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DOUBLE-CONTOUR GEODESIC SHELLS WITH TETRAHEDRAL PYRAMIDS

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ABSTRACT

Geodesic domes are self-supporting spatial structures without additional support. When these are broken down into their elements, geodesic lines are used. In this area, the research into geometric models of double-contour geodesic shells is conducted. The features of these works include the use of trihedral and hexagonal pyramidal structures as the first contour of the geodesic shell. However, for the formation of such structures, tetrahedral pyramids can also be used. The purpose of this research is to develop methods, algorithms and software for double-contour geodesic shells with tetrahedral pyramids. To solve these problems, the duality principle was used, in addition to methods of analytic geometry and programming in language GDL embedded in ArchiCAD. As a result, the ArchiCAD parametric object of a double-contour geodesic shell with tetrahedral pyramids has been developed. This program can be used in the architectural and structural design of unique buildings.

Keywords: geodesic domes, double-contour geodesic sells, a duality principle, ArchiCAD parametric object, programming language GDL.

INTRODUCTION

Domes are self-supporting spatial structures without additional support. Geodesic domes are a class of domes that, when broken down into their elements, use geodesic lines.

Double-contour geodesic domes, in addition to the basic lamellar contour, have a rod contour that provides the durability and stability of the structure.

The configuration of the second contour can repeat the configuration of the first contour [1] or it can differ from it. In the latter case, the second contour can be based on heuristic approaches or the principle of the duality of polyhedrons/networks [2].

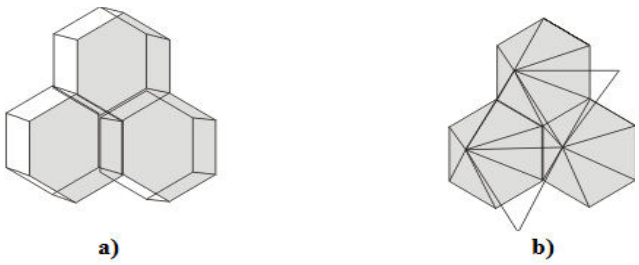

Figure-1. Configurations of the second contour: identical configuration with racks a) different configuration b).

The duality principle has been widely applied in technical appendices. In the field of the theory of polyhedron duality in Platonic solids, there is the developed DPM mechanism of the creation of a dual body [3]. It is established that each polyhedron P has a dual polyhedron P*, which means that P* is similar to P. The algorithm of the construction of dual convex polyhedrons [4] is developed. If to consider non-convex polyhedrons, the dual polyhedron turns out also not to be convex and it has self-intersecting faces. For this case, it expands the concept of a polyhedron duality, having admitted there to be self-intersecting faces.

In the mechanics area, the topological optimisation of truss is considered by the means of defining a subset of rods on the given discrete lattice. As the criterion functions, either the minimum compliance of the structure or the minimum volume of the structure is used. It is shown that these formulations of the optimisation problem are dual - solutions to both criteria will be equivalent for both problems [5].

In the field of geometry and topology, duality in non-polyhedral bodies is considered. The duality principle extends to the bodies with flat faces - polyliners. These polyliners bind to the structure of alternating nodes[6]. The triangular grid for the purpose of surface representation is considered. The duality grid which edges turn out by the means of moving the double vertices along a gradient of a function of weight is being built. It is established that this approach applies to self-supporting structures [7]. The application of the Minkowski sum and the difference of the convex polyhedrons is considered. An exact and effective algorithm for the computing of the Minkowski difference for polyhedrons is developed. Its duality to the contributing vertices concept of the Minkowski operations is shown [8].An oriented polygonal surface is considered. It is offered for the purpose of the storage of this surface to use the new combinatorial data structure PDER (vertices and faces, which are dual with the vertices). This ensures that the surface is a 2-manifold and that it has the property of primary / dual efficiency [9].

In the field of nonlinear optimisation, the problem of the optimisation of elastic structures is considered. For this problem decision, the method of penal functions is applied. It is underlined that there is a possibility of numerical instability in this method. It has been offered to solve this problem using Lagrangian duality [10].

