# FINITE SINE INTEGRAL TRANSFORM DYNAMIC ANALYSIS OF FREE DAMPED ORTHOTROPIC PLATE ON ELASTIC SUBGRADE 

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#### Abstract

In this study, the dynamic behaviour of orthotropic type of plate, without loading but with effect of damping, was analysed. The mathematical model governing such phenomenon was evaluated using finite sine integral transform method. Analytical approach was adopted throughout the study. Numerical examples were, however, carried out and it was observed that the present method is very simple to apply and performed well for the type of plate considered in this study, going by its efficiency. The results obtained are consistent with the ones in the literature. The finite integral transform is an easy method to use.


Keywords: finite sine integral transform, dynamic analysis, orthotropic plate, elastic subgrade, damping.

## 1. INTRODUCTION

Many authors have studied orthotropic rectangular plates extensively especially in Engineering applications. There has been difficulty in obtaining exact solutions which can satisfy the equations and the boundary conditions of most of these plates [1].

Methods such as the finite difference method, finite element method, finite strip method, integral equation method, differential quadrature element method, method of discrete singular convolution, method of differential quadrature, meshless method and spline element method, have been used by different researchers to solve the boundary value problem of plates [2, 3, 4]

In the present study, a double finite sine integral transform method is adopted to analyse the dynamic behaviour of free damped orthotropic plate on an elastic subgrade. To the best of knowledge of the authors, such transform method has not been used to analyse such plates supported by a foundation $[5,6,7,8,9]$.

Free vibrations of an elastic body, such as elastic rectangular plates, are called natural vibrations and occur at a frequency called the natural frequency. As against forced vibration, natural vibrations happen at frequency of applied force, thus the name forced frequency. When both are equal, the amplitude of vibration increases. A phenomenon is known as resonance [ $4,6,10,11]$.

## 2. FORMULATION OF PROBLEM

In line with the classical plate theory, the governing equation of motion for free damped orthotropic plate on a Winkler foundation is $[5,7,8]$ :
$D \frac{\partial^{4} w}{\partial x^{4}}+2 D \frac{\partial^{4} w}{\partial^{2} x \partial^{2} y}+D \frac{\partial^{4} w}{\partial y^{4}}+\rho h \frac{\partial^{2} w}{\partial t^{2}}+k w(x, y, t)+2 M \gamma \frac{\partial w}{\partial t}=0$

Where,
$\begin{array}{ll}\gamma & =\text { viscous damping coefficient } \\ \mathrm{w} & =\text { out of plane displacement } \\ \mathrm{t} & =\text { time in seconds }\end{array}$
$\mathrm{k} \quad=$ foundation stiffness (reaction coefficients of foundation)
h = thickness of plate
$\rho \quad=$ density of plate
D $\quad=\mathrm{Eh}^{3} / 12\left(1-\mathrm{v}^{2}\right)$ is the Flexural rigidity of plate
$\mathrm{E} \quad=$ Young's modulus

### 2.1 Assumptions

The following assumptions were made in this study [3, 4]:
a) The plate is of constant cross-section
b) The plate is continuously supported by Winkler foundation
c) The orthotropic plate is elastic.
d) The plate exhibits harmonic vibration
e) Very small natural frequency

The following holds for free plate [1]:
$Q_{x}=-D \frac{\partial}{\partial x}\left(\frac{\partial^{2} W}{\partial x^{2}}+\frac{\partial^{2} W}{\partial y^{2}}\right)=0$ at $x=0$ and $x=a$
$M_{x}=-D\left(\frac{\partial^{2} W}{\partial x^{2}}+\frac{\partial^{2} W}{\partial y^{2}}\right)=0$ at $x=0$ and $x=a$
$M_{x y}=-D(1-v)\left(\frac{\partial^{2} W}{\partial x \partial y}\right)=0$ at $x=0$ and $x=a ; y=0$ and $y=b$
$Q_{y}=-D \frac{\partial}{\partial y}\left(\frac{\partial^{2} W}{\partial x^{2}}+\frac{\partial^{2} W}{\partial y^{2}}\right)=0$ at $\mathrm{y}=0$ and $\mathrm{y}=a$
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$$
\begin{align*}
& M_{y}=-D\left(\frac{\partial^{2} W}{\partial y^{2}}+\frac{\partial^{2} W}{\partial x^{2}}\right)=0 \text { at } \mathrm{y}=0 \text { and } \mathrm{y}=b  \tag{6}\\
& V_{x}=Q_{x}+\frac{\partial M_{x y}}{\partial y}=0  \tag{7}\\
& V_{y}=Q_{y}+\frac{\partial M_{x y}}{\partial x}=0 \tag{8}
\end{align*}
$$

