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THE EULER-MACLAURIN METHOD WITH EXPONENTIAL FILTER TO REDUCE FLUCTUATIONS IN REACTIVITY CALCULATION

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ABSTRACT

This paper presents a new study to reduce fluctuations in the calculation of reactivity. The inverse equation of point kinetics is solved with an approximation in the neutron density history using the Euler-Maclaurin method. An exponential filter is implemented to reduce the fluctuations in the neutron population density. These fluctuations are simulated using a Gaussian noise with zero mean and unity variance around the average power.

Keywords: Bernoulli number, inverse point kinetics equations, reactivity, neutron population density.

INTRODUCTION

Numerous contributions to nuclear physics have been made throughout history; one of the most important was the discovery of nuclear fission on December 19, 1938 - a few months before the break of the World War II - by Lise Meitner, Otto Hahn and Fritz Strassman at the Kaiser Wilhem Institute in Berlin, Germany. The phenomenon of nuclear fission describes the splitting of atoms like Uranium, which is an element of special interest due to its fissile properties. Since it was discovered that the fission process releases a large amount of energy, Enrico Fermi and his colleagues at the University of Chicago in 1942, devised a mechanism that could take advantage of the *200 MeV* of energy released in the fission process, while the chain reaction was controlled.

It is necessary to control the neutrons which originate in the chain reaction, for this, it is important to know the reactivity, a parameter that is vital for the safe operation of the nuclear power plant. The control of the neutron population is done by means of the control rods, which are made of neutron absorbing materials. The up and down movement of these control rods modifies the reactivity in the reactor core according to the operational requirements. Many works developed over the years have focused on finding numerical methods to precisely calculate the reactivity value [1, 4]. Recent work considers the fluctuations in the neutron population density signal [5]. Accuracy and computational cost determine whether these methods can be implemented in real-time.

THEORETICAL ASPECTS

The equations of point kinetics describe the time evolution of the neutron population density [6]. These equations are a set of seven equations, of which six describe the concentration of precursors and one the population density of neutrons:

$$\frac{dP(t)}{dt} = \left[\frac{\rho(t)-\beta}{\Lambda}\right]P(t) + \sum_{i=1}^{6}\lambda_i C_i(t) \tag{1}$$

$$\frac{dC_i(t)}{dt} + \lambda_i C_i(t) = \frac{\beta_i}{\Lambda}P(t)\,,\ i = 1,2,\ldots,6 \tag{2}$$

In equations (1) and (2), $C_i(t)$describe the concentration of the *i-th* group of backward neutron precursors, $P(t)$ is the density of the neutron population, $\rho(t)$ is the reactivity, Λ is the neutron generation time, $\beta_i$is the *i-th* fraction of delayed neutrons β is the total effective fractionof delayed neutrons and $\lambda_i$is the decay constant of the *i-th* group of backward neutron precursors.

In order to solve equations (1) and (2), the initial conditions around the neutron population density and the concentration of precursors, which make the reactor a critical one, must be considered, which means that the neutron population density is constant, therefore, its reactivity is zero.

It is possible to obtain an expression for reactivity by solving equations (1) and (2) using the aforementioned initial conditions:

$$\rho(t) = \beta + \frac{\Lambda}{P(t)}\frac{dP(t)}{dt} - \frac{\langle P_0\rangle}{P(t)}\sum_{i=1}^{6}\beta_i\, e^{-\lambda_i t} - \frac{1}{P(t)}\sum_{i=1}^{6}\int_0^t \lambda_i\beta_i e^{-\lambda_i(t-t')}P(t')\,dt' \tag{3}$$

Equation (3) is not directly used in real-time calculations since it is difficult to implement as it stands. For this, it is necessary to discretize the integral term on the right-hand side, which is known as the historical of the neutron population density [7].

PROPOSED METHOD

The Euler-Maclaurin formula [8] allows a transition from continuous space to discrete space which facilitates solving equation (3):

$$\int_0^n F(x)dx = \sum_{y=1}^{n-1}F[y] + \frac{1}{2}[F[0]+F[n]] - \sum_{k=1}^{\infty}\frac{B_k}{(2k)!}\left[F^{(2k-1)}[n] - F^{(2k-1)}[0]\right] \tag{4}$$

In equation (4), the term $B_k$represents the Bernoulli numbers.

