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STATISTICAL ANALYSIS OF CONCRETE REINFORCEMENT STRENGTH FOR ANALYSIS OF REINFORCED CONCRETE CONSTRUCTION

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ABSTRACT

The article deals with the determination of static characteristics of concrete reinforcement for selected cross-sections. The aim of the paper is to provide a comprehensive processing of a set of laboratory data that can be used in probabilistic calculation or stochastic modeling. Specifically, three test series are processed that differ in diameter. At the same time, the selected diameters also have different manufacturing methods for B500A and B500B. Part of the contribution is regression analysis and determination of the confidence band. The test results are compared with the declared values of the manufacturer and the design standard.

Keywords: reinforced concrete, reinforcement, tensile strength, regression analysis, probability, reliability.

INTRODUCTION

For the design of reinforced concrete structures [1] it is important to determine the mechanical properties of concrete and concrete reinforcement [2]. In both cases, it is important that the character of these mechanical properties is random [3], which can be described, for example, by Gaussian normal distribution. However, statistical characteristics are used for the design of the construction. Most often, for example, the 5 percent quantile denoting the characteristic value [4], [5]. Detailed knowledge from laboratory tests can also be used for probabilistic calculation [6]. The use is especially important for the appropriate determination of input parameters in nonlinear calculation [7], [8]. Typical tasks include slab analysis [9], [10], [11], where reinforcement significantly affects the overall load-bearing capacity and failure. It also includes detailed analysis of concrete beams [12], bottom reinforcement bars of details [13] or steel beam [14].

Among the most important mechanical properties of concrete reinforcement are the yield strength, tensile strength and ductility. These characteristics can be found in the standards ČSN EN 1992-1-1 [1], ČSN EN 10080 [4] and ČSN 42 0139 [5], ČSN EN 10027 [15], [16] or also at the manufacturer of concrete reinforcement.

The basic characteristics given by the standard ČSN EN 1992-1-1 [1] describing the behavior of concrete reinforcement are:

- yield strength (f_{yk} or $f_{0,2k}$),
- maximum actual yield strength ($f_{y,max}$)
- tensile strength (f_t),
- ductility (ε_{uk} a f_t / f_{yk}),
- bendability,
- cohesion characteristics (f_R),
- cross-sectional dimensions and tolerances,
- fatigue strength,
- weldability,
- shear and shape strength for welded nets and lattices.

The most important mechanical properties of concrete reinforcement are determined on the basis of a tear strength test in laboratories. In particular, tensile strength f_u, when this value is exceeded the test specimen breaks. Other characteristics that can be determined on the basis of a tear strength test are the yield strength of f_y or 0.2% yield strength $f_{y,\ 0.2}$, which is determined in the case of concrete reinforcement where the yield strength (typically cold formed concrete reinforcement) is not clear at first sight (Figure-1 (a) and Figure-1 (b)). In this case we are looking for a contract value of the yield strength, which corresponds to 0.2% permanent elongation of the test sample. It is also possible to evaluate the ductility and the modulus of elasticity based on concrete reinforcement tests. The ductility is expressed as the ratio between the measured concrete reinforcement elongation to the original length of the specimen and the modulus of elasticity E_s is based on the validity of Hook's law, where part of the working diagram is linear, as the ratio of stress to strain.

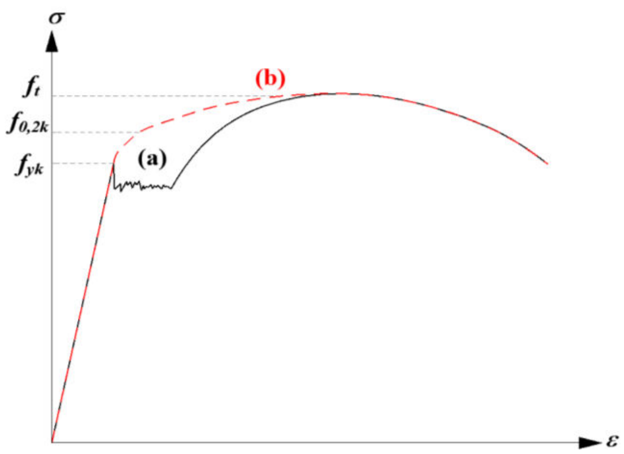

Figure-1. Stress-strain diagrams of typical concrete reinforcing steels: (a) hot-rolled steel, (b) cold-formed steel.