METHOD OF CONFIGURATION DETERMINING

For the determining of a configuration of the second contour of geodesic shells, first, it is offered to use a principle the duality of networks of the first and second contours. Second, it is offered to use a principle of uniformity (equal strength) in the network of the second contour. Thirdly, it is offered to use a principle of the connectivity of a network in the second contour.

Accordingly, the principle of duality a networks assumes there to be conformity between the elements of two networks. For example, the face of one network corresponds to the vertex of another. In this case, we will put in conformity a top of a tetrahedral pyramid of the first contour and a vertex of the second contour.

The principle of uniformity of the second contour assumes the same value of the degrees of the graph nodes that correspond to the same number of arcs converging in the node. In this case, this will ensure that there is no excessive network concentration in some of the nodes.

The principle of connectivity of the second contour will provide equal strength in the second contour of the shell. Consequently, in order to implement this principle, it is necessary to solve the problem of ensuring connectivity for the rods located across the borders of the Moebius triangle.

Stage 1. The initial parametric object is a single-contour geodesic shell of class I1; 4 (see Figure-2, a)

The task is not to use the initial plates, but instead to form pyramids. The shell surface consists mainly of flat quadrangular plates. It is necessary to find the coordinates of the centre of the quadrangle - P1 as an average value of corresponding coordinate (X, Y, Z) of all of the vertices of a quadrangle.

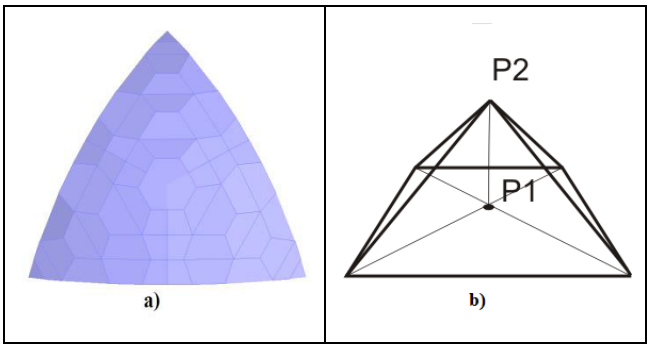

Figure-2. The scheme of the formation of parametric object I1; P4: initial object I1; 4, the scheme of formation of a pyramid for object I1; P4 (where the following notation is used: I - icosahedron, 1 - single-contour shell, 4 - quadrangular plates, P4 - tetrahedral pyramids).

Next, we lift the point P1 perpendicularly to this quadrangle to the height of the pyramid h. To perform the lifting of point P1, we use the transition formulas from the spherical coordinate system to the Cartesian and back. The origin of this system of coordinates corresponds to the sphere centre. We transform the Cartesian coordinates of P1 into a spherical system.Next, we move the P1 along the ray connecting the centre of the sphere and P1 on the height of a pyramid in the spherical coordinate system. We get point P2. Next, we perform the inverse transformation on the Cartesian coordinate system.

$$X = r * \sin(\theta) * \cos(\varphi)$$
$$Y = r * \sin(\theta) * \sin(\varphi)$$
$$Z = r * \cos(\theta)$$
$$r = \sqrt{X^2 + Y^2 + Z^2}$$
$$Fi = Arctg\frac{Y}{X}$$
$$Et = Arctg\left(\frac{\sqrt{X^2 + Y^2}}{Z}\right)$$

Where X, Y, Z - the Cartesian coordinates, *r, Fi, Et* - spherical coordinates (*r* - sphere radius, *Fi* - a longitude, *Et* - polar distance).

Stage 2. Next it is necessary to generate the second contour connecting part of the tops of the pyramids being guided by the formulated principles. The configuration of the second contour is that which includes the rods settling down across the borders of a triangle of Moebius. Therefore it is necessary to solve the problem of the visualisation of rods of the second contour belonging to various triangles of Moebius.

The program of the parametric object has only one Moebius triangle. Other Moebius triangles are generated by the turns of this initial triangle. Therefore it is necessary to perform a visualisation only on the part of the rod that belongs to the initial Moebius triangle. Consequently, it is necessary to provide a connection for the parts of the rod belonging to various triangles. This is possible to reach using methods of analytic geometry [11].