Where,
$Q_{x}$ is the shear force along $\mathrm{x}-\mathrm{axis}$
$M_{x}$ is the bending moment along x -axis
$M_{y}$ is the bending moment along y - axis
$Q_{y}$ is the shear force along y - axis
$M_{x y}$ is the torsional moment
$V_{x}$ is the effective shear force along x - axis $V_{y}$ is the effective shear force along y - axis $v$ is the poison's ratio

The boundary conditions of the plate can be expressed as $[3,5]$ :

$$
\begin{equation*}
\left.W\right|_{x=0, a}=0,\left.\quad W\right|_{y=0, b}=0,\left.\quad \frac{\partial W}{\partial x}\right|_{x=0, a}=0,\left.\quad \frac{\partial W}{\partial x}\right|_{y=0, b}=0 \tag{9}
\end{equation*}
$$

For harmonic vibration [1],

$$
\begin{equation*}
w(x, y, t)=W(x, y) \sin \omega t \tag{10}
\end{equation*}
$$

Where $\omega$ is the natural circular frequency of the plate, $a$ and $b$ are the plate dimensions, while $W(x, y)$ is the shape function describing the modes of the vibration. Differentiating equation (10) gives;

$$
\begin{align*}
& \frac{\partial w}{\partial t}=\omega W(x, y) \cos \omega t  \tag{11}\\
& \frac{\partial^{2} w}{\partial t^{2}}=-\omega^{2} W(x, y) \sin \omega t \tag{12}
\end{align*}
$$

Substituting equations (11) and (12) into equation (1) gives;
$D \frac{\partial^{4} W}{\partial x^{4}}+2 D \frac{\partial^{4} W}{\partial^{2} x \partial^{2} y}+D \frac{\partial^{4} W}{\partial y^{4}}+\rho h\left(-\omega^{2} W(x, y) \sin \omega t\right)+k W(x, y)+2 M \gamma \omega W(x, y) \cot \omega t=0$
For very small $\omega$;
$D \frac{\partial^{4} W}{\partial x^{4}}+2 D \frac{\partial^{4} W}{\partial^{2} x \partial^{2} y}+D \frac{\partial^{4} W}{\partial y^{4}}+\left\{k-\rho h \omega^{2}\right\} W(x, y)+2 M \gamma \omega W(x, y) \cot \omega t=0$

Dividing through by D and replacing $\rho h_{\text {with M, }}$, the mass density per unit area, without x and y , equation (14) becomes;

$$
\begin{align*}
& \frac{\partial^{4} W}{\partial x^{4}}+2 \frac{\partial^{4} W}{\partial^{2} x \partial^{2} y}+\frac{\partial^{4} W}{\partial y^{4}}+\frac{\left\{k-M \omega^{2}\right\}}{D} W+\frac{2 M \gamma \omega}{D} W \cot \omega t=0  \tag{15}\\
& \frac{\partial^{4} W}{\partial x^{4}}+2 \frac{\partial^{4} W}{\partial^{2} x \partial^{2} y}+\frac{\partial^{4} W}{\partial y^{4}}+\lambda_{1} W+\lambda_{2} W \cot \omega t=0 \tag{16}
\end{align*}
$$

Where
$\lambda_{1}=\frac{\left\{k-M \omega^{2}\right\}}{D}$
$\lambda_{2}=\frac{2 M \gamma \omega}{D}$

By carefully chosen the values of natural circular frequency of the plate and time such that $\cot \omega t$ equals 1 and $\omega t \neq 0$. This implies $\omega t=2 \pi t$ radian and makes equation (16) to become;
$\frac{\partial^{4} W}{\partial x^{4}}+2 \frac{\partial^{4} W}{\partial^{2} x \partial^{2} y}+\frac{\partial^{4} W}{\partial y^{4}}+\lambda W=0$
where $\lambda=\left(\lambda_{1}+\lambda_{2}\right)$

## 3. PROBLEM SOLUTION

Finite sine integral transform method was adopted in this study.