The test specimen is clamped vertically into the jaws of the tear machine and a constant increase in axial tensile force causes the specimen to be stressed until it is completely broken, i.e. the tensile strength limit f_u is reached. During the test, the maximum achieved force F_{max} is recorded, on the basis of which it is possible to determine the tensile strength of the test sample f_u according to the relation:

$$f_u = \frac{F_{\max}}{A} = \frac{F_{\max}}{\frac{\pi \cdot d^2}{4}} \tag{1}$$

where A is the area of concrete reinforcement, F_{max} is the maximum force achieved in the test press and d is the diameter of the reinforcement.

In Table-1 shows all types of concrete reinforcement and its corresponding min. yield strength f_{yk}, respectively contractual yield strength $f_{0.2k}$ according to ČSN EN 1992-1-1 [1]. Table-2 [5] describes a typical designation for concrete reinforcement and Table-3 states the mechanical properties of concrete reinforcement according to ČSN 42 0139 [5].

Table-1. Properties of concrete reinforcement according to ČSN EN 1992-1-1 [1].

Product		Rods and aligned coils		
Ductility class		**A**	**B**	**C**
Characteristic yield strength f_{yk} or $f_{0.2k}$ [MPa]		400 to 600		
Minimum value $k = (f_t/f_y)_k$		≥ 1.05	≥ 1.08	≥ 1.15 < 1.35
Characteristic value of strain at maximum force ε_{uk} [%]		≥ 2.5	≥ 5.0	≥ 7.5
Flexibility		Bending test / return bending test		
Shear strength		-		
Max. deviation from nominal weight (single member) [%]	Nominal wire size [mm] ≤ 8 > 8	± 6 ± 4.5		

Table-2. Numerical designation of steel for concrete reinforcement according to ČSN 42 0139 [5].

steel sign	B420B	B500A	B500B	B550A	B550B
number signification	1.0429	1.0438	1.0439	1.0448	1.0449

Table-3. Mechanical properties according to ČSN 42 0139 (example) [5].

Steel sign	Basic mechanical properties		
	R_e [MPa]	R_m/R_e [-]	A_{gt} [%]
B420B	420	1.08	5
B500A	500	1.05	2.5
B500B	500	1.08	5
B550A	550	1.05	2.5
B550B	550	1.08	5

EXPERIMENTAL MEASUREMENTS

This paper deals with statistical analysis of tensile strength data of concrete reinforcement. These values were measured at testing facilities located in the laboratory of the Faculty of Civil Engineering. For the statistical set, 3 sets of concrete reinforcement were selected, which differed mainly in their diameter and ductility class. The first two test sets contained test specimens of B500A concrete reinforcement with a diameter of 6 and 8 mm. The third test set was made of concrete reinforcement B500B with a diameter of 10 mm. Each set contained a total of 20 test samples. A total of 60 test specimens were tested.

Part of the paper is evaluation of statistical values for each set of test samples, i.e. mean value, median, standard deviation, variance, quantiles etc., determination of probability density and distribution function for normal distribution. The last part will be a regression analysis of the dependence of the diameter of the concrete reinforcement and the maximum achieved force during the test in the press and the evaluation of the reliability band with a probability of occurrence of 95%.

Figures 2, 3 and 4 show the working diagrams on the basis of experiments.