By making $k = 1$ in equation (4), we get:

$$\int_0^n F(x)dx = \sum_{y=1}^{n-1}F[y] + \frac{1}{2}[F[0]+F[n]] - \frac{B_1}{2!}\left[F^{(1)}[n] - F^{(1)}[0]\right] \tag{5}$$

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In equation (5) B_1 takes the value of 1/6.

It is possible to simplify equation (5) by using the following expressions:

$$F(t') = h_i(t - t')P(t') \tag{6}$$

$$F[y] = h_i[n - y]P[y] \tag{7}$$

With equation (6) being a continuous version of equation (7).

The term h_i in equations (6) and (7) represents the system's response to a unit impulse [9] defined by:

$$h_i(t - t') = \lambda_i\beta_i e^{-\lambda_i(t-t')} \tag{8}$$

By deriving equation (7) once, we get:

$$F^{(1)}[y] = h_i^{(1)}[n - y]P[y] + h_i[n - y]P^{(1)}[y] \tag{9}$$

Evaluating equations (7) and (9) at $y = n$, $y = 0$, and replacing the result in equation (5) we get:

$$\int_0^n h_i(t - t')P(t')dt' = \sum_{y=1}^{n} h_i[n - y]P[y] - \frac{1}{2}[h_i[n]P[0] + h_i[0]P[n]]] - \frac{1}{12}\left[h_i^{(1)}[0]P[n] + h_i[0]P^{(1)}[n] - h_i^{(1)}[n]P[0] - h_i[n]P^{(1)}[0]\right] \tag{10}$$

Now, in order to deduce an expression that describes the reactivity with the Euler-Maclaurin formula, equation (10) must be replaced into equation (3) to obtain:

$$\rho[n] = \beta + \frac{\Lambda}{P[n]}P^{(1)}[n] - \frac{\langle P_0\rangle}{P[n]}\sum_{i=1}^{6}\beta_i\, e^{-\lambda_i nT} - \frac{T}{P[n]}\sum_{i=1}^{6}\left[\sum_{y=1}^{n} h_i[n - y]P[y] - \frac{1}{2}[h_i[n]P[0] + h_i[0]P[n]]]\right] + \frac{T^2}{12P[n]}\sum_{i=1}^{6}\left[h_i^{(1)}[0]P[n] + h_i[0]P^{(1)}[n] - h_i^{(1)}[n]P[0] - h_i[n]P^{(1)}[0]\right] \tag{11}$$

Where the first four terms represent a FIR filter and the last term represents the correction of the Euler-Maclaurin method with the first Bernoulli number.

To analyse fluctuations in the neutron population density signal, firstly, random noise with Gaussian distribution is used, which is applied around the average of the neutron population density [10]:

$$\overline{P_i} = \frac{1}{N}\sum_{j=1}^{N} P_j \tag{12}$$

Then, a filter should be implemented to reduce fluctuations. A well-known one is the first-order low-pass filter [11]:

$$P_i = P_{i-1} + \frac{T}{T+\tau}(\overline{P_i} - P_{i-1}) \tag{13}$$

Where T is the time step, τ is the filtering constant, which takes a value of $\tau = 1.5s$ in this work.

The other filter considered in this work is the exponential filter [12]. This filter adjusts the values of the neutron population density signal to an exponential form:

$$y(x) = Ce^{Ax} \tag{14}$$

Where A and B are constants.