To solve the partial differential equation (19), a double finite sine integral transform approach was utilized. $\mathrm{W}(\mathrm{x}, \mathrm{y})$ is defined within a rectangular domain $0<\mathrm{x}<\mathrm{a}$ and $0<y<b$.

Following Rui Li's work, a double sine integral transform is defined as [6]:
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$W_{m n}=\iint W(x, y) \sin \alpha_{m} x \sin \beta_{n} y d x d y$
$m=1,2.3 \ldots$ and $n=1,2,3 \ldots$
$W(x, y)=\frac{4}{a b} \sum_{1}^{\infty} \sum_{1}^{\infty} W_{m n} \sin \alpha_{m} x \sin \beta_{n} y$
where,

$$
\begin{equation*}
\alpha_{m}=\frac{m \pi}{a} \text { and } \beta_{n}=\frac{n \pi}{b} \tag{21}
\end{equation*}
$$

The double sine integral transform of the first, third and second partial derivative terms in equation (19) can be written respectively, as follows $[6,8]$ :

$$
\begin{aligned}
& \int_{0}^{a} \int_{0}^{b} \frac{\partial^{4} W}{\partial x^{4}} \sin \alpha_{m} x \sin \beta_{n} y d x d y=\alpha_{m}^{4} \bar{W}(m, n)-\alpha_{m} \int_{0}^{b}(-1)^{m}\left\{\left.\frac{\partial^{2} W}{\partial x^{2}}\right|_{x=a}-\left.\frac{\partial^{2} W}{\partial x^{2}}\right|_{x=0}\right\} \sin \beta_{n} y d y \\
& +\alpha_{m}^{3} \int_{0}^{b}\left[\left.(-1)^{m} W\right|_{x=a}-\left.W\right|_{x=0}\right] \sin \beta_{n} y d y
\end{aligned}
$$

$$
\int_{0}^{a} \int_{0}^{b} \frac{\partial^{4} W}{\partial y^{4}} \sin \alpha_{m} x \sin \beta_{n} y d x d y=\beta_{m}^{4} \bar{W}(m, n)-\beta_{n} \int_{0}^{b}(-1)^{n}\left\{\left.\frac{\partial^{2} W}{\partial y^{2}}\right|_{y=b}-\left.\frac{\partial^{2} W}{\partial y^{2}}\right|_{y=0}\right\} \sin \alpha_{m} x d x
$$

$$
\begin{equation*}
+\beta_{n}^{3} \int_{0}^{b}\left[\left.(-1)^{n} W\right|_{y=b}-\left.W\right|_{y=0}\right] \sin \alpha_{m} x d x \tag{23}
\end{equation*}
$$

$\int_{0}^{a} \int_{0}^{b} \frac{\partial^{4} W}{\partial x^{2} \partial y^{2}} \sin \alpha_{m} x \sin \beta_{n} y d x d y=\alpha_{m}^{2} \beta_{n}^{2} \bar{W}(m, n)-\alpha_{m} \int_{0}^{b}\left[\left.(-1)^{n} \frac{\partial^{2} W}{\partial y^{2}}\right|_{x=a}-\left.\frac{\partial^{2} W}{\partial y^{2}}\right|_{x=0}\right] \sin \beta_{n} y d y$
$+\alpha_{m}^{3} \beta_{n} \int_{0}^{a}\left[\left.(-1)^{n} W\right|_{y=b}-\left.W\right|_{y=0}\right] \sin \alpha_{m} x d x$
Imposing the boundary conditions, described by equation (9) on equation (22), (23) and (24) to simplify the expressions, with the right hand sides becoming, respectively:

$$
\begin{align*}
& \alpha_{m}^{4} W_{m n}-\alpha_{m} \int_{0}^{b}\left[\left.(-1)^{m} \frac{\partial^{2} W}{\partial x^{2}}\right|_{x-a}-\left.\frac{\partial^{2} W}{\partial x^{2}}\right|_{x-0}\right] \sin \beta_{n} y d y  \tag{25}\\
& \beta_{n}^{4} W_{m n}-\beta_{n} \int_{0}^{a}\left[\left.(-1)^{n} \frac{\partial^{2} W}{\partial y^{2}}\right|_{y-b}-\left.\frac{\partial^{2} W}{\partial y^{2}}\right|_{y-0}\right] \sin \alpha_{m} x d x \tag{26}
\end{align*}
$$

$$
\begin{equation*}
\alpha_{m}^{2} \beta_{n}^{2} W_{m n} \tag{27}
\end{equation*}
$$