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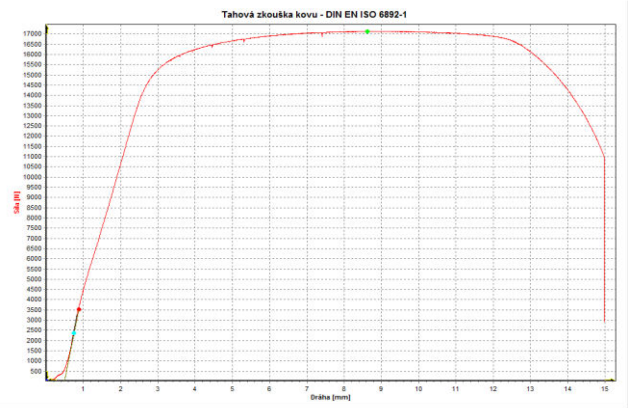

Figure-2. Working diagram of concrete reinforcement Ø 6 mm without significant yield strength.

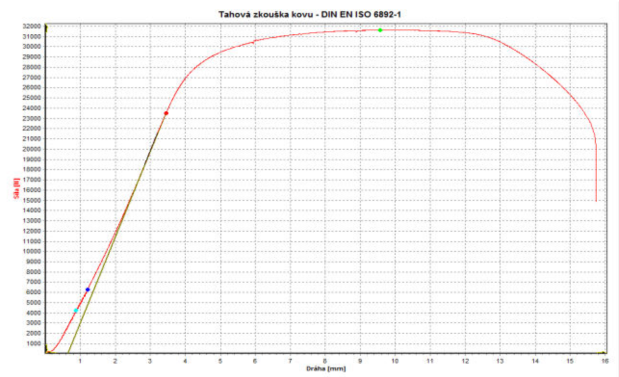

Figure-3. Working diagram of concrete reinforcement Ø 8 mm without significant yield strength.

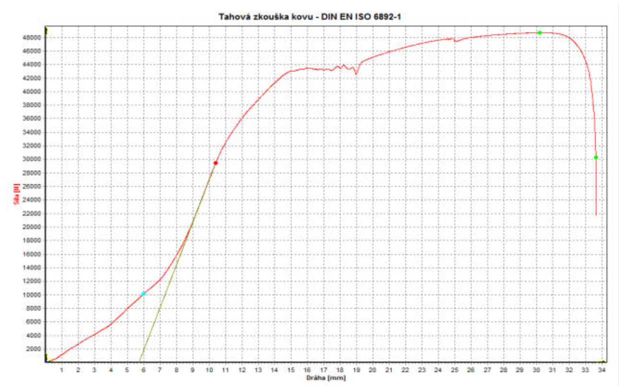

Figure-4. Working diagram of concrete reinforcement Ø 10 mm with significant yield strength.

STATISTICAL ANALYSIS

The default statistically set containing the measured maximum force in the test press F_{max} and the calculated tensile strength f_u is shown in Table-4.

Table-4. Maximum force F_{max} [kN] and tensile strength f_u [MPa].

Sample number	**Ø 6**	**Ø 8**	**Ø 10**	**Ø 6**	**Ø 8**	**Ø 10**
1	17130	30629	48721	605.8	609.3	620.3
2	16088	31635	49946	569.0	629.4	635.9
3	16502	31419	50956	583.6	625.1	648.8
4	16286	30436	49196	576.0	605.5	626.4
5	16807	30745	49780	594.4	611.7	633.8
6	16816	31132	51544	594.7	619.4	656.3
7	17306	31030	51037	612.1	617.3	649.8
8	17162	31132	50534	607.0	619.4	643.4
9	16071	31033	50682	568.4	617.4	645.3
10	17391	31163	50224	615.1	620.0	639.5
11	17517	31374	49169	619.5	624.2	626.0
12	16924	31222	51014	598.6	621.1	649.5
13	17435	30795	49555	616.6	612.6	631.0
14	17382	30943	51549	614.8	615.6	656.3
Sample number	Ø 6	Ø 8	Ø 10	Ø 6	Ø 8	Ø 10
15	16232	31568	51171	574.1	628.0	651.5
16	16578	31518	50970	586.3	627.0	649.0
17	16210	31042	50951	573.3	617.6	648.7
18	16726	31351	51441	591.6	623.7	655.0
19	17336	30961	49353	613.1	615.9	628.4
20	16839	31392	49439	595.6	624.5	629.5

An illustration of a concrete reinforcement test specimen where the tensile strength has been reached is shown in Figure-5. Figure-6 shows a test machine located in the laboratory at the Faculty of Civil Engineering.