Linearizing equation (14) we get an expression of the form:

$$Y = AX + B \tag{15}$$

Where

$$Y = \ln(y) \quad , \quad X = x \quad , \; B = \ln(C) \tag{16}$$

The constants A and B can be calculated by the least squares method based on the normal Gaussian equations:

$$\left(\sum_{k=1}^{N} x_k^2\right)A + \left(\sum_{k=1}^{N} x_k\right)B = \sum_{k=1}^{N} x_k y_k \tag{17}$$

$$\left(\sum_{k=1}^{N} x_k\right)A + NB = \sum_{k=1}^{N} y_k \tag{18}$$

The constant C can be found from equation (16), thus obtaining:

$$C = e^B \tag{19}$$

RESULTS AND DISCUSSIONS

This section presents the results of the simulations of reactivity. The neutron population density takes the form $P(t) = \exp(\omega t)$ with an angular frequency ω varying from *0.00243* to *52.80352*. In the numerical simulations, the term $P^{(1)}[n]$ from equation (21) is neglected since its analytical solution is taken as the reference method. The number of seeds to generate the random noise is $2^{31}-1$ and the physical constants used are those typical of ^{235}U, represented in Table-1. The time step T varies between $T = 0.01$ s and $T = 0.1$ s. For the filtering of the neutron population density, a sampling window with a number of $n = 10$ samples is applied to each filter.

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Table-1. Physical constants typical of ^{235}U.

Group	1	2	3	4	5	6
λ_i	0.0127	0.0317	0.115	0.311	1.4	3.87
β_i	0.000266	0.001491	0.001316	0.002849	0.000896	0.000182
$\Lambda=2*10^{-5}$ [s]						
$\beta=7*10^{-3}$						

Tables 2-3 show a comparison of the results, for average error and maximum difference of the Euler-Maclaurin (E-M) method using the low-pass and exponential filters. A time step of *T = 0.01 s* and a standard deviation of $\sigma = 0.001$ are used. It is possible to see that the average errors improve in accuracy as the value of ω increases. It is evident that for ω values of *0.12353* onwards, the maximum difference improves markedly. Although the method supports noise for large ω values, it is observed that the low-pass filter does not reduce fluctuations appreciably, contrary to what happens when the exponential filter is used.

Table-2. Mean error for a neutron population density of the form $P(t) = \exp(\omega t)$ with different values of ω and standard deviation σ = 0.001, with a time step of T = 0.01 s.

P(t)=exp(ωt) , T=0.01s , sigma=0.001		Mean Absolute Error		
		E-M	E-M with Low Pass Filter, TAU=1.5	E-M with Exponential Filter, n=10
ω=0.00243	t_f=1000 s	0.54	0.05	0.23
ω=0.006881	t_f=500 s	0.52	0.18	0.22
ω=0.01046	t_f=800 s	0.50	0.16	0.21
ω=0.02817	t_f=600 s	0.44	0.39	0.19
ω=0.12353	t_f=300 s	0.32	1.43	0.14
ω=1.00847	t_f=150 s	0.12	2.99	0.05
ω=11.6442	t_f=60 s	0.02	3.33	0.008
ω=52.80352	t_f=10 s	0.06	5.77	0.01

Table-3. Maximum difference for a neutron population density of the form $P(t) = \exp(\omega t)$ with different values of ω and standard deviation σ = 0.001, with a time step of T = 0.01 s.

P(t)=exp(ωt) , T=0.01s , sigma=0.001		Max Diff [pcm]		
		E-M	E-M with Low Pass Filter, TAU=1.5	E-M with Exponential Filter, n=10
ω=0.00243	t_f=1000 s	3.25 in t=319.76 s	1.53 in t=2.8 s	1.73 in t=345.79 s
ω=0.006881	t_f=500 s	3.10 in t=319.76 s	4.24 in t=2.8 s	1.65 in t=345.79 s
ω=0.01046	t_f=800 s	3.01 in t=319.76 s	6.38 in t=2.8 s	1.60 in t=345.79 s
ω=0.02817	t_f=600 s	2.67 in t=319.76s	16.46 in t=2.6 s	1.41 in t=345.79 s
ω=0.12353	t_f=300 s	2.14 in t=1.58 s	60.27 in t=2 s	0.94 in t=3.9 s
ω=1.00847	t_f=150 s	1.49 in t=0.1 s	228.08 in t=0.8 s	0.74 in t=0.1 s
ω=11.6442	t_f=60 s	1.12 in t=0.01 s	452.41 in t=0.15 s	0.41 in t=0.1 s
ω=52.80352	t_f=10 s	7.71 in t=0.01 s	322.45 in t=0.04 s	0.45 in t=0.02 s