Substitution expressions (25), (26) and (27) into equation (19) gives:

$$
\begin{align*}
& \left\{\alpha_{m}^{4} W_{m n}-\alpha_{m} \int_{0}^{b}\left[\left.(-1)^{m} \frac{\partial^{2} W}{\partial x^{2}}\right|_{x=a}-\left.\frac{\partial^{2} W}{\partial x^{2}}\right|_{x=0}\right] \sin \beta_{n} y d y\right\}+2\left\{\alpha_{m}^{2} \beta_{n}^{2} W_{m n}\right\} \\
& +\left\{\beta_{n}^{4} W_{m n}-\beta_{n} \int_{0}^{a}\left[\left.(-1)^{n} \frac{\partial^{2} W}{\partial y^{2}}\right|_{y=b}-\left.\frac{\partial^{2} W}{\partial y^{2}}\right|_{y=0}\right] \sin \alpha_{m} x d x\right\}+\lambda W_{n n}=0 \tag{28}
\end{align*}
$$

Which can be written as:
$\alpha_{m} \int_{0}^{b}\left[\left.(-1)^{m} \frac{\partial^{2} W}{\partial x^{2}}\right|_{x=a}-\left.\frac{\partial^{2} W}{\partial x^{2}}\right|_{x=0}\right] \sin \beta_{n} y d y+\beta_{n} \int_{0}^{a}\left[\left.(-1)^{n} \frac{\partial^{2} W}{\partial y^{2}}\right|_{y=b}-\left.\frac{\partial^{2} W}{\partial y^{2}}\right|_{y=0}\right] \sin \alpha_{m} x d x=$

$$
\begin{equation*}
\left[\alpha_{m}^{4}+2 \alpha_{m}^{2} \beta_{n}^{2}+\beta_{n}^{4}+\lambda\right] W_{m n} \tag{29}
\end{equation*}
$$

To be succinct, let
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$I_{m}=\left.\int_{0}^{a} \frac{\partial^{2} W}{\partial y^{2}}\right|_{y=b} \sin \alpha_{m} x d x$
$J_{m}=\left.\int_{0}^{a} \frac{\partial^{2} W}{\partial y^{2}}\right|_{y=0} \sin \alpha_{m} x d x$

$$
\begin{align*}
& K_{n}=\left.\int_{0}^{a} \frac{\partial^{2} W}{\partial x^{2}}\right|_{x=a} \sin \beta_{n} y d y  \tag{32}\\
& L_{n}=\left.\int_{0}^{a} \frac{\partial^{2} W}{\partial x^{2}}\right|_{x=0} \sin \beta_{n} y d y \tag{33}
\end{align*}
$$

Substituting equations (30) - (33) into equation (29) gives;
$\alpha_{m}\left[(-1)^{m} K_{n}-L_{n}\right]+\beta_{n}\left[(-1)^{n} I_{m}-J_{m}\right]=\left[\alpha_{m}^{4}+2 \alpha_{m}^{2} \beta_{n}^{2}+\beta_{n}^{4}+\lambda\right] W_{m n}$
This implies,
$W_{m n}=\frac{1}{\alpha_{m}^{4}+2 \alpha_{m}^{2} \beta_{n}^{2}+\beta_{n}^{4}+\lambda}\left\{\alpha_{m}\left[(-1)^{m} K_{n}-L_{n}\right]+\beta_{n}\left[(-1)^{n} I_{m}-J_{m}\right]\right\}$
$m=1,2,3, \ldots$
$n=1,2,3, \ldots$