Figure-5. The samples concrete reinforcement after tear test.

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Figure-6. The sample concrete reinforcement placed in the press.

In Table-5, all the statistical characteristics that were determined for data file of tests.

Table-5. Statistical characteristics.

Tensile strength	**Ø 6**	**Ø 8**	**Ø 10**
Mean value μ	595.48	619.23	641.22
Mean value error *error*	3.85	1.43	2.55
Median $x_{0.5}$	595.15	619.35	644.36
Modus	-	619.35	-
Standard deviation σ	17.20	6.37	11.40
Selection variance s^2	295.96	40.64	129.99
Sharpness α	-1.33	-0.32	-1.30
Skewness β	-0.21	-0.37	-0.32
Variation range (max-min)	51.14	23.85	36.01
Min	568.40	605.50	620.34
Max	619.54	629.36	656.34
Total	11909.67	12384.64	12824.48
Number	20.00	20.00	20.00
Reliability level (95%)	8.05	2.98	5.34

Other necessary statistical characteristics to evaluate the tensile strength of concrete reinforcement are quantiles, which are typical in that they divide the set statistically into two parts. One part of the set contains values smaller than the selected quantile and the other part is larger.

The basic quantiles for data set evaluation are the lower quartile corresponding to the probability of occurrence p = 0.25 (25%), the upper quartile corresponding to the probability of occurrence p = 0.75 (75%) and the median with probability of occurrence p = 0.5 (50 %). Median is typical in that it divides the dataset into two equal parts.

In the case of determining material properties, the quantiles decisive are:

- Corresponding probability of occurrence p = 0.05 (5%) - the value needed to determine the characteristic value.
- Corresponding probability of occurrence p = 0.01 (1%).
- Corresponding probability of occurrence p = 0.001 (0.1%) - value needed to determine the dominant design value.
- Corresponding probability of occurrence p = 0.1 (10%) - value corresponding to probability of non-dominant design value.

Significant quantiles for evaluation of concrete reinforcement strength are given in Table-6.

Table-6. Quantiles.

Quantiles	**Diameter of concrete reinforcement**		
	Ø 6	**Ø 8**	**Ø 10**
0.1%	568.41	605.58	620.44
1%	568.51	606.23	621.42
5.0%	568.97	609.15	625.75
10.0%	572.88	611.42	626.35
25.0%	581.73	615.86	630.58
50.0%	595.15	619.35	644.36
75.0%	612.34	624.26	649.60
95.0%	616.78	628.09	656.28
99%	618.99	629.11	656.33
99.9%	619.48	629.33	656.34

Another part of the paper is to evaluate the probability density of a normal distribution according to the relation:

$$f(x) = \frac{1}{\sigma \cdot \sqrt{2 \cdot \pi}} \cdot e^{-\left(\frac{x-\mu}{\sqrt{2}\,\sigma}\right)^2} \quad pro\ x\left(-\infty, \infty\right) \qquad (2)$$

After insertion the statistical characteristics into relationship (2), i.e. the mean value μ and the standard

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deviation σ, we get the corresponding function corresponding to the individual data series.

It was also determined the distribution function corresponding to the normal probability distribution according to the equation:

$$F(x)=\frac{1}{\sigma\cdot\sqrt{2\cdot\pi}}\cdot\int_{-\infty}^{x} e^{-\left(\frac{t-\mu}{\sqrt{2}\sigma}\right)^2} dt \tag{3}$$

After insertion the statistical characteristics into relationship (3), i.e. the mean value μ and the standard deviation σ, we get the corresponding function corresponding to the individual data series.