In Tables 4-5, the time step is increased to *T = 0.01 s* while keeping the standard deviation constant at $\sigma = 0.001$. Accuracy in the average error is increased for $\omega = 52.80352$ when the exponential filter is used to reduce the fluctuations in the neutron population density signal; it is observed that the E-M method by itself does not support the noise and that the application of the low-pass filter causes the method to lose its effectiveness. In turn, when the exponential filter is used, it is evident that the average error decreases markedly from *2.19 x 10^{18} pcm* to *29.10*

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pcm. With respect to the maximum differences, it is noticeable that for a value of $\omega = 1.00847$ the proposed method goes from a value of *1.09 pcm* to *250.02 pcm* making use of the low-pass filter, which indicates that this filter cannot reduce the fluctuations. In comparison, the exponential filter reduces the fluctuations to a value of *0.47 pcm.*1.00847

Table-4. Mean error for a neutron population density of the form $P(t) = \exp(\omega t)$ with different values of ω and standard deviation σ = 0.001, with a time step of T = 0.1 s.

P(t)=exp(ωt) , T=0.1s , sigma=0.001		Mean Absolute Error		
		E-M	E-M with Low Pass Filter, TAU=1.5	E-M with Exponential Filter, n=10
ω=0.00243	t_f=1000 s	0.53	0.09	0.22
ω=0.006881	t_f=500 s	0.51	0.25	0.20
ω=0.01046	t_f=800 s	0.49	0.28	0.20
ω=0.02817	t_f=600 s	0.44	0.69	0.17
ω=0.12353	t_f=300 s	0.30	2.60	0.12
ω=1.00847	t_f=150 s	0.11	10.12	0.05
ω=11.6442	t_f=60 s	13.27	137.18	3.79
ω=52.80352	t_f=10 s	$2.19*10^{18}$	$8.60*10^{19}$	29.10

Table-5. Maximum difference for a neutron population density of the form $P(t) = \exp(\omega t)$ with different values of ω and standard deviation σ = 0.001, with a time step of T = 0.1 s.

P(t)=exp(ωt) , T=0.1s , sigma=0.001		Max Diff [pcm]		
		E-M	E-M with Low Pass Filter, TAU=1.5	E-M with Exponential Filter. n=10
ω=0.00243	t_f=1000 s	2.56 in t=283.5 s	1.79 in t=4 s	1.24 in t=117 s
ω=0.006881	t_f=500 s	2.44 in t=283.5 s	4.69 in t=3 s	1.19 in t=117 s
ω=0.01046	t_f=800 s	2.36 in t=283.5 s	7.04 in t=3 s	1.15 in t=117 s
ω=0.02817	t_f=600 s	2.08 in t=283.5 s	18.07 in t=3 s	1.04 in t=117 s
ω=0.12353	t_f=300 s	1.45 in t=283.5 s	66.00 in t=2 s	0.78 in t=117 s
ω=1.00847	t_f=150 s	1.09 in t=1 s	250.02 in t=1 s	0.47 in t=1 s
ω=11.6442	t_f=60 s	391.77 in t=38.1	2144.75 in t=23 s	29.19 in t=30 s
ω=52.80352	t_f=10 s	$4.83*10^{19}$ in t=0.9 s	$1.09*10^{21}$ in t=0.9 s	259.69 in t=0.1 s

Tables 6-7 show the results for average error and the maximum differences from the numerical simulations using the E-M method with the low-pass and exponential filters. For this case, the standard deviation is increased to σ = 0.01 with a time step of T = 0.01 s in the calculation of reactivity. It is observed that for the value of $\omega = 11.6442$, the average error decreases greatly, from 0.20 pcm to 0.08 pcm with the exponential filter. For this same value of ω, the maximum difference in the calculation of the reactivity, it can be seen that the low-pass filter does not decrease the fluctuations, on the contrary, increasing them to 452.42 pcm, in contrast, the maximum difference with the exponential filter was 4.08 pcm.