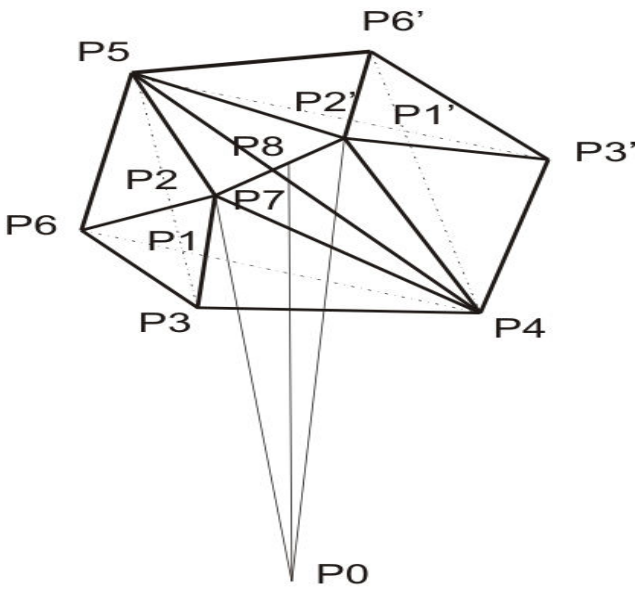

Figure-3. The scheme of the formation of a connected rod of the second contour (P2 and P2 - tops of the pyramids, P0 - the centre of the circumscribed sphere, P1 and P1' - the centres of quadrangles, P3, P4, P5, P6, P3', P6' - corners of quadrangles, P7 - projection P2 on P4-P5, P8 - projection P2 on P0-P7 (the middle of rod P2-P2')).

The problem of the calculation of the foot of the perpendicular drawn from a given point to a given line is known. Accordingly, we use our decisions related to this problem to develop an algorithm for ensuring the connectivity of the second contour (see Figure-3).

Algorithm. Calculating the centre of a rod.

Step 1. We will calculate the coordinates of the foot of the perpendicular drawn from point P2 to straight line P4-P5. Thus, we will get point P7.

Step 2. We will calculate the coordinates of the foot of the perpendicular drawn from point P2 to straight line P0-P7. Thus, we will get point P8. It will be the middle of rod P2-P2'.

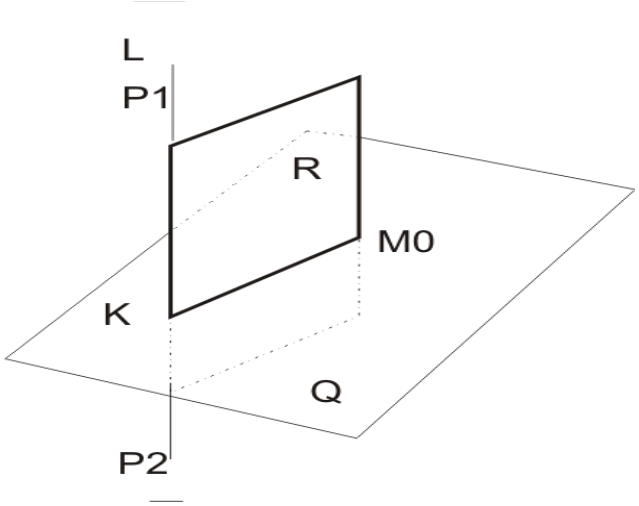

Figure-4. The scheme used for the calculation of the coordinates of the foot of the perpendicular (M0 - a point in space, L - a straight line, P1, P2 - the points on a straight line, K - the foot of the perpendicular).

Point M0 and straight line L is located in space. It is necessary to find the foot of the perpendicular drawn from this point to straight line L. Plane R passes through point M0 and line L. Plane Q passes through point M0 and point K, and this plane is perpendicular to line L. It is known that the coordinates of point K can be found by solving together a system of three equations representing line L(1), plane Q(2), and plane R(3).

It is used following the sub-algorithm at at each step of the main algorithm (see Figure-4).

Sub-algorithm. The calculation of the foot of the perpendicular.