Figures 7 and 8 are graphs of probability density and distribution functions for each data set (Ø 6, 8, and 10 mm), which are offset by the median value to the center of the coordinate system, that the differences more apparent. This figure shows the influence of the standard deviation σ on the probability density curve, respectively distribution function. The Figure-9 and 10 shows the influence of the mean value μ on the probability density curve, respectively distribution function.

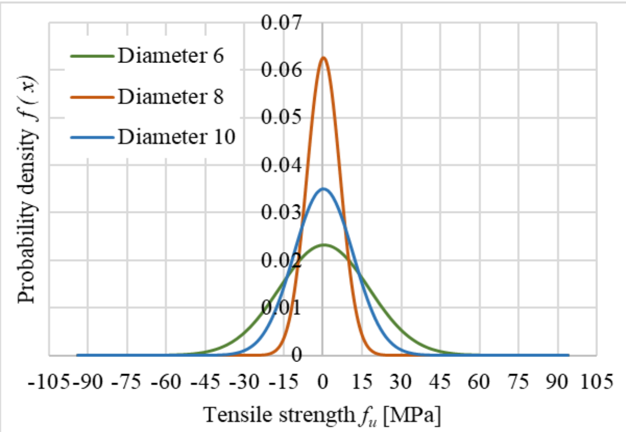

Figure-7. Probability density for a data set corresponding to a diameter of 6, 8 and 10 mm.

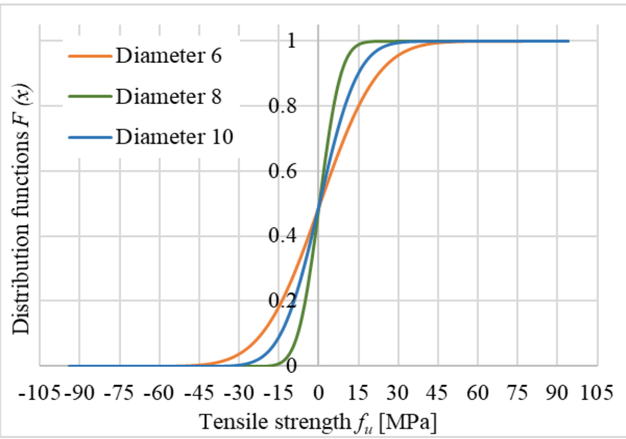

Figure-8. Distribution function for data set corresponding to diameter 6, 8 and 10 mm.

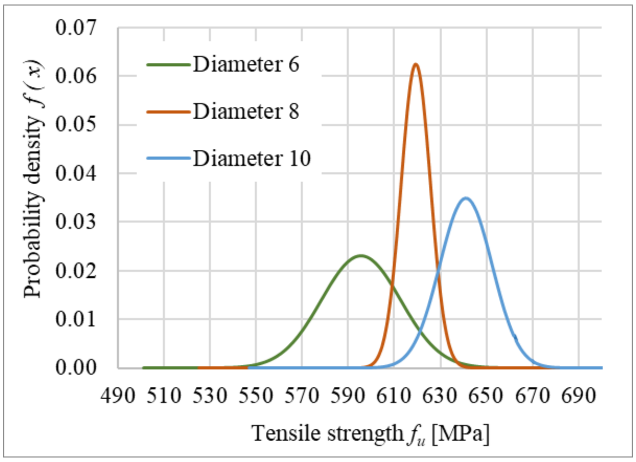

Figure-9. Probability density for a data set corresponding to a diameter of 6, 8 and 10 mm.

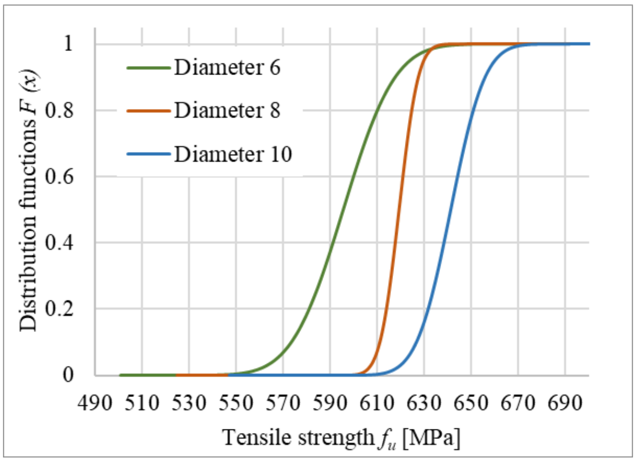

Figure-10. Distribution function for data set corresponding to diameter 6, 8 and 10 mm.