Following Rui Li et al work [8], by substituting equation (21) into the boundary condition represented by equation (9) gives;

$$
\begin{equation*}
\sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{4}{a b} \alpha_{m} W_{m n} \sin \beta_{n} y=0 \tag{36}
\end{equation*}
$$

$$
\begin{align*}
& \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{4}{a b} \beta_{n} W_{m n} \sin \alpha_{m} x=0  \tag{38}\\
& \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{4}{a b}(-1)^{n} \beta_{n} W_{m n} \sin \alpha_{m} x=0 \tag{39}
\end{align*}
$$

$$
\begin{equation*}
\sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{4}{a b}(-1)^{m} \alpha_{m} W_{m n} \sin \beta_{n} y=0 \tag{37}
\end{equation*}
$$

Substituting equation (35) into equations (36) (39) gives
$\sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{4}{a b} \alpha_{m}\left[\frac{1}{\alpha_{m}^{4}+2 \alpha_{m}^{2} \beta_{n}^{2}+\beta_{n}^{4}+\lambda}\left\{\alpha_{m}\left[(-1)^{m} K_{n}-L_{n}\right]+\beta_{n}\left[(-1)^{n} I_{m}-J_{m}\right]\right\}\right] \sin \beta_{n} y=0$
$\sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{4}{a b}(-1)^{m} \alpha_{m}\left[\frac{1}{\alpha_{m}^{4}+2 \alpha_{m}^{2} \beta_{n}^{2}+\beta_{n}^{4}+\lambda}\left\{\alpha_{m}\left[(-1)^{m} K_{n}-L_{n}\right]+\beta_{n}\left[(-1)^{n} I_{m}-J_{m}\right]\right\}\right] \sin \beta_{n} y=0$
$\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{4}{a b} \beta_{n}\left[\frac{1}{\alpha_{m}^{4}+2 \alpha_{m}^{2} \beta_{n}^{2}+\beta_{n}^{4}+\lambda}\left\{\alpha_{m}\left[(-1)^{m} K_{n}-L_{n}\right]+\beta_{n}\left[(-1)^{n} I_{m}-J_{m}\right]\right\}\right] \sin \alpha_{m} x=0$
$\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{4}{a b}(-1)^{n} \beta_{n}\left[\frac{1}{\alpha_{m}^{4}+2 \alpha_{m}^{2} \beta_{n}^{2}+\beta_{n}^{4}+\lambda}\left\{\alpha_{m}\left[(-1)^{m} K_{n}-L_{n}\right]+\beta_{n}\left[(-1)^{n} I_{m}-J_{m}\right]\right\}\right] \sin \alpha_{m} x=0$

Equations (40) - (43) are four infinite systems of linear simultaneous equations with respect to unknown constants $\mathrm{K}_{\mathrm{n}}, \mathrm{L}_{\mathrm{n}}, \mathrm{I}_{\mathrm{m}}$, and $\mathrm{J}_{\mathrm{m}}(\mathrm{m}=1,2,3, \ldots ; \mathrm{n}=1,2,3, \ldots)$. Usually, a finite number of terms in each set of equations are considered and the resulting sets of finite number of simultaneous equations are solved to determine the constants. In this study however, numerical examples were exploited.

## 4. NUMERICAL EXAMPLES AND RESULTS OF THE PROBLEM

For the purpose of numerical evaluation the following values were used in order to find the values of the constants $K_{n}, L_{n}, I_{m}$, and $J_{m}$. The values were used to evaluate the out of plane displacement of the system. The geometrical and material properties are taken to be: $\mathrm{a}=4 \mathrm{~m}$, $\mathrm{b}=5 \mathrm{~m}, \mathrm{v}=0.15, \mathrm{~h}=0.3 \mathrm{~m}, \mathrm{E}=70 \mathrm{GPa}$. Tables below show the values of parameters needed to solve for the constants in equations (40) - (43).

