EXPERIMENTAL AND NUMERICAL DETERMINATION OF ULTIMATE STRENGTH OF THE DENTED PLATE UNDER AXIAL COMPRESSION THROUGH 3D INTERPOLATABLE IMPERFECTION SURFACE MODEL

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ABSTRACT

The geometrical imperfections present on the shell structures can be classified into distributed geometrical imperfections (present on the intact plate) and local geometrical imperfections. One of the local geometrical imperfections is dent. In the present work, a new approach was adopted to map the measured geometrical imperfections on FE mesh of perfect plate model using 3D interpolatable surface model option of Matlab, to model imperfect plate models. Here, eight test plate specimens of size 500mm x500mmx5 mm (2 intact plates, 2 transversely dented plates, 2 longitudinally dented plates and 2 dented plates with approximately 45° dent orientation) were fabricated in order to study the combined effect of distributed geometrical imperfections and centrally located dent on their ultimate strengths. Before testing the specimens for the ultimate strength experimentally, geometrical imperfections present on the plate specimens were recorded using 3D scanner and this actual geometrical imperfections were mapped on the FE meshes using 3D interpolatable geometrical imperfection surface model. These imperfect FE plate models are analyzed for their ultimate strengths under axial compression with simply supported boundary conditions using Non-linear FE analysis of ANSYS. The results obtained from numerical analysis and experiments are compared and are found to be match each other reasonably.

Keywords: imperfections, FE analysis, dent, buckling experiment, matlab.

1. INTRODUCTION

Koiter was the first, who realise effect of imperfection on buckling strength of shell structures and proved its effect through analytical approach [8]. Only after few decades, this effect was realised by many researchers. Researchers Arbocz and Starnes [5], Teng [34] and Simitses [31], explained that the huge deviations between theoretical and experimental buckling strength of shell structures were due to imperfection present in shell structures and categorized this imperfections into the following categories: Geometric imperfections (i.e. out - of Straightness, ovality, dents, cylindricity, swells, circularity, etc.); structural imperfections (i.e. residual stress and material in homogeneity, building defects such as small holes, cutouts, and delaminations); and loading imperfections (i.e., non- uniform edge load distribution, unintended edge distribution). Further, it was also concluded that geometrical imperfections category has most detrimental effect.

In case of cylindrical shell structure, the out-of-roundness was taken as quantifying parameters for imperfections in early 1950s. According to Singer and Abramovich [21], Arabocz was the first researcher developed the first automated imperfection measurement system for entire thin shell structures in the year 1968, at the Guggenheim Aeronautical Laboratory of the California Institute of Technology (GALCIT) based on a contactless, capacitive sensor.

Coppa [3] developed measurement system taking measurement from outside of the shell structure. Singer and Abramovich [21] presented an excellent overview about the geometric measurements of thin-walled shell structures by then earlier researchers. Esong *et al.* [14] developed laser based non-contact measurement system to record initial geometric imperfections of thin-walled shell structure. A similar approach was followed by Elghazouli *et al.* [1], Spagnoli *et al.* [32] and Bisagni and Potito Cordisco [9].

Bernard *et al.* [6] adopted CMM for measuring imperfections. Athianna and Palaninathan [22] used contact type imperfection measurement system using LVDT. Jin *et al.* [20] adopted disk beam probe based optical method for the imperfection measurement system. Paor [12] developed a measuring rig using four dial gauges to measure geometric imperfections of a shell structure. Lyssakow *et al. [*24] developed a geometric and thickness imperfection measurement setup using two laser sensors system for thin-walled structures.

In recent days, most of the researchers are using FEA to determine the buckling strength of the thin shell structures. Hence, in order to transfer the actual imperfections measurement accurately to FE mesh measured imperfections were characterised mostly by Fourier series to capture the inherent dominant characteristic harmonic components of imperfections of manufacturing / fabrication process of the shell structures (for example Arabocz and Hol [4], Chryssanthopoulos and Poggi [10, 11], Bernard *et al.* [6], Paik *et al.* [25], Athiannan and Palaninathan [24], etc..). Other approaches followed by researchers were statistically characterizing imperfection measurement using single or multi-mode combination of Eigen mode shape(s) (for example Pircher *et al.* [27], Featherston [15, 17], Lin and Teng [23], etc...)