Step 1. Calculating the coefficients of a linear equation system.

$$\text{Line L} \quad \frac{X-X_1}{l_1}=\frac{Y-Y_1}{m_1}=\frac{Z-Z_1}{n_1} \tag{1}$$

$$\text{Plane Q} \quad l_1(X-X_0)+m_1(Y-Y_0)+n_1(Z-Z_0)=0 \tag{2}$$

$$\text{Plane R} \begin{bmatrix} X-X_0 & Y-Y_0 & Z-Z_0 \\ X_1-X_0 & Y_1-Y_0 & Z_0-Z_1 \\ l_1 & m_1 & n \end{bmatrix} \tag{3}$$

Where $l_1=X_2-X_1$, $m_1=Y_2-Y_1$, $n_1=Z_2-Z_1$.

For example, after some of the transformations involved in the calculation of the foot of the perpendicular drawn from the given point P2 to the given straight line P4-P5, the linear equations system look like:

$$\text{Line L} \quad X\frac{1}{l_1}-Y\frac{1}{m_1}-Z\frac{1}{n_1}+\left(-X_5\frac{1}{l_1}+Y_5\frac{1}{m_1}+Z_5\frac{1}{n_1}\right)=0$$

$$\text{Plane Q} \quad l_1(X-X_2)+m_1(Y-Y_2)+n_1(Z-Z_2)=0$$

$$\text{Plane R} \quad XK_{31}-YK_{32}+ZK_{33}+\left(-X_2K_{31}+YK_{32}-Z_2K_{33}\right)=0$$

Where $l_1=X_4-X_5$, $m_1=Y_4-Y_5$, $n_1=Z_4-Z_5$,

$K_{31}=(Y_5-Y_2)n_1-(Z_5-Z_2)m_1$,

$K_{32}=(Y_5-Y_2)n_1-(Z_5-Z_2)l_1$,

$K_{33}=(Y_5-Y_2)m_1-(Z_5-Z_2)l_1$.

Step 2. The solution of the linear equation system.

$$\left.\begin{aligned} a_{11}x+a_{12}y+a_{13}z=b_1 \\ a_{21}x+a_{22}y+a_{23}z=b_2 \\ a_{31}x+a_{32}y+a_{33}z=b_3 \end{aligned}\right\} \tag{4}$$

To solve the system of linear equations (4), we used Cramer's rule (the calculation of the coordinates of point P7) [12].

Implementation in THE GDL language

In NNGASU, the system of the automated architectural designing and strength analysis of the geodesic domes and shells [13, 14] is developed. It includes a library of ArchiCAD parametric objects of the geometrical models of the geodesic shells in language GDL. This contains objects of various classes of the geodesic shells. In the library GeoDomeLib v.1.0, parametric objects of double-contour shells of classes I2; P3 with trihedral pyramids and I2 P6 with hexahedral pyramids have been presented. However, there was no program implementation of a library object of class I2; P4. Therefore there arose the task of developing ArchiCAD parametric objects of the geometrical models of geodesic double-contour shells with tetrahedral pyramids.

Stage 1. At first, we comment on the code fragments which drawing plates. For the computing of the coordinates of the centre of a quadrangle, we use the subroutine which computes the coordinates of the centre as an average value of the coordinates of all four vertices of a quadrangle. Next, we will perform a recalculation of the coordinates in the spherical system from Cartesian and back using subroutine 9500. At the input, the subroutine

receives the coordinates of the point laying in the plane of a quadrangle and at the output;it computes the coordinates of the point lifted on some of the given height.

Listing.1. The subroutine Computation of point P2.

```
9500:
IF XTMP=0 THEN XTMP=0.00001
FITMP=atn(YTMP/XTMP)
ETTMP=atn(sqr(XTMP^2+YTMP^2)/ZTMP)
RTMP=1+hhh/r
! offset on the radius
XTMP=RTMP*sin(ETTMP)*cos(FITMP)
YTMP=RTMP*sin(ETTMP)*sin(FITMP)
IF YTMP<0 THEN YTMP=1*abs(YTMP)
ZTMP=RTMP*cos(ETTMP)
RETURN
```

Then we form pyramid faces. To do this, we use the slab function of GDL, which receives the three vertices of the formed face as parameters. Since there are four faces in each pyramid, the slab function will be called upon four times.