Table-1. Values of $\alpha_{m}$ and $\beta_{n}$ at different values of $m$ and $n$.

| x | m | n | $\alpha_{m}$ | $\beta_{n}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 1 | 0.25 | 0.20 |
| 1 | 2 | 2 | 0.50 | 0.40 |
| 1 | 3 | 3 | 0.75 | 0.60 |
| 1 | 4 | 4 | 1.00 | 0.80 |
| 1 | 5 | 5 | 1.25 | 1.00 |
| 2 | 1 | 1 | 0.50 | 0.40 |
| 2 | 2 | 2 | 1.00 | 0.80 |
| 2 | 3 | 3 | 1.50 | 1.20 |
| 2 | 4 | 4 | 2.00 | 1.60 |
| 2 | 5 | 5 | 2.50 | 2.00 |
| 3 | 1 | 1 | 0.75 | 0-60 |
| 3 | 2 | 2 | 1.50 | 1.20 |
| 3 | 3 | 3 | 2.25 | 1.80 |
| 3 | 4 | 4 | 3.00 | 2.40 |
| 3 | 5 | 5 | 3.75 | 3.00 |
| 4 | 1 | 1 | 1.00 | 0.80 |
| 4 | 2 | 2 | 2.00 | 1.60 |
| 4 | 3 | 3 | 3.00 | 2.40 |
| 4 | 4 | 4 | 4.00 | 3.20 |
| 4 | 5 | 5 | 5.00 | 4.00 |

Table-2. Values of $\lambda_{\text {at different values of the parameters for } t=1}$.

| $\mathbf{k}$ | $\gamma$ | $\mathbf{D}$ | $\mathbf{M}$ | $\omega$ | $\lambda_{1}$ | $\lambda_{2}$ | $\lambda$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.05 | 10 | 0.16 | 0.5 | 360 | $-404,999.69$ | 22,500 | $-382,499.69$ |
| 0.05 | 100 | 0.16 | 0.5 | 360 | $-404,999.69$ | 225000 | $-179,999.69$ |
| 0.05 | 300 | 0.16 | 0.5 | 360 | $-404,999.69$ | 675,000 | $270,000.31$ |
| 0.05 | 500 | 0.16 | 0.5 | 360 | $-404,999.69$ | $1,125,000$ | $720,000.31$ |
| 0.05 | 800 | 0.16 | 0.5 | 360 | $-404,999.69$ | $1,800,000$ | 1395000.31 |
| 0.04 | 10 | 0.16 | 0.5 | 360 | $-404,999.75$ | 22,500 | $-382,499.75$ |
| 0.06 | 10 | 0.16 | 0.5 | 360 | $-404,999.63$ | 22,500 | $-382,499.63$ |
| 0.08 | 10 | 0.16 | 0.5 | 360 | $-404,999.50$ | 22,500 | $-382,499.50$ |
| 1.00 | 10 | 0.16 | 0.5 | 360 | $-404,993.75$ | 22,500 | $-382,493.75$ |
| 1.20 | 10 | 0.16 | 0.5 | 360 | $-404,992.50$ | 22,500 | $-382,492.50$ |
| 0.05 | 10 | 0.16 | 0.5 | 360 | $-404,999.69$ | 22,500 | $-382,499.69$ |
| 0.05 | 10 | 0.16 | 1.00 | 360 | $-809,999.69$ | 45,000 | $-764,999.69$ |
| 0.05 | 10 | 0.16 | 2.00 | 360 | $-1,619,999.69$ | 90,000 | $-1,529,999.69$ |
| 0.05 | 10 | 0.16 | 2.50 | 360 | $-2,024,999.69$ | 112,500 | $-1,912,499.69$ |
| 0.05 | 10 | 0.16 | 3.00 | 360 | $-2,429,999.69$ | 135,000 | $-2,294,999.69$ |

Substituting the values of $\alpha_{m}, \beta_{n}$ and $\lambda$, at different times into equations (40) - (43) gives:

