VOL. 15, NO. 3, FEBRUARY 2020 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

STUDY OF FLUCTUATIONS WITH THE MATRIX FORMULATION METHOD

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ABSTRACT

This article presents a study for the calculation of reactivity using the matrix formulation method for reducing fluctuations in the density of the neutron population present in a nuclear reactor. A first-order low-pass delay filter was used, assuming random noise with Gaussian distribution around the mean value of the neutron population density. Studies were made of the different values that the density of neutrons could have, considering an exponential form, a specific time step to carry out experiments with different filtration constants. The results show how it is possible to reduce fluctuations, obtaining a very low average error, using the low- pass filter with the matrix formulation method.

Keywords: low-pass filter, matrix formulation, point kinetics, nuclear reactors.

INTRODUCTION

Reactivity is a very important parameter in the operation of a nuclear reactor, and knowing this parameter allows programming of the movement of the control rods, preventing accidents [1], and keeping the chain reaction controlled. The calculation of the reactivity is done using digital meters, where it is necessary to know the neutron population to solve the inverse point kinetic equation.

In literature there are various numerical methods for calculating reactivity without considering fluctuations [2], [3]. In other works, these fluctuations, produced by the movement of the control rods or by the same physical phenomenon, have been taken into account, generating signals with noise, making it difficult to calculate reactivity[4], [5].

Different filters can be found to reduce fluctuations, such as the first order low-pass delay filter [6] with the use of the Laplace transformation there is the FIR Filter [7], and the Savitzky-Golay filter [8]. Another study uses the Kalman filter to estimate reactivity [9] and up to now the most recent with the Fourier Discrete Transformation using two filters; the Savitzky-Golay filter and the first-order low-pass delay filter [10]. A recent method using matrix formulation [11], was used to calculate reactivity without taking into account the fluctuations, therefore in this work it is proposed to reduce the fluctuations in the calculation of the reactivity using that formulation with the first order low-pass delay filter, considering Gaussian noise around a mean value of neutron population density.

This document is organized as follows: in the second section, we present the point kinetic equations and their inverse, the random noise and the low-pass filter are exposed. In the third section a brief presentation of the matrix formulation method is made, and finally a presentation of the results and conclusions.

THEORETICAL ASPECTS

To know the neutron population, there is a system of coupled differential equations from the neutron diffusion equation [12]. This physical-mathematical system is described as:

$$\frac{dn(t)}{dt} = \frac{\rho(t)-\beta}{\Lambda} n(t) + \sum_{i=1}^{m} \lambda_i C_i(t) \tag{1}$$

$$\frac{dC_i(t)}{dt} = \frac{\beta_i}{\Lambda} n(t) - \lambda_i C_i(t) \qquad i = 1,2,\ldots,m \tag{2}$$

With the following initial conditions:

$$n(t=0) = n_0 \tag{3}$$

$$C_i(t=0) = \frac{\beta_i}{\Lambda\lambda_i} n_0 \tag{4}$$

Where $n(t)$ is the neutron population density, ρ is the reactivity, Λ is mean neutron generation time, $C_i(t)$ the concentration of the *i-th* group of delayed neutron precursors, β is the effective total fraction for delayed neutron precursors, $\beta_i$is the *i-th* fraction of delayed neutrons and λ_i is the decay constant of the *i-th* group of delayed neutron precursors.

To establish the reactivity; based on equation (1) it is possible to rewrite the equation according to the density of the neutron population and the concentration of precursors, in the following way:

$$\rho(t) = \beta + \frac{\Lambda}{n(t)}\frac{dn(t)}{dt} - \frac{\Lambda}{n(t)}\sum_{i=1}^{m} \lambda_i C_i(t) \tag{5}$$

Equation (5) is called the inverse point kinetic method. By solving equation (2), the concentration of precursors is obtained,

$$C_i(t) = \frac{\beta_i}{\Lambda}\left[\frac{e^{-\lambda_i t}}{\lambda_i}\langle n_0\rangle + \int_0^{t'} e^{\lambda_i(t'-t)} n_{(t')} dt'\right] = \frac{\beta_i}{\Lambda} H_i(t) \tag{6}$$

Where, $H_i(t)$ is the historic of neutron density, equation (6) can be analytically solved by different forms of neutron density $n(t)$.