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Table-6. Mean error for a neutron population density of the form $P(t) = \exp(\omega t)$ with different values of ω and standard deviation $\sigma = 0.01$, with a step size of $T = 0.01$ s.

P(t)=exp(ωt) , T=0.01s , sigma=0.01		Mean Absolute Error		
		E-M	E-M with Low Pass Filter, TAU=1.5	E-M with Exponential Filter, n=10
ω=0.00243	t_f=1000 s	5.43	0.24	2.34
ω=0.006881	t_f=500 s	5.21	0.34	2.25
ω=0.01046	t_f=800 s	5.04	0.33	2.17
ω=0.02817	t_f=600 s	4.49	0.53	1.93
ω=0.12353	t_f=300 s	3.23	1.53	1.39
ω=1.00847	t_f=150 s	1.22	3.02	0.52
ω=11.6442	t_f=60 s	0.20	3.34	0.08
ω=52.80352	t_f=10 s	0.40	6.09	0.17

Table-7. Maximum difference for a neutron population density of the form $P(t) = \exp(\omega t)$ with different values of ω and standard deviation $\sigma = 0.01$, with a time step of $T = 0.01$ s.

P(t)=exp(ωt) , T=0.01s , sigma=0.01		Max Diff [pcm]		
		E-M	E-M with Low-Pass Filter, TAU=1.5	E-M with Exponential Filter, n=10
ω=0.00243	t_f=1000 s	34.02 in t=319.76 s	1.95 in t=1.6 s	16.96 in t=345.79 s
ω=0.006881	t_f=500 s	32.50 in t=319.76 s	4.55 in t=2.2 s	16.19 in t=345.79
ω=0.01046	t_f=800 s	31.49 in t=319.76 s	6.66 in t=2.2 s	15.68 in t=345.79 s
ω=0.02817	t_f=600 s	27.95 in t=319.76 s	16.69 in t=2.2 s	13.88 in t=345.79 s
ω=0.12353	t_f=300 s	22.13 in t=1.58 s	60.54 in t=2.1 s	9.26 in t=3.9 s
ω=1.00847	t_f=150 s	14.59 in t=0.1 s	228.31 in t=0.8 s	7.38 in t=0.1 s
ω=11.6442	t_f=60 s	11.44 in t=0.01 s	452.42 in t=0.15 s	4.08 in t=0.1 s
ω=52.80352	t_f=10 s	94.38 in t=0.01 s	320.22 in t=0.04 s	5.11 in t=2.3 s

Tables 8-9 show the mean error and maximum difference, respectively. A time step of $T = 0.1$ s is used and the standard deviation is the same as in the previous case, $\sigma = 0.01$. A significant reduction in the average error is observed for $\omega > 0.02817$, specifically for the case with $\omega = 52.80352$ for an exponential filter.

Table-8. Mean error for a neutron population density of the form $P(t) = \exp(\omega t)$ with different values of ω and standard deviation $\sigma = 0.01$, with a time step of $T = 0.1$ s.

P(t)=exp(wt) , T=0.1s , sigma=0.01		Mean Absolute Error		
		E-M	E-M with Low Pass Filter, TAU=1.5	E-M with Exponential Filter, n=10
w=0.00243	t_f=1000 s	5.35	0.65	2.19
w=0.006881	t_f=500 s	5.10	0.74	2.05
w=0.01046	t_f=800 s	4.97	0.72	2.02
w=0.02817	t_f=600 s	4.42	0.95	1.77
w=0.12353	t_f=300 s	3.09	2.63	1.21
w=1.00847	t_f=150 s	1.18	10.13	0.46
w=11.6442	t_f=60 s	100.41	$1.14*10^{03}$	8.12
w=52.80352	t_f=10 s	$2.19*10^{19}$	$8.62*10^{20}$	39.79

Table-9. Maximum difference for a neutron population density of the form $P(t) = \exp(\omega t)$ with different values of ω and standard deviation $\sigma = 0.01$, with a time step of $T = 0.1$ s.