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Hence in this work efforts are made to transfer the actual geometrical imperfections to FE mesh, using the 3D interpolatable surface model of Matlab. In this present work, four types of test specimens were used to determine the ultimate strength of the dented plate which has both distributed and local geometrical imperfections. It was also planned to study the effects of orientation of dent on the ultimate strength of the plate.

Totally eight test specimens along with distributed geometrical imperfections namely A1& A2 (without dent), B1&B2 (with horizontal dent), C1&C2 (with vertical dent) and D&D2 (inclined dent with orientation of approximately 45°) were fabricated , using 500 mm x 500 mm x 5mm square HT-32 steel plate. Before taking experimentation geometrical imperfections on the specimens were recorded by using 3D scanner and this imperfections data were modeled as 3D interpolatable surface model option available in the Matlab. Using this surface model of imperfection, amplitudes on all the nodal points of the FE model are obtained and imperfect FE Plate models are generated with actual measured imperfections by adding the amplitude of imperfections on the all the nodes of the perfect FE plate Models. These generated model are analyzed using non-liner FE model of ANSYS, including both material and geometrical non - linearities and results are compared with experimental results.

2. DETERMINATION OF YIELD STRENGTH OF THE PLATE MATERIAL

In order to determine the yield strength of the HT-32 steel plate material taken for study, three dog bone shaped specimens were cut from the plate with 5mm thickness according to the ASTM - E8 and tested using universal testing machine INSTRON. The average value of the yield strength obtained by testing of specimens is taken as yield strength σy= 293.16 MPa.

3. MATERIAL MODELING

The material model used for FE Analysis is elastic perfect plastic curve with zero strain hardening as shown in Figure-1. With the Young's modulus (E) = 2 x 10^5 N/mm^2 and Yield stress (σy) = 293.16 MPa, Poisson's ratio (v) = 0.3 and tangential modulus E_T=0.

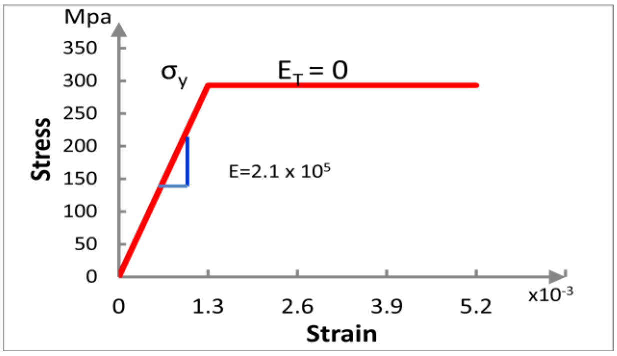

Figure-1. Stress-strain curve used for FE analysis.

This assumption of elastic and perfect plastic nature of the material modeling is reasonable as strain hardening separated from elastic zone by plastic yield for structural carbon steel [13].

4. PREPARATION OF TEST SPECIMEN

Guedes Soares [18], Saad-Eldeen *et al.* [30], in their work mentioned that the practical ranges of plate slenderness ratio (Eq.1) for marine applications are in the range of 1 to 5.

$$\beta = \frac{b}{t}\sqrt{\frac{\sigma_y}{E}} \qquad (1)$$

where b = width of the plate , t= thickness of the plate, E= Young's modulus of the material and σ_y = yield stress of the material. Hence in this work 500 mm square HT32 steel plate with 5 mm thickness taken for the study (whose β value is 3.82).

Totally eight test specimens were prepared, namely A1, A2 intact plates (without dent), B1, B2 - horizontally dented plates, C1,C2 - vertically dented plates and D1, D2 - dented plates with approximately 45° dent orientation . In all the cases of dented plates, dents were formed at centre of the plate as reduction buckling strength of the plate at this location will be higher compared to other locations of the plate [29]. The tool - dent intender used to form the dents is shown in Figure-2 and is made of EN24.