Listing. 2. An example of the formation of a pyramid of the first contour.

```
!tria 6-300-10
! pyramid faces
slab 3,c,
r*x[6],r*y[6],r*z[6],
r*x[300],r*y[300],r*z[300],
r*x[10],r*y[10],r*z[10]
!tria 10-300-12
slab 3,c,
r*x[10],r*y[10],r*z[10],
r*x[300],r*y[300],r*z[300],
r*x[12],r*y[12],r*z[12]
!tria 12-300-7
slab 3,c,
r*x[12],r*y[12],r*z[12],
r*x[300],r*y[300],r*z[300],
r*x[7],r*y[7],r*z[7]
!tria 7-300-6
slab 3,c,
r*x[7],r*y[7],r*z[7],
r*x[300],r*y[300],r*z[300],
r*x[6],r*y[6],r*z[6]
```

Stage 2. After lifting all of the points and forming the faces of the pyramids, we transfer control to the program code fragment that generate the second contour. We will perform the formation of the second contour inside the Moebius triangle by adding rods connecting the tops of some of the tetrahedral pyramids. To do this, we use the lin _ function of GDL which receives two vertices of the formed line as parameters.

Listing.3. An example of the formation of the internal rods of the second contour.

```
lin_ r*x[335],r*y[335],r*z[335],
r*x[302],r*y[302],r*z[302]
lin_ -r*x[335],r*y[335],r*z[335],
r*x[302],r*y[302],r*z[302]
lin_ -r*x[335],r*y[335],r*z[335],
r*x[335],r*y[335],r*z[335]
lin_ r*x[342],r*y[342],r*z[342],
r*x[335],r*y[335],r*z[335]
lin_ r*x[336],r*y[336],r*z[336],
r*x[300],r*y[300],r*z[300]
lin_ r*x[304],r*y[304],r*z[304],
r*x[300],r*y[300],r*z[300]
lin_ -r*x[336],r*y[336],r*z[336],
-r*x[300],r*y[300],r*z[300]
lin_ -r*x[304],r*y[304],r*z[304],
-r*x[300],r*y[300],r*z[300]
lin_ r*x[302],r*y[302],r*z[302],
r*x[300],r*y[300],r*z[300]
lin_ r*x[302],r*y[302],r*z[302],
-r*x[300],r*y[300],r*z[300]
lin_ r*x[336],r*y[336],r*z[336],
r*x[335],r*y[335],r*z[335]
```

It is necessary to solve the problem of ensuring connectivity for the rods located at the boundary of the Moebius triangle.

The calculation of the foot of the perpendicular for step 1 of the main algorithm performed in subroutine 449700. The input data for the subroutine is the coordinates of points P4, P5, P2. The output is the coordinates of point P7.

Listing.4. Subroutine Point on the line.

```
449700:
!1 equation
LSTR1= XSTR4-XSTR1: MSTR1=YSTR4-YSTR1:
NSTR1=ZSTR4-ZSTR1
ASTR11=1/LSTR1: ASTR12=-1/MSTR1: ASTR13=-
1/NSTR1
BSTR1=-
XSTR1/LSTR1+YSTR1/MSTR1+ZSTR1/NSTR1
!2 equation
ASTR21=LSTR1: ASTR22=MSTR1: ASTR23=NSTR1
BSTR2=-XSTR0*LSTR1-YSTR0*MSTR1-
ZSTR0*NSTR1
!3 equation
ALFSTR31=(YSTR1-YSTR0)*NSTR1-(ZSTR1-
ZSTR0)*MSTR1
ALFSTR32=(XSTR1-XSTR0)*NSTR1-(ZSTR1-
ZSTR0)*LSTR1
ALFSTR33=(XSTR1-XSTR0)*MSTR1-(YSTR1-
YSTR0)*LSTR1
ASTR31=ALFSTR31: ASTR32=-ALFSTR32:
ASTR33=ALFSTR33
BSTR3=-
XSTR0*ALFSTR31+YSTR0*ALFSTR32+ZSTR0*ALFS
TR33
! Decision of linear equation system
GOSUB 449800
!output point3 XSTR3, YSTR3, ZSTR3
XSTR7=XSTR3: YSTR7=YSTR3: ZSTR7=ZSTR3

RETURN
```

The calculation of the foot of the perpendicular for step 2 of the main algorithm performed in subroutine 4410000. The input data for the subroutine is the coordinates of points P7, P0, P2. The output is the

coordinates of point P8.This subroutine (Middle of the external rod) is similar to the subroutine of the Point on the line.