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It is possible to simulate the random noise of Gaussian distribution around of the mean value of neutron population density, in the following way:

$$\bar{n}_i = \frac{1}{N}\sum_{j=1}^{N} n_j \tag{7}$$

Where $\bar{n}_i$ is the mean density of neutron population, and N the number of samples, for the reduction of fluctuations the first order low-pass delay filter is presented, and expressed as:

$$n_i = n_{i-1} + \frac{h}{h+\tau}(\bar{n}_i - n_{i-1}) \tag{8}$$

Where h is the time step and τ time constant of the filter which is commonly taken the same as $\tau = 0.1$[13].

PROPOSED METHOD

In this work, the Matrix Formulation method is put into practice [11]. This method avoids the neutron density history of equation (6), therefore its computational cost decreases. This method thus consists of solving the system of differential equations of the concentration of precursors by means of equation (2), using the matrix form, which reduces it to a single homogeneous first order differential equation, described as follows:

$$\frac{d\vec{x}(t)}{dt} = S(t)\vec{x}(t), \ \ \vec{x}(0) = \vec{x}_0 \tag{9}$$

where $\frac{d\vec{x}(t)}{dt}$ and $\vec{x}(t)$ are vectorial functions with dimensions of $m+1$, with their initial conditions n_0 and x_0. The matrix function $S(t)$ is of dimensions $(m+1)\times(m+1)$, and are described as:

$$\frac{d\vec{x}(t)}{dt} = \begin{bmatrix} n(t) \\ C_1(t) \\ C_2(t) \\ \vdots \\ C_{m(t)} \end{bmatrix}, ; \vec{x}(t) = \begin{bmatrix} n(t) \\ C_1(t) \\ C_2(t) \\ \vdots \\ C_{m(t)} \end{bmatrix}; \ \vec{x}(0) = n(0)\begin{bmatrix} 1 \\ \beta_1/\Lambda\lambda_1 \\ \beta_2/\Lambda\lambda_2 \\ \vdots \\ \beta_m/\Lambda\lambda_m \end{bmatrix} \tag{10}$$

$$S(t) = \begin{bmatrix} \frac{n'(t)}{n(t)} & 0 & 0 & \dots & 0 \\ \frac{\beta_1}{\Lambda} & -\lambda_1 & 0 & \dots & 0 \\ \frac{\beta_2}{\Lambda} & 0 & -\lambda_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\beta_m}{\Lambda} & 0 & 0 & \dots & -\lambda_m \end{bmatrix} \tag{11}$$

The homogenous system of equation (9) is an initial value problem which is solved at any moment in time. The solution to equation (9) is as follows:

$$\vec{x}_{n+1} = X_n e^{D_n} {X_n}^{-1}\vec{x}_n \tag{12}$$

RESULTS

The results of the different numerical experiments for the calculation of the reactivity of equation (5) are presented below, taking into account the random noise of equation (7). Also shown are the maximum differences and the average errors for neutron population density of the form $n(t) = e^{wt}$ with a time step h = 0.01s. The matrix formulation method given by equation (12) with a low-pass filter is used to reduce fluctuations, using equation (8). For this simulation, we considered kinetic parameters, such as the decay constant λ_i equal to (0.0127, 0.0317, 0.115, 0.311, 1.4 and 3.87 s^{-1}), the delayed neutron fractions β_i are (0.000266, 0.001491, 0.002849, 0.000896 and 0.000182) and the generation time $\Lambda = 2 \times 10^{-5}$ s. The low- pass filter is used with different τ; (0.1, 0.5, 1 and 1.5), and a random Gaussian noise is used around the mean neutron density, with different degrees of noise σ given as (0.001, 0.01 and 0.1).

Table-1 shows the maximum differences for a step size $h = 0.01$s and standard deviation of $\sigma = 0.001$, with respect to the matrix formulation method (MF). When applying the low-pass filter for the first values of w: 0.006881 and 0.01046, the $\tau = 0.1$ and $\tau = 0.5$ make it possible to reduce the maximum error, for example for $w = 0.01046$, the maximum error with noise gives a value of 3.06 pcm, but with $\tau = 0.1$, it is reduced by 1.21 pcm.With this, for the range w of 0.006881 to 0.02817 the low-pass filter which best reduces fluctuations is for$\tau = 0.1$, since by increasing the value of τ, there is no improvement in its precision. It can be noted that by increasing the value of w, the low-pass filter has difficulty in reducing the maximum difference, and this is notable in the case of $w = 0.12353$, where the maximum error with noise is 1.62 pcm, and by applying the low-pass filter, the least error is for $\tau = 0.1$, giving an error of 9.57 pcm, for values $w \geq 0.12353$, the low-pass filter is not successful in reducing the fluctuations and is therefore not recommended to use the filter.