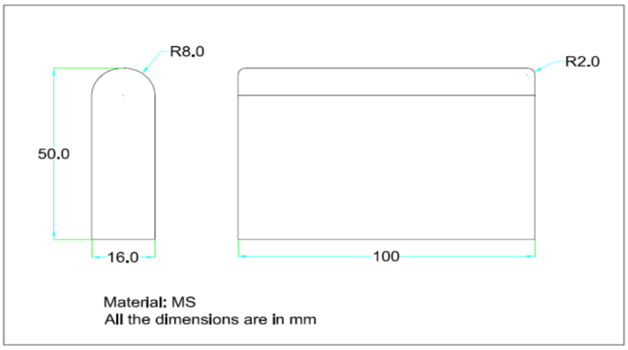

Figure-2. Dent intender.

This dent intender was fitted on the ram of 100 Tons of Hydraulic press and using this, dents were formed on the specimens by maintain approximately 10 mm dent depth. but there will be a variation on the actual depth depth of dent formed on the specimens , because of spring back effect of the plate material and inherent deviation in the dent formation process. The sample photographs of the dented plate specimens are shown in Figure-3.

5. MEASUREMENT OF GEOMETRIC IMPERFECTIONS

In order of to obtain accurate measurements of the geometrical imperfections present on the test specimens, 3D scanning machine was used and is shown

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in Figure-4. After scanning the each test specimen, scanned data were imported to geometric software, which gave x,y and z co-ordinates of the measured geometrical imperfections on the test specimens. Scanning of imperfections was carried out at interval of 10 mm along the both side of the plate. Further in order to capture the dent geometry more accurately, near the dent region scanning was done at an interval of 2 mm.

The mid plane of the scanned data of geometrical imperfections were read using Matlab software and then from which interpolatable 3D surface model was generated using the Matlab building function interp with cubic spline option. The accuracy of the matlab generated surface model of imperfection was verified by comparing with scanned data and error was found to be less than 1%.

(a) (b) (c)

Figure-3. Photographs of sample plates (a) Thin plate with horizontal dent, (b) Thin plate with vertical dent (c) Thin plate with inclined dent (approximately 45 °)

Figure-4. Scanning of Dented Plate using 3D scanner.

For smooth FE modeling and analysis of dented FE plate model using element shell 281 of ANSYS requires imperfection data at minimum of 7000 nodes points. But there is a limitation for measuring imperfection data such as (i) number of imperfection data, (ii) the space between line of scanning, (iii) data required at the required node location of FE modeling, etc... These above said facts prompt for formation of 3D interpolatable surface model of the imperfections. The sample of grid points of Matlab surface model and nodes points of the FE model without imperfections are shown in Figure-5. The sample of Matlab imperfection surface model and FE Nodal disaplacement for imperections are shwns in Figure-6.

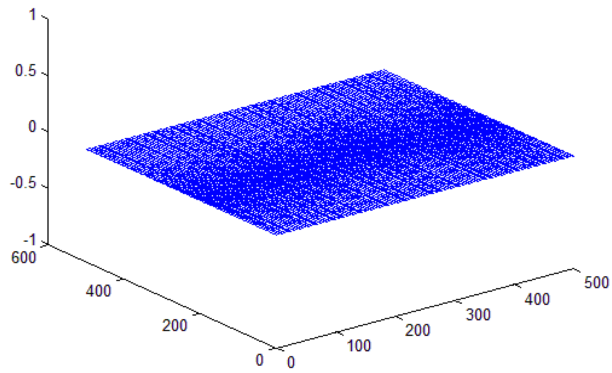

Figure-5. (a) Grid points thin plate for Matlab surface model.