The decision of the linear equation system from the three equations is performed in subroutine 449800. The input data for the subroutine is the coefficient values a11, a12, a13, a21, a22, a23, a31, a32, a33. The output is the roots X3, Y3, Z3.

Listing.5. Subroutine Decision of the linear equation system.

```
449800:
! determinants
DSTR1=ASTR11*(ASTR22*ASTR33-
ASTR23*ASTR32)
DSTR2=(ASTR12)*(ASTR21*ASTR33-
ASTR23*ASTR31)
DSTR3=(ASTR13)*(ASTR21*ASTR32-
ASTR22*ASTR31)
DSTR=DSTR1-DSTR2+DSTR3
DSTRX1=(-BSTR1)*(ASTR22*ASTR33-
ASTR23*ASTR32)
DSTRX2=ASTR12*(-BSTR2*ASTR33-ASTR23*(-
BSTR3))
DSTRX3=ASTR13*(-BSTR2*ASTR32-ASTR22*(-
BSTR3))
DSTRX=DSTRX1-DSTRX2+DSTRX3
DSTRY1=ASTR11*((-BSTR2)*ASTR33-ASTR23*(-
BSTR3))
DSTRY2=(-BSTR1)*(ASTR21*ASTR33-
ASTR23*ASTR31)
DSTRY3=ASTR13*(ASTR21*(-BSTR3)-(-
BSTR2)*ASTR31)
DSTRY=DSTRY1-DSTRY2+DSTRY3
DSTRZ1=ASTR11*(ASTR22*(-BSTR3)-(-
BSTR2)*ASTR32)
DSTRZ2=ASTR12*(ASTR21*(-BSTR3)-(-
BSTR2)*ASTR31)
DSTRZ3=(-BSTR1)*(ASTR21*ASTR32-
ASTR22*ASTR31)
DSTRZ=DSTRZ1-DSTRZ2+DSTRZ3
!coordinates
XSTR3=DSTRX/DSTR: YSTR3=DSTRY/DSTR:
ZSTR3=DSTRZ/DSTR
RETURN
```

As a result, first, the parametric object of single-contour geodesic shells of class I1; P4 (the program GDL_System_L1.gsm) is developed. With its help, it is possible to form geometrical models of single-contour geodesic shells with tetrahedral pyramids (without the second contour).

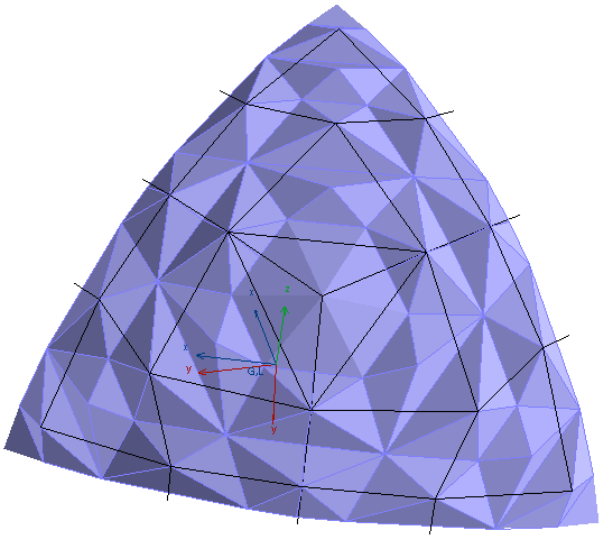

Figure-5. Geometric model of one Moebius triangle of a class I2;P4 geodesic shell.

Second, the parametric object of double-contour geodesic shells of class I2; P4 (the program GDL_ System_L2.gsm) is developed. With its help, it is possible to form geometrical models of double-contour geodesic shells with tetrahedral pyramids as the first contour and a rod network as the second contour. You thus get a geometric model corresponding to one Moebius triangle (see Figure-5.) or a shell in the form of a whole sphere (see Figure-6.).