P(t)=exp(wt) , T=0.1s , sigma=0.01		**Max Diff [pcm]**		
		E-M	**E-M with Low Pass Filter, TAU=1.5**	**E-M with Exponential Filter, n=10**
w=0.00243	t_f=1000 s	25.13 in t=782.8 s	3.66 in t=5 s	12.59 in t=117 s
w=0.006881	t_f=500 s	24.17 in t=15.8 s	6.34 in t=4 s	12.08 in t=117 s
w=0.01046	t_f=800 s	23.64 in t=15.8 s	8.59 in t=4 s	11.74 in t=117 s
w=0.02817	t_f=600 s	21.32 in t=15.8 s	18.91 in t=4 s	10.54 in t=117 s
w=0.12353	t_f=300 s	14.21 in t=1 s	66.16 in t=2 s	7.92 in t=117 s
w=1.00847	t_f=150 s	10.57 in t=0.1 s	249.85 in t=1 s	4.65 in t=1 s
w=11.6442	t_f=60 s	$1.45*10^4$ in t=45.3 s	$2.09*10^4$ in t=22.9 s	20.20 in t=0.1 s
w=52.80352	t_f=10 s	$4.83*10^{20}$ in t=0.9 s	$1.09*10^{22}$ in t=0.9 s	330.05 in t=0.1 s

CONCLUSIONS

An exponential filter was applied to reduce fluctuations in the calculation of reactivity using the Euler-Maclaurin (E-M) method to solve the inverse equation of point kinetics. In this work, fluctuations in the neutron population density signal were taken into account, which were simulated with Gaussian noise with a seed of random number generation of $2^{31}-1$. These fluctuations were solved using the exponential filter which is obtained using the least squares method. Several numerical experiments were performed by fixing the form of the neutron population density $P(t) = \exp(\omega t)$ for different values of ω.

When comparing the effectiveness of the Euler-Maclaurin method with a first-order-of-delay low-pass filter and the exponential filter, better results in the reduction of fluctuations can be obtained by using the latter, which we recommend for real-time measurements.

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REFERENCES

[1] Hoogengoom J. E., Van Der Sluijs, A. R. 1988. Neutron source strength determination for on-line reactivity measurements. Ann. Nucl. Energy. 15:553-559.

[2] Binney S. E., Bakir A. I. M. 1989. Desing and development of a personal computer based reactivity meter for a nuclear reactor. Nucl. Technol. 85:12-21.

[3] Ansari S. A. 1991. Development of on-line reactivity meter for nuclear reactors. IEEE Trans. Nucl. Sci. 38: 946-958.

[4] Malmir H., Vosoughi N. 2013. On-line reactivity calculation using Lagrange method. Ann. Nucl. Energy. 62: 463-467.

[5] Suescún D. D., Ule D. G. and Escobar F. H. 2020. Reduction of reactivity fluctuation with the Euler-Maclaurin method. ARPN J. Eng. App. Sci. 15: 96-103.

[6] Duderstadt J. J., Hamilton L. J. 1976. Nuclear Reactor Analysis, second ed. John Wiley & Sons Inc., New York.

[7] Stacey W. 2018. Nuclear Reactor Physics. Weinheim, Germany: Wiley-VCH Verlag.

[8] Kuen Y. K. 2010. Applied complex variable for scientist and engineers. Second ed. Cambridge.

[9] Haykin S., Veen B. V. 1999. Signal and System. Wiley, New York.

[10] Kitano A., Itagaki M., Narita M. 2000. Memorial-index-based inverse kinectics method for continuous measurement of reactivity and source strength. J. Nucl. Sci. Technol. 37: 53-59.

[11] Shimazu Y., Nakano Y., Tahara Y., *et al*. 1987. Development of a compact digital reactivity meter and reactor physics data processor. Nucl. Technol. 77: 247: 254.

[12] Mathews J. H., Fink K. D. 2000. Numerical methods using matlab, Third ed. Prentice Hall.