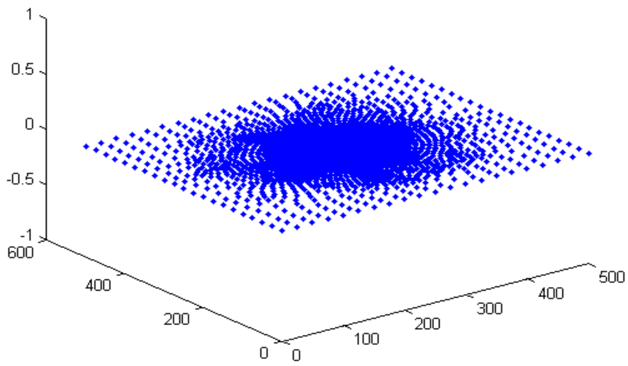

Figure-5. (b) FE nodes for thin plate.

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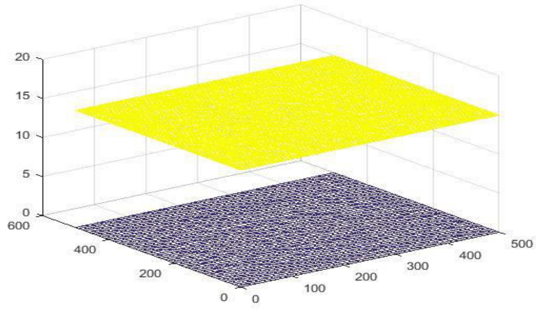
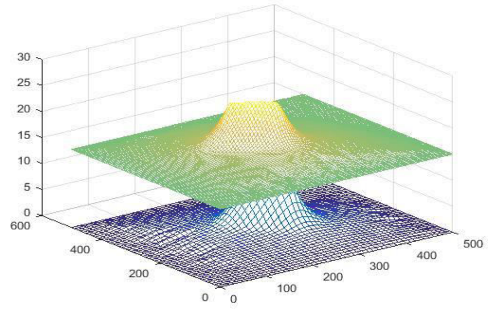
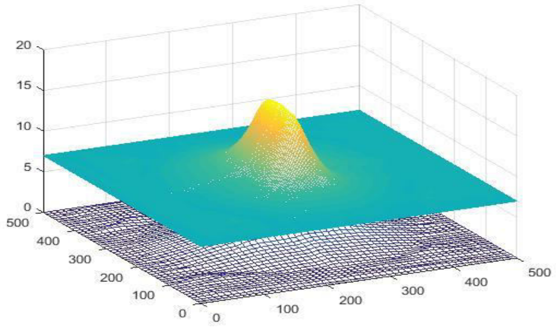

(6a) plain plate A1


(6b) horizontal dented plate B1


(6c) vertical dented plate C1

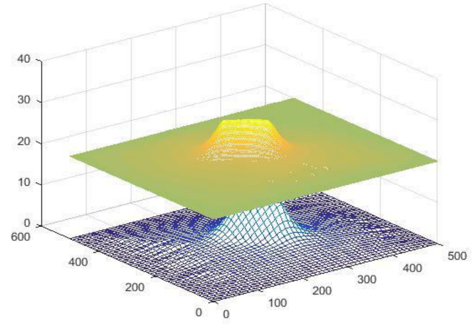

(6d) inclined dented plate D1

Figure-6. Imperfection surface modeling using MATLAB 3D cubic spline surface of different test specimens.

6. BOUNDARY CONDITIONS

Boundary conditions used for FE analysis is shown in Figure-7. The same boundary conditions adopted by Suneel Kumar *et al*. [33], Jana and Bhaskar [19] and Raviprakash *et al*. [28] were adopted for the present FE analysis and experimental determinations of bucking strength of the test specimens.