$$
\begin{align*}
& \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{4}{a b} \alpha_{m}\left[\frac{1}{\alpha_{m}^{4}+2 \alpha_{m}^{2} \beta_{n}^{2}+\beta_{n}^{4}+\lambda}\left\{\alpha_{m}\left[(-1)^{m} K_{n}-L_{n}\right]+\beta_{n}\left[(-1)^{n} I_{m}-J_{m}\right]\right\}\right] \sin \beta_{n} y=0  \tag{44}\\
& \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{4}{a b}(-1)^{m} \alpha_{m}\left[\frac{1}{\alpha_{m}^{4}+2 \alpha_{m}^{2} \beta_{n}^{2}+\beta_{n}^{4}+\lambda}\left\{\alpha_{m}\left[(-1)^{m} K_{n}-L_{n}\right]+\beta_{n}\left[(-1)^{n} I_{m}-J_{m}\right]\right\}\right] \sin \beta_{n} y=0  \tag{45}\\
& \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{4}{a b} \beta_{n}\left[\frac{1}{\alpha_{m}^{4}+2 \alpha_{m}^{2} \beta_{n}^{2}+\beta_{n}^{4}+\lambda}\left\{\alpha_{m}\left[(-1)^{m} K_{n}-L_{n}\right]+\beta_{n}\left[(-1)^{n} I_{m}-J_{m}\right]\right\}\right] \sin \alpha_{m} x=0  \tag{46}\\
& \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{4}{a b}(-1)^{n} \beta_{n}\left[\frac{1}{\alpha_{m}^{4}+2 \alpha_{m}^{2} \beta_{n}^{2}+\beta_{n}^{4}+\lambda}\left\{\alpha_{m}\left[(-1)^{m} K_{n}-L_{n}\right]+\beta_{n}\left[(-1)^{n} I_{m}-J_{m}\right]\right\}\right] \sin \alpha_{m} x=0 \tag{47}
\end{align*}
$$

Equations (44) - (47) implies:
$\alpha_{m}\left[(-1)^{m} K_{n}-L_{n}\right]+\beta_{n}\left[(-1)^{n} I_{m}-J_{m}\right]=0$
For the purpose of calculation, it is assumed for n $=1,2,3,4$, and $m=1,2,3,4$, the values of $K_{n}, L_{n}, I_{m}$ and $J_{m}$ are constant since $\alpha_{m}, \beta_{n}$ are very small.

For $\mathrm{n}=1$ and $\mathrm{m}=1$

$$
\begin{equation*}
\left[0.25\left[-K_{n}-L_{n}\right]+0.2\left[-I_{m}-J_{m}\right]\right]=0 \tag{49}
\end{equation*}
$$

For $\mathrm{n}=2$ and $\mathrm{m}=2$,

$$
\begin{equation*}
\left[0.5\left[K_{n}-L_{n}\right]+0.4\left[I_{m}-J_{m}\right]\right]=0 \tag{50}
\end{equation*}
$$

For $\mathrm{n}=3$ and $\mathrm{m}=3$,
$\left[0.75\left[-K_{n}-L_{n}\right]+0.6\left[-I_{m}-J_{m}\right]\right]=0$
For $\mathrm{n}=4$ and $\mathrm{m}=4$,
$\left[1\left[K_{n}-L_{n}\right]+0.8\left[I_{m}-J_{m}\right]\right]=0$

Solving equations (52) - (55) simultaneously yielded the following particular results:

$$
\begin{aligned}
& J=1, I=1, K=5+L \\
& J=2, I=2, K=10+L \\
& J=3, I=3, K=15+L \\
& J=4, I=4, K=20+L \\
& J=5, I=5, K=25+L
\end{aligned}
$$

So for a particular value of $L$, say 7 , gives:

$$
\begin{aligned}
& J=1, I=1, K=12, L=7 \\
& J=2, I=2, K=17, L=7 \\
& J=3, I=3, K=22, L=7 \\
& J=4, I=4, K=27, L=7 \\
& J=5, I=5, K=32, L=7
\end{aligned}
$$

From equation (35):

$$
\begin{equation*}
W_{m n}=Q\left\{\alpha_{m}\left[(-1)^{m} K_{n}-L_{n}\right]+\beta_{n}\left[(-1)^{n} I_{m}-J_{m}\right]\right\} \tag{53}
\end{equation*}
$$

Which can be written as;

$$
\begin{equation*}
W_{m n}=Q\left\{\alpha_{m}\left[(-1)^{m} K-L\right]+\beta_{n}\left[(-1)^{n} I-J\right]\right\} \tag{54}
\end{equation*}
$$

With the assumption that the values of $\mathrm{K}, \mathrm{L}, \mathrm{I}$ and $j$ remains constant at any particular values of $m$ and $n$. Where,

$$
\begin{equation*}
Q=\frac{1}{\alpha_{m}^{4}+2 \alpha_{m}^{2} \beta_{n}^{2}+\beta_{n}^{4}+\lambda} \tag{55}
\end{equation*}
$$

