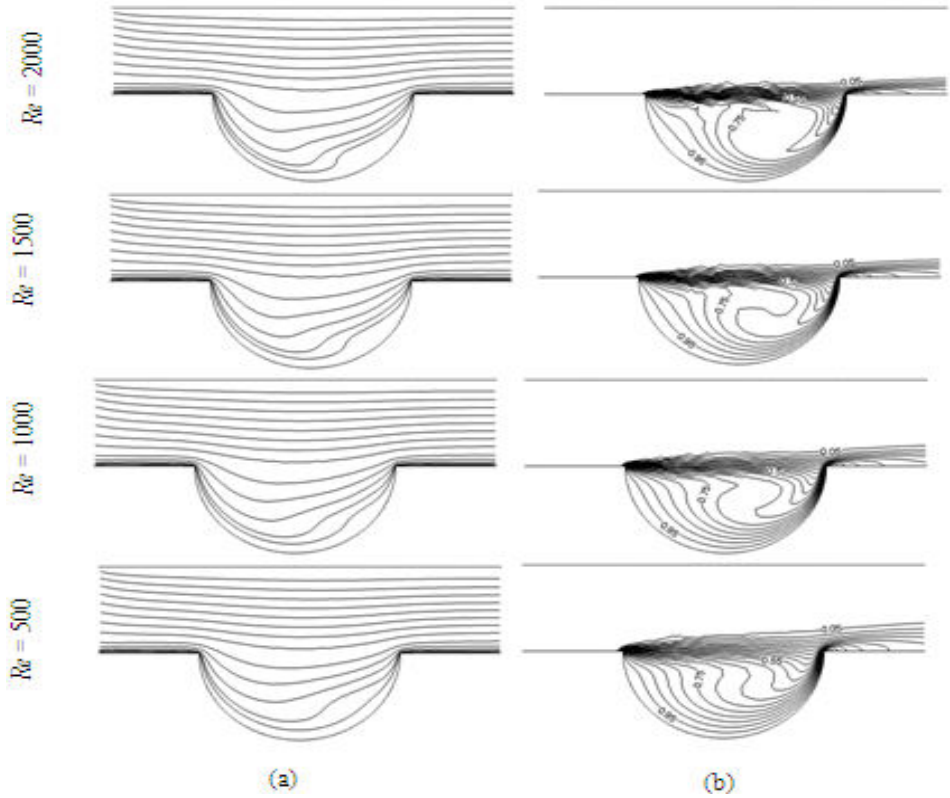

Figure-7. (a) Streamlines and (b) isotherms contours for $J = 1$, $Pr = 0.7$, Ha = 10 and Ra = 10^5 at selected values of Reynolds number *Re*.

The effect of Rayleigh number on average Nusselt number, temperature at the exit, pressure and temperature gradient is also seen in Figure-8 (a)-(d). As shown in Figure-8 (a), overall heat transfer increases with increasing of Reynolds number. This figure also indicates the regions of domination of forced convection. For higher Rayleigh and Reynolds numbers, maximum average Nusselt number is obtained. Temperature values are decreased with increasing of Reynolds number and the lowest value is observed at the highest value of Reynolds number and Ra = 10^3 as shown in Figure-8 (b). Pressure is positive for lower values of Rayleigh number and negative for Ra = 10^5 at Re = 100. For Re = 500, pressure distribution is indifferent to the variation of Rayleigh number and it decreases with increasing of Reynolds number as seen in Figure-8 (c). Temperature gradient in the domain versus Reynolds number for different Rayleigh numbers is plotted in Figure-8 (d). It is clearly seen that temperature gradient smoothly increases increasing values of Re and Ra.