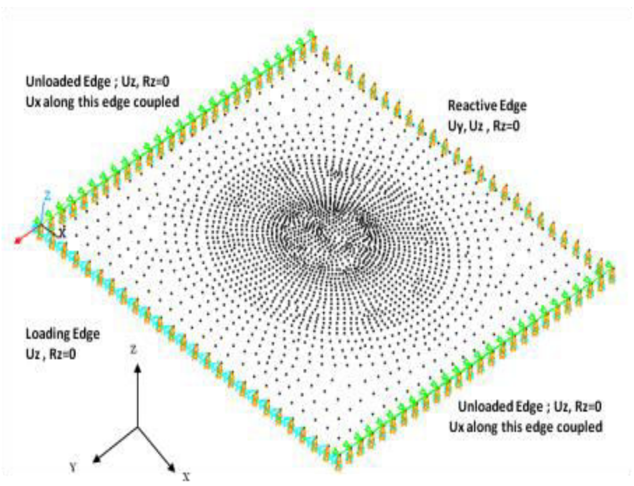


Figure-7. Boundary conditions.

In order to restrain the rigid body motion of the FE model, all the nodes along the four edges of the plate are constrained for deflection and rotation along the thickness direction i.e., Uz = Rz=0.

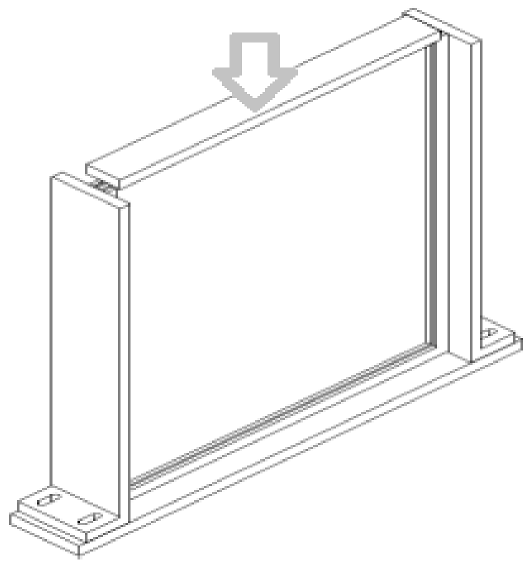


Figure-8. CAD 3D model of the test rig.

Further all the nodes along unloaded edges are coupled for in-plane displacement (Ux) to simulate lateral (in-plane) restraining effect of the adjoining structure on an isolated square plate [33]. In order to imply the above said boundary conditions a test rig was developed and the overall 3D model of the test rig given in Figure-8.

Figure-9. Photograph view of developed test rig.

7. TESTING OF TEST SPECIMEN FOR BUCKLING STRENGTH

FIE -1000 computerized UTM with capacity of 1000kN was used to determine the ultimate strength of the test specimens. Before performing compression test on UTM, the following checking and initial settings were carried out.

7.1 Initial setting on the machine

Parallelism and face out of both top and bottom platens were checked by using the micron dial indicator and it was found that there are no face out on the working surface of the platen. The parallelism limit of 10 micron between the platens was ensured.

7.2 Initial setting the specimens

Before applying load on the test rig, the parallelism between top edge of the specimen and top platen was checked by using the feeler gauge and it was found that within the tolerance limit of 30 microns. To ensure extremely slow loading on the test rig, first, the upward movement lower arm controlled at a rate of 1.0 mm per minute [16] and results sampled at a rate 5kN increase. A micron dial indicator with its magnetic base was mounted on the mechanical surface of the bottom ram and the measuring probe of the dial indicator was made to touches the machined surface of the upper fixed ram as shown in Figure-9. On loading, as the lower ram moves upward the dial indicator shows the reduction in gap between the two platens which is nothing between the edge displacements applied on the test rig on loading.

7.3 Experimental procedure adopted

As soon as the upper edge of the test rig touches top platen, at which increase in micron dial indicator reading stops for a while and that micron dial indicator reading (R1) was noted. Uniform displacement loading from the bottom platen was applied on the test rig gradually until limit load condition of the test specimen was reached. For every 5 kN loading on UTM, the axial edge displacement was noted in order to plot load vs. displacement. As soon as the load applied reaches the limit load condition (at which arm of the live dial indicator of the UTM tends to return back on further loading) both the limit load value on the dial indicator of the UTM and micron dial indicator value (R2) was noted simultaneously. The difference between R1 and R2 was taken as total axial edge displacement.