REGRESSION ANALYSIS AND BELT OF RELIABILITY

The default statistical data for determining dependence, so-called regression analysis are available in Table-4. Regression analysis deals with determining the dependence of two variables *x* and *y*. In this case, it is a matter of determining the dependence of the diameter of the concrete reinforcement on their maximum force developed in the testing device. In total, the data set contained 60 values, which were determined on the tear test. For each concrete reinforcement diameter (6, 8, 10) 20 samples were tested.

The general equation for linear regression with one explanatory variable for any x, i.e. the diameter of reinforcement d is:

$$\hat{Y}(x)=b_0-b_1\cdot x \tag{4}$$

Statistical quantity describing the quality of the regression model, respectively force dependence between individual quantities, in this case the dependence of the achieved maximum force in the test press F_{max} and the diameter d of the concrete reinforcement, is the index of

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determination I^2. The determination index I^2 is calculated using the relation:

$$I^2 = \frac{SS_{\hat{Y}}}{SS_Y} \tag{5}$$

where $SS_{\hat{Y}}$ is the sum of the squares of the model and SS_Y is the total sum of squares.

In this case, the index of determination is close to 1 (more precisely 0.991), so we can assume that the regression model is of good quality and the studied dependence has a strong meaning.

Another statistical characteristic describing the direction of relations of two variables and the mutual relationship between them is the so-called correlation coefficient. In this regression model, the correlation number was positive with the value 0.995, so it is possible to assume that the selected monitored variables grow and fall together and it is also a number close to 1, so they have a strong degree of dependence. The correlation coefficient is calculated according to the formula:

$$\sqrt{I^2} = \sqrt{\frac{SS_{\hat{Y}}}{SS_Y}} \tag{6}$$

The statistics describing the regression model are shown in Table-7. The coefficients to build the linear regression and the factors describing the significance of the model are shown in Table-8. Figure-11 shows the regression analysis describing the dependence of the maximum achieved force in the press and the diameter of the reinforcement.

Table-7. Regression analysis.

Correlation coefficient (Multiple R)	0.995
Determination index I^2	0.991
Modified index determination I^2	0.990741
Mean value error	1334.2
Number of observations	60

Table-8. Coefficients for linear regression equation.

Coefficients		Mean value error S_{bi}	t-stat	p-value
b_0	-34274.6	861.2034	-39.7985	8.32E-44
b_1	8381.175	105.4755	79.46091	6.85E-61

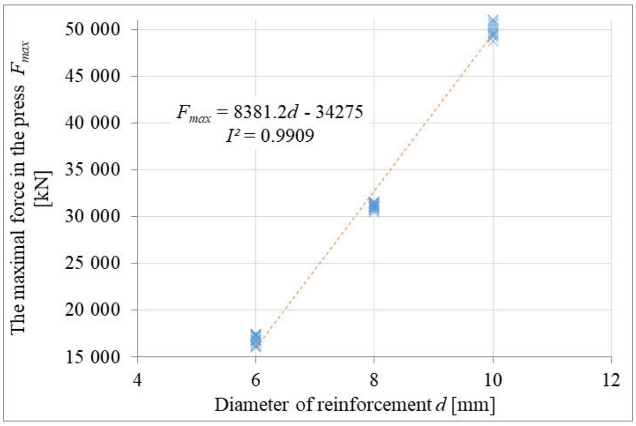

Figure-11. Regression analysis.