Table-3. Values of ${ }^{W_{m n}}$, for different values of the parameters and fixed value of $\lambda$.

| $\mathbf{m}$ | $\mathbf{n}$ | $\alpha_{m}$ | $\beta_{n}$ | $\lambda$ | $Q$ | ${ }^{1} W_{m n}$ | ${ }^{2} W_{m n}$ | ${ }^{3} W_{m n}$ | ${ }^{4} W_{m n}$ | ${ }^{5} W_{m n}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 0.25 | 0.20 | 0.5 | 1.96 | -10.09 | -13.33 | -16.36 | -18.93 | -19.89 |
| 2 | 2 | 0.50 | 0.40 | 0.5 | 1.50 | 3.75 | 7.50 | 11.25 | 15.00 | 18.75 |
| 3 | 3 | 0.75 | 0.60 | 0.5 | 0.74 | -15.45 | -15.10 | -18.76 | -22.42 | -22.53 |
| 4 | 4 | 1.00 | 0.80 | 0.5 | 0.31 | 1.55 | 3.10 | 4.65 | 3.10 | 7.75 |
| 5 | 5 | 1.25 | 1.00 | 0.5 | 0.14 | -3.61 | -4.76 | -5.92 | -7.07 | -7.11 |

Where,
${ }^{1} W_{m n}$ : Values of $W_{m n}$, for $J=1, I=1, K=12, L=7$ and fixed value of $\lambda$
${ }^{2} W_{m n}$ : Values of $W_{m n}$, for $J=2, I=2, K=17, L=7$ and fixed value of $\lambda$
${ }^{3} W_{m n}$ : Values of $W_{m n}$, for $J=3, I=3, K=22, L=7$ and fixed value of $\lambda$
${ }^{4} W_{m n}$ : Values of $W_{m n}$, for $J=4, I=4, K=27, L=7$ and fixed value of $\lambda$
${ }^{5} W_{m n}$ : Values of $W_{m n}$, for $J=5, I=5, K=32, L=7$ and fixed value of $\lambda$

The shape function describing the mode of vibration of the plate under consideration shows that the values of the parameters and constants have significant effect on the dynamic response of the plate supported by Winkler foundation. The maximum amplitude of the vibration of the plate at different values of constants
considered, J, I, K, and L, occurred when $\mathrm{m}=2$ and $\mathrm{n}=2$. The highest maximum amplitude of all the ones considered occurred at the highest values of the constants. The trend showed that the higher the values of the constants, the higher the maximum amplitude.

Two dimensional of plate deflection for different values of J, I, K, and L have been depicted in Figure-1. The trend of shape function describing the modes of vibration of the plate have been depicted in Figure-2.
${ }^{1} W_{m n},{ }^{2} W_{m n},{ }^{3} W_{m n},{ }^{4} W_{m n}$, and ${ }^{5} W_{m n}$ are represented respectively, in the Figure-1, by 1W, 2W, 3W, 4W, and
5 W . The Figure shows that ${ }^{5} W_{m n}$ has the highest maximum amplitude.

Series 1, Series 2, Series 3, Series 4, and Series 5 in Figure-2 represent trend of the shape function as the values of the constants; J, I, K, and L, increase. It can observed that series 2 and series 4 , when the values of $n$ and $m$ are positive, exhibits positive trends. Series1, series 3 and series 3 when the values of $n$ and $m$ are odd shows negative trend.


Figure-1. Comparing the description of the vibration of the plate at different constants values.
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Figure-2. The trend of the vibration as $n$ and $m$ increase.

## 5. CONCLUSIONS

Dynamic Analysis of Free Damped Orthotropic Plate on Elastic Subgrade was carried out in this study using Finite Sine Integral Transform. The type of elastic subgrade considered was the Winkler type. The fourth order partial differential equation governing such system was considered and transformed into a simpler algebraic form, which was easier to handle in terms of computation. The shape function describing the mode of vibration of the type of plate considered was analysed. The infinite series obtained described the characteristic equations. The series were truncated to finite number of terms and solved analytically to obtain shape functions at different values of the parameters and constants. Displacements have been demonstrated in two dimensions. The present method performed well for free damped orthotropic plate going by its efficiency. It is also very simple to apply.

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