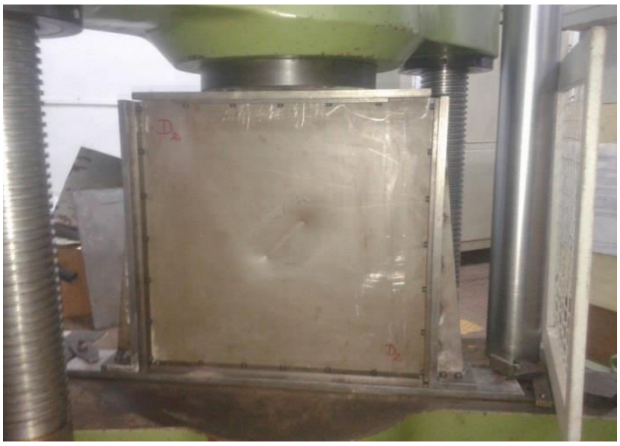


Figure-10. Photograph of the test setup with inclined dented plate D2 compressed axially on the UTM machine.

8. FE ANLYSIS

In this work FE models the test specimens are generated by using Shell 281 of ANSYS. Shell 281is an eight node - quadrilateral shell element able to modal curved features which accounts for membrane, bending and transverse shear effect on its calculation. This element can handle plasticity, large strain and displacement, stress stiffening effects also in its computation. A sample FE mesh and isometric view of the dent plate FE model are shown in Figure-11. Using the boundary conditions said in section 5, uniform displacement load applied at top edges of the plate model and model was analyzed using the Non-Liner static analysis of the ANSYS including material and geometric non-linear.

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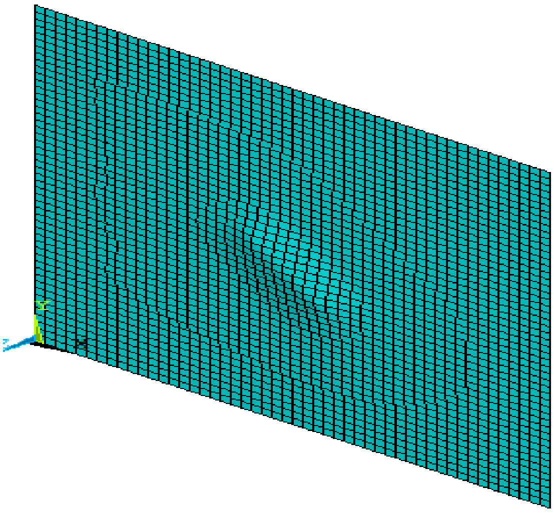
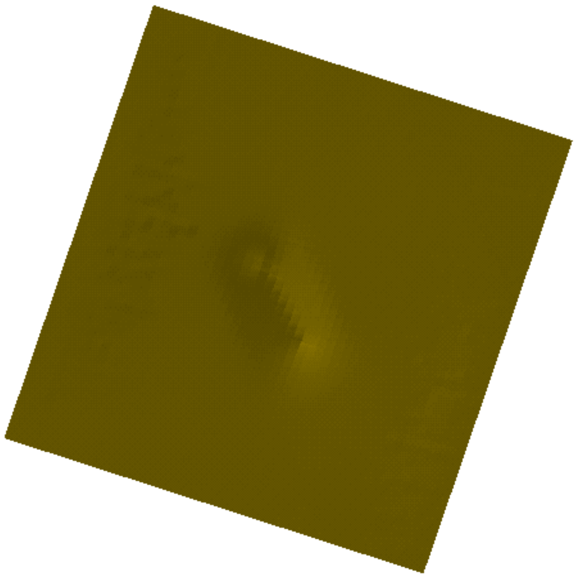


(a)



(b)

Figure-11. (a) FE mesh view of dented thin plate, (b) isometric view of dented thin plate.