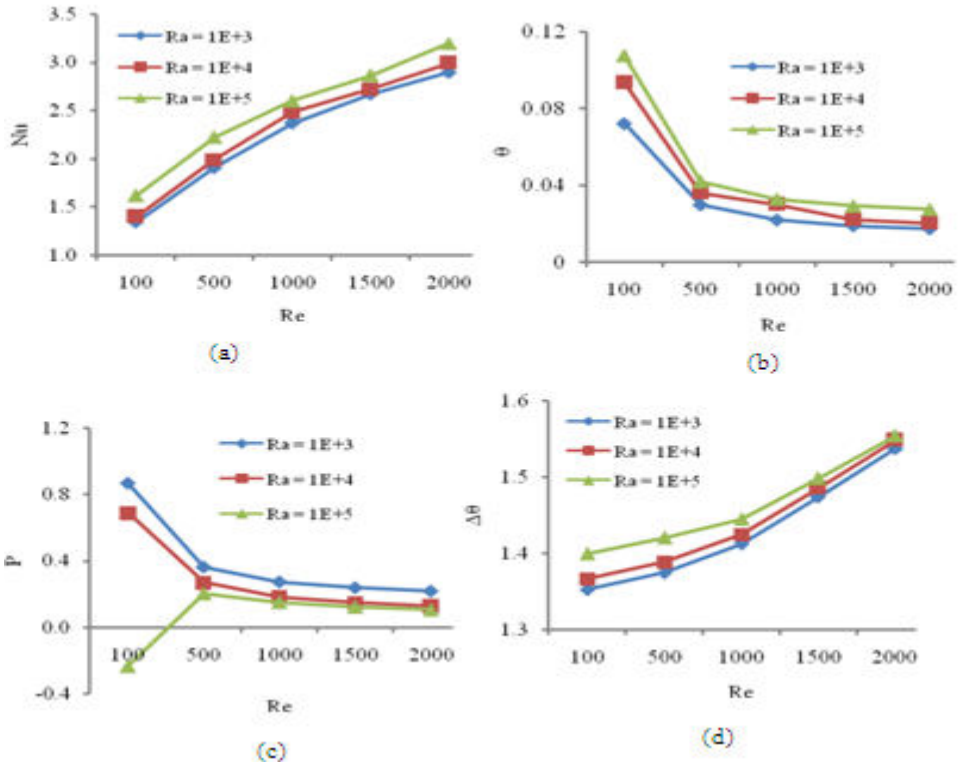

Figure-8. (a) Average Nusselt number, (b) fluid temperature at the outlet of the channel (c) average pressure in the domain, and (d) temperature gradient in the domain versus Reynolds number Re for different Ra and J = 1, Ha = 10 and Pr = 0.7.

5. CONCLUSIONS

A numerical analysis has been performed on mixed convection heat transfer in an open duct with a semi-circular heated cavity.

Some important findings can be drawn as follows:

- Increasing of joule heating parameter affects flow field and temperature distribution for only higher Rayleigh number. Average heat transfer rate decreases and fluid temperature at the exit port as well as temperature gradient in the domain increases with increasing joule heating parameter.
- Effect of Prandtl number on thermal field is significant for highest value of Ra. Overall heat transfer rate increases and fluid temperature at the outlet of the channel decreases with increasing of Prandtl number.
- Thermal field significantly depends on Reynolds number. Average heat transfer rate increases and fluid temperature at the outlet of the channel decreases with increasing of Reynolds number.
- Temperature gradient in the domain linearly increases with increasing of all the mentioned parameters except at the highest value of Ra for the considered values of joule heating parameter.

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NOMENCLATURE

B_0	strength of the magnetic field, (Tesla)
CBC	convective boundary condition
g	gravitational acceleration (ms^{-2})
Ha	Hartmann number
J	joule heating parameter
L	length of the channel (m)
Ls	length of the semi-circle (m)
Nu	average Nusselt number
P	dimensional pressure ($kgm^{-1}s^{-2}$)
P	non-dimensional pressure
Pr	Prandtl number
Ra	Rayleigh number
Re	Reynolds number

T	dimensional temperature (K)
u, v	velocity components (ms^{-1})
U, V	non-dimensional velocity components
w	height of the channel (m)
x, y	Cartesian coordinates (m)
X, Y	non-dimensional coordinates
Greek symbols	
α	thermal diffusivity (m^2s^{-1})
β	thermal expansion coefficient (K^{-1})
μ	dynamic viscosity (kg $m^{-1}s^{-1}$)
ν	kinematic viscosity (m^2s^{-1})
θ	non-dimensional temperature
ρ	density (kg m^{-3})
σ	electrical conductivity (Sm^{-1})
ψ	stream function
Subscripts	
h	heated wall
i	inlet state

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