Table-1. Maximum differences in pcm for the neutron density form, n(t)=exp(wt), with h=0.01 s and $\sigma = 0.001$. Using the first-order delay low-pass filter.

		Maximum Differences (pcm)				
w	t(s)	Noise	$\tau = 0.1$	$\tau = 0.5$	$\tau = 1$	$\tau = 1.5$
0.006881	500	3.15	0.92	2.02	3.26	4.29
0.01046	800	3.06	1.21	2.99	4.88	6.44
0.016957	300	2.69	1.73	4.74	7.78	10.25
0.02817	600	2.76	2.62	7.70	12.60	16.55
0.12353	300	1.62	9.57	29.95	47.43	60.36

In Table-2, the average errors for a time step h = 0.01s with a standard deviation of σ = 0.001, with respect to the MF method, are shown. Unlike the maximum difference in Table 1, the mean errors show good results without using a filter. When applying the low-pass filter for the first two values of w: 0.006881 and 0.01046, with all the proposed τ, succeed in reducing the mean errors. As w increases, the τ = 1 and τ = 1.5 fail to reduce the fluctuations, being τ = 0.1 which presents the most precision in the mean errors, in contrast to τ = 0.5, τ = 1 and τ = 1.5. Therefore, it can be seen that for the w between 0.006881 and 0.02817, there is a decrease in the mean error, the reduction being greater when τ = 0.1, however for large w such as 0.12353 it is recommended not to use the low-pass filter, because the fluctuations are too great.

Table-2. Mean errors in pcm for the neutron density form, n(t)=exp(wt), with h=0.01 s and $\sigma = 0.001$. Using the first-order delay low-pass filter.

		Mean absolute errors (pcm)				
w	t(s)	Noise	$\tau = 0.1$	$\tau = 0.5$	$\tau = 1$	$\tau = 1.5$
0.006881	500	0.52	0.10	0.11	0.16	0.21
0.01046	800	0.50	0.10	0.11	0.15	0.19
0.016957	300	0.50	0.14	0.25	0.43	0.60
0.02817	600	0.45	0.14	0.21	0. 34	0.46
0.12353	300	0.23	0.26	0.57	1.02	1. 45

Figure-1 shows the mean error result obtained for a numerical experiment using the low-pass filter with w = 0.016957, with the degree of noise σ and time step$h$ proposed n Table-2 with τ = 0.1. Its maximum difference is 1.73 pcm and the average error of 0.14 pcm, which gives a good indication of the low-pass filter for this type of case.

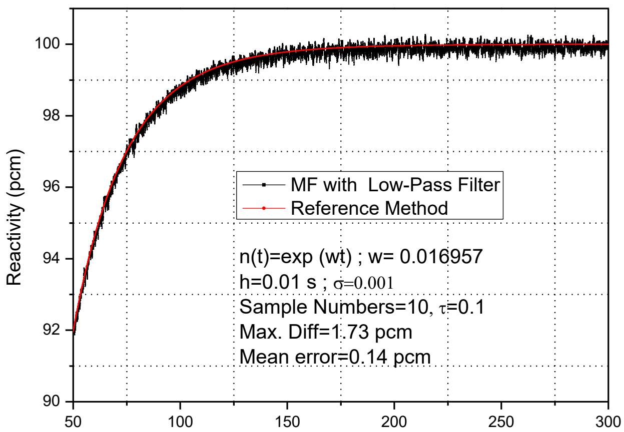

Figure-1. Variation in reactivity for a neutron population density of the form n(t)=exp(wt), with h=0.01 s, $\sigma = 0.001$ for $w = 0.016957$ with low-pass filter of τ =0.1.

Table-3 shows the maximum differences of the MF method for the same time step h = 0.01s as presented in Tables 1 and 2, but its standard deviation is now increased to σ = 0.01. It can be observed that for the

VOL. 15, NO. 3, FEBRUARY 2020 ISSN 1819-6608
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values of w from 0.006881 to 0.02817, all the τ proposed notably decrease the noise. For w = 0.12353, its maximum difference with only noise, that is, without using the filter, is 16.77 pcm. However, using the low-pass filter with τ = 0.1, the difference decreases to 12.59 pcm. It is also shown that for this w; by increasing the filter for others τ, the results become less favorable. With this, it is proven that for w greater than 0.12353, the filter does not give good results for reducing the maximum differences.

The same parameters in Table-3 result in the mean errors observed in Table-4. For the two smaller values of w, that is w = 0.006881 and w = 0.01046 with filter for τ = 1 is the most favorable, but as the frequency increases, τ should decrease. Therefore, for larger w than those presented in this article, the low-pass filter cannot reduce the mean error. Even so, for the values of w from 0.006881 to 0.12353, it reduces the mean errors.