9. RESIULTS AND DISCUSSIONS

By comparing the above Table-1 and Figure-12, it can be noted that experimental load vs. displacement curve fairly matches with each other by maximum with18% error. The deviations between experimental ultimate strength values and FE ultimate strength values are due to not accounting the factors such as residual stress induced during plastic deformation of the plate during dent formation, plate thickness variation, the numerical error, while transferring the measured imperfections value into the FE plate model, the variation in even E and σy at dent region due to strain hardening effect in the numerical analysis. The deviations reported in this work are comparable with deviations reported in the published references like Bisagni [7] and Tennyson *et al.* [35]. From above discussion, it is clear that the error between in experimental and numerical results can be reduced by incorporating above said factors in the FE models.

Table-1. Experimental and FE analysis Results.

Thin plate with different type of dent	Specimen No	Max. imperfection depth (in mm)	Ultimate strength (in kN)		% of Error on Ultimate Strength
			Experimental	FE Model	
Thin plate without dent	A1	0.06	293.6	339.1	13.4%
	A2	0.05	286.9	339.5	15.5%
Thin plate with vertical dent	C1	9.42	282.0	338.0	16.6%
	C2	9.64	280.0	332.8	15.9%
Thin plate with inclined dent	D1	9.58	280.7	337.8	16.9%
	D2	9.65	278.2	332.1	16.2%
Thin plate with horizontal dent	B1	9.12	264.2	322.0	18.0%
	B2	9.05	267.0	325.5	18.0%

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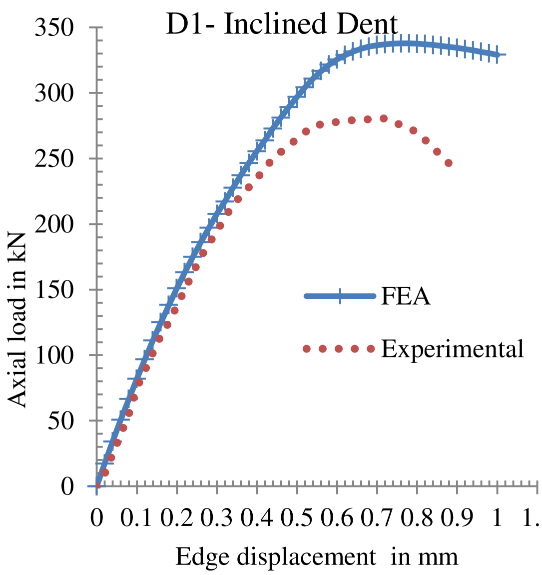
Table-1 compares the experimental value of the dented plates with corresponding the ultimate strength of the dented plate obtained from the FE analysis.



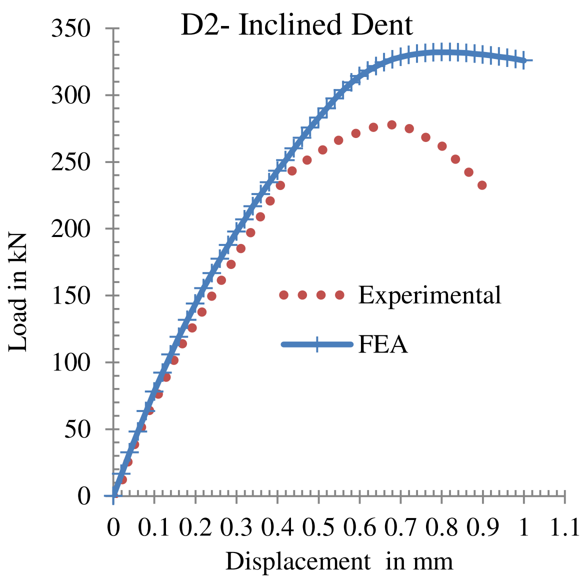


Figure-12. Comparison of load vs edge displacement curves of FEA and experimental results of thin plates with a inclined dent.