Furthermore, a 95% confidence band was created around the regression line for maximum achieved force in the press depending on the diameter of the concrete reinforcement. This confidence band can be used to predict the maximum force F_{max} achieved in a press with a probability of 95%. The upper limit of the confidence band is calculated according to the formula:

$$\hat{Y}(x) + S_{\hat{Y}} \cdot t_{1-\frac{\alpha}{2}} \cdot (n-2) \tag{7}$$

And the lower limit according to the relation:

$$\hat{Y}(x) - S_{\hat{Y}} \cdot t_{1-\frac{\alpha}{2}} \cdot (n-2) \tag{8}$$

where $\hat{Y}(x)$ is a regression function, $S_{\hat{Y}}$ is the sum of the squares of the model, α is reliability. The upper and lower limits of the confidence band are shown in Figure-12.

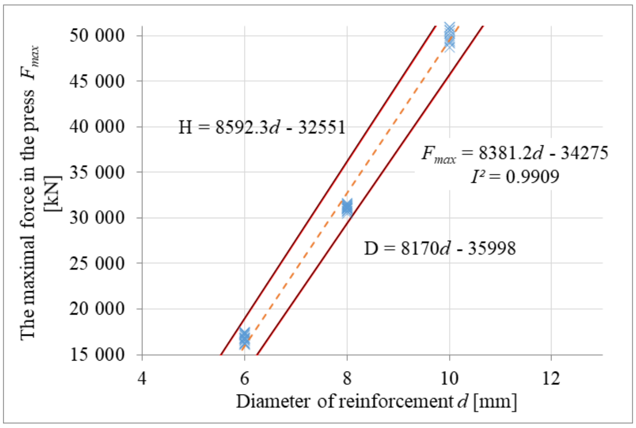

Figure-12. 95% Reliability belt.

CONCLUSIONS AND DISCUSSIONS

The experimentally determined values of tensile strength f_u of concrete reinforcement were compared with the values in the standard ČSN 42 0139 [5]. The values calculated on the basis of the experiment, i.e., the 5% quantile of the tensile strength, which corresponds to the characteristic value of the tensile strength, are larger by about 10-20%. The comparison is shown in Table-9. The standard gives values lower than those obtained from the experiment. When designing reinforced concrete structures according to standards, a sufficient margin is created.

The paper also included evaluation of probability density and distribution function. The probability density for the reinforcement tensile strength of the B500A Ø 6 had the highest standard deviation σ = 17.20 and the largest sampling variance s^2 = 295.96. The lowest variance and hence the smallest standard deviation was observed for the B500A reinforcement Ø 8, i.e. σ = 6.37 and s^2 = 40.64 and the probability density graph has the highest sharpness. In all cases, there was a flat probability density distribution with a slight tilt to the right side, as the sharpness and skewness values were negative.

A regression analysis was also performed to determine the dependence of the maximum achieved force F_{max} in the test press and the diameter d of the concrete reinforcement. The regression model showed high statistical significance, which corresponds to the value of the determination index I^2 = 0.991. The dependence of force F_{max} on the diameter of reinforcement d is thus given by:

$$F_{max} = 8381.2d - 34275 \tag{9}$$

where d is the diameter of the reinforcement in mm and F_{max} is the maximum force for the calculation of the tensile strength of the concrete reinforcement in kN. The authors want to use the results of the research and the knowledge from the examinations in the design and analysis of engineering structures [17] using high performance computing.

Table-9. Comparison of experimentally determined and tensile strength values given in the standard.

Standard ČSN 42 0139					**Experiment**		**Difference from standard [%]**
Signification	**Diameter [mm]**	**Yield strength R_e [MPa]**	**R_m / R_e [-]**	**Tensile strength R_m [MPa]**	**Mean strength value in tensile R_m [MPa]**	**5% quantile Tensile strength R_m [MPa]**	
B500A	6	500	1.05	525	595.5	568.97	13.4
B500A	8	500	1.05	525	619.2	609.15	17.9
B500B	10	500	1.08	540	641.2	625.75	18.7

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