Table-3. Maximum differences in pcm for the form of neutron density, n(t)=exp(wt), with h=0.01 s and $\sigma = 0.01$. Using the first-order delay low-pass filter.

		Maximum Differences (pcm)				
w	**t(s)**	**noise**	$\boldsymbol{\tau = 0.1}$	$\boldsymbol{\tau = 0.5}$	$\boldsymbol{\tau = 1}$	$\boldsymbol{\tau = 1.5}$
0.006881	500	32.83	5.60	3.23	4.19	4.90
0.01046	800	31.82	5.45	4.18	5.77	7.01
0.016957	300	27.66	5.07	5.92	8.61	10.78
0.02817	600	28.28	5.94	8.88	13.40	17.07
0.12353	300	16.77	12.59	30.93	48.36	60.89

Table-4. Mean errors in pcm for the form of neutron density, n(t)=exp(wt), with h=0.01 s and $\sigma = 0.01$. Using the first-order delay low-pass filter.

		Mean absolute errors (pcm)				
w	**t(s)**	**Noise**	$\boldsymbol{\tau = 0.1}$	$\boldsymbol{\tau = 0.5}$	$\boldsymbol{\tau = 1}$	$\boldsymbol{\tau = 1.5}$
0.006881	500	5.22	0.96	0.54	0.49	0.51
0.01046	800	5.06	0.92	0.52	0.47	0.48
0.016957	300	4.85	0.91	0.62	0.70	0.87
0.02817	600	4.50	0.84	0.57	0.61	0.71
0.12353	300	2.29	0.70	0.83	1.22	1.63

Figure-2 shows the mean errors of w= 0.006881 with noise σ = 0.01 and time step $h = 0.01\,s$ proposed in Table-4 for a filter constant τ = 1. The mean noise error of the MF method yields a value of 5.22 pcm. When filtering, its average error decreases to 0.49 pcm. Therefore, the low-pass filter behaves well for this exposed w.

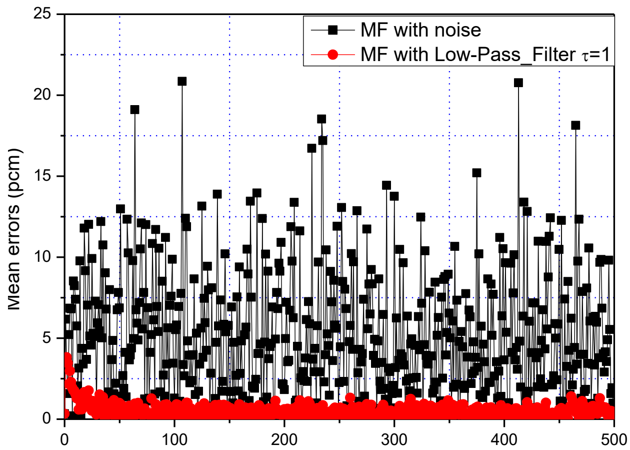

Figure-2. Mean errors for a neutron population density of the form n(t)=exp(wt), with h=0.01 s, $\sigma = 0.01$ for a w= 0.006881 with low-pass filter of τ =0.1.

In Table-5 the same time step of the two previous numerical experiments is maintained, but its standard deviation still increases to σ = 0.1. For all the exposed w

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there are too many fluctuations in their maximum difference. It is understood that for the smallest values of; w = 0.006881, the MF method generates a maximum difference of 599. 9365 pcm, when applying the low-pass filter, although all τ decrease the fluctuations, that which best filters is τ=1.5 which produces a maximum difference of 13.74 pcm. Something similar occurs with w = 0.01046. As w increases it is necessary to decrease τ; as for example for the last w=0.12353, it generates a maximum difference of 253.82 pcm, but when applying the low-pass filter with only τ=0.5 it succeeds in reducing it to 42.30 pcm. Therefore, when the standard deviation is high, as w increases, the low-pass filter can reduce fluctuations in part by using a progressively smaller τ, to the point that this filter is no longer optimal for the MF noise method.

Table-5. Maximum differences in pcm for the form of neutron density, n(t)=exp(wt), with h=0.01 s and $\sigma = 0.1$. Using the first-order delay low-pass filter.

		Maximum Differences (pcm)				
w	t(s)	Noise	$\tau = 0.1$	$\tau = 0.5$	$\tau = 1$	$\tau = 1.5$
0.006881	500	599.93	60.20	25. 83	17.52	13.74
0.01046	800	581.27	58.30	25. 07	16.97	15.24
0.016957	300	467.24	49.31	21. 42	18.59	18.73
0.02817	600	515.86	51.68	23. 30	23.33	24.61
0.12353	300	253.82	46.51	42. 30	57.83	68.25

On the other hand, to analyze the mean errors for the standard deviation at σ = 0.1 in Table-6, it can be seen that it is possible to decrease with the low pass filter of τ = 1.5 for the w from 0.006881 to 0.02817, then for w = 0.12353, the filtering constant that performs best is τ = 1.

Table-6. Mean errors in pcm for the form of neutron density, n(t)=exp(wt), with h=0.01 s and $\sigma = 0.1$. Using the first-order delay low-pass filter.

		Mean absolute errors (pcm)				
w	t(s)	noise	$\tau = 0.1$	$\tau = 0.5$	$\tau = 1$	$\tau = 1.5$
0.006881	500	53. 41	9. 68	5. 11	3. 99	3. 60
0.01046	800	51. 74	9. 26	4. 90	3. 83	3. 46
0.016957	300	49. 61	8. 97	4. 72	3. 76	3. 54
0.02817	600	46. 01	8. 26	4. 42	3. 57	3. 35
0.12353	300	23. 30	5. 99	3. 54	3. 31	3. 52

Figure-3 shows the mean errors of the different filtering constants τ with w = 0.02817 and standard deviation σ = 0.1. The mean error with noise using MF is 46.01 pcm, and the best low-pass filter to reduce this mean error is achieved for τ = 1.5, with a value of 3.35 pcm.

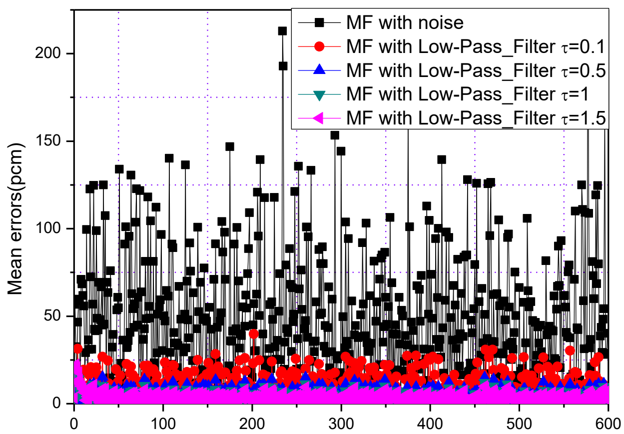

Figure-3. Mean errors for a neutron population density of the form n(t)=exp(wt), with h=0.01 s, $\sigma = 0.01$ for a w= 0.02817 with low-pass filter of τ =0.1, τ =0.5, τ =1 and τ =1.5.

ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

In summary, the different numerical experiments in this study for the specific case of a form of exponential neutron density for a step size $h = 0.01$s, with the matrix formulation method (MF) applying the low-pass filter, show that: for Tables 1 and 2 with a standard deviation σ = 0.001 their results are favorable for the entire specified range of w, with a filtering constant τ = 0.1. When the standard deviation begins to increase, as shown in Tables 3 and 4 with a deviation σ = 0.01, the improvement for the maximum differences and mean errors varies according to its w. For the first two w, its maximum difference is favorable for a τ = 0.5 but in its average errors, the most precise numerical experiment is for a τ = 1. In this same case as w increases, the most favorable τ in the maximum differences is τ = 0.1 and for the average errors it is τ = 0.5. Now for the final increase in deviation of σ = 0.1, seen in Tables 5 and 6, the favorable τ in the maximum differences is τ = 0.5 and for its mean errors is τ = 1.

CONCLUSIONS

The matrix formulation method for the specific case where its time step is equal to a $h = 0.01$s, presents good results for small *w* between 0.006881 and 0.12353. Whenever its standard deviation is increased, the filtering constant τ needs to be reduced; therefore for larger *w* there is the possibility that the low-pass filter will not reduce these fluctuations. Good results were observed for τ = 1.5 when the standard deviation is σ = 0.1. However, for large *w*; that is, values between 1.00847 to 52.80352, the low-pass filter fails to reduce error due to large fluctuations. For this reason, it is not recommended to use a low-pass filter for this type of *w*. Research with another type of filter for reducing fluctuations in the calculation of reactivity is recommended.

ACKNOWLEDGEMENTS

This research was carried out thanks to the academic and financial support from the Computational Physics Research Group, the Applied Physics Research Group (FIASUR) and the Physics Department at the Surcolombian University, Neiva, Colombia.

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