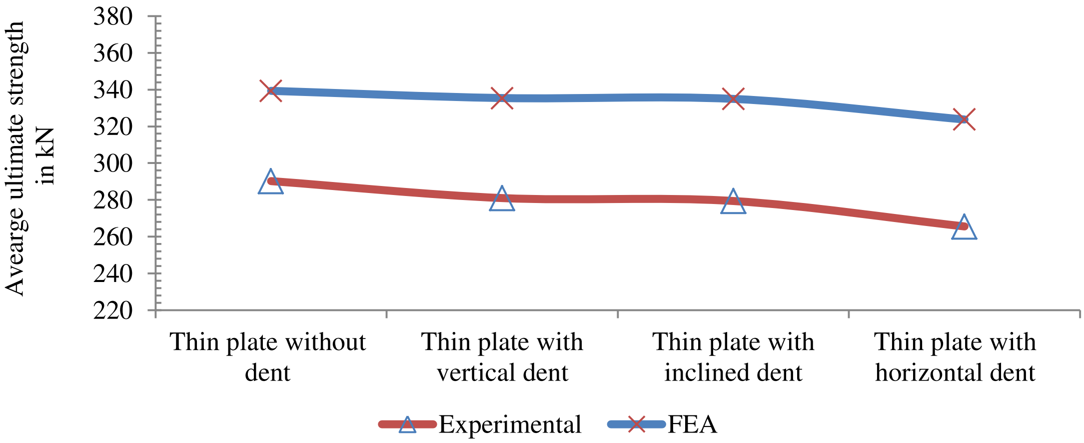


Figure-13. Comparison of average ultimate strengths of dented plates obtained from experimental and FE analysist.

Figure-13 compares the average experimental ultimate strength and average ultimate strength obtained from the FE models for each case of the dented plate models and intact plate (without dent) taken for the study.

The Figure-13 shows that, FE ultimate strength values are greater than corresponding the average experimental values (with maximum error of 18 %), but experimental and numerical results show the same trend. In general, ultimate strength of the intact plates (with only distributed geometrical imperfections) showed higher ultimate strength than dented plates and also plate with longitudinal dent showed higher strength than that of transversely dented plates. Further can be noted that the effect of local deformation is meager compared to distributed geometrical imperfection present in the intact plate. Further, a slight deviation is noted for edge displacement at which the ultimate strength obtained by experimental and numerical analysis.

From the above discussion, it can be that concluded that distributed geometrical imperfections have more dominant effect than local geometrical imperfections namely dent. The maximum deviation obtained between longitudinal dent and transverse dent 5.8% and 3.6 % respectively for experimental and FE analysis. The deviation between the average ultimate strength of intact plate model and longitudinal dented plate is 3.3 % and 1.2 % respectively for experimental and FE Analysis.

The reduction in ultimate strength by combined effect of distributed geometrical imperfection and dent is meagerly increases by maximum of 9.3% and 4.8 %

respectively from experimental and numerical results for the dented plate taken for study with average dent depth of 9.4 mm. Further it can be noted that, in case of dented plates, as dent depth almost equal, we can clearly visualize the effect of angle of orientation of dent on the ultimate strength of the plate. i.e., as the angle of orientation increases from transverse dent to longitudinal dent orientation, the ultimate strength of the dented plate also increases and same trend for dent orientation on ultimate strength of the plate structures was also reported in the published research works (Raviprakash [28], Peroumal *et al*. [26]).

10. CONCLUSIONS

The following conclusions are derived from experimetal and numerical results obtained from the present work.

a) The approach followed for transferring the actual geometrical imperfections to FE models using 3D interpolatable surface imperfections model is succussfully implemented with error less than 1%

b) The devaitions between the experimental results and numerical results are reasonble and it can be improved by including residual stress and thickness variation, etc., in the FE model.

c) Distributted geometrical imperfections are more dominent effect on ultimate strength of the plate than that by local geometrical imperfection-dent.

d) The reduction in ultimate strength by combined effect of distributed geometrical imperfections and a dent meagerly increases.

e) Longitudinal dented plate showed higher strength compared to transverse dented plate.

f) As the dent depth is almost same in all the dented plates taken for study, the effect of orientation of the dent on the ultimate strength can be clearly visualize. i.e., as the dent orientation increases from transverse orientation to longitudinal orientation reduction of ultimate strength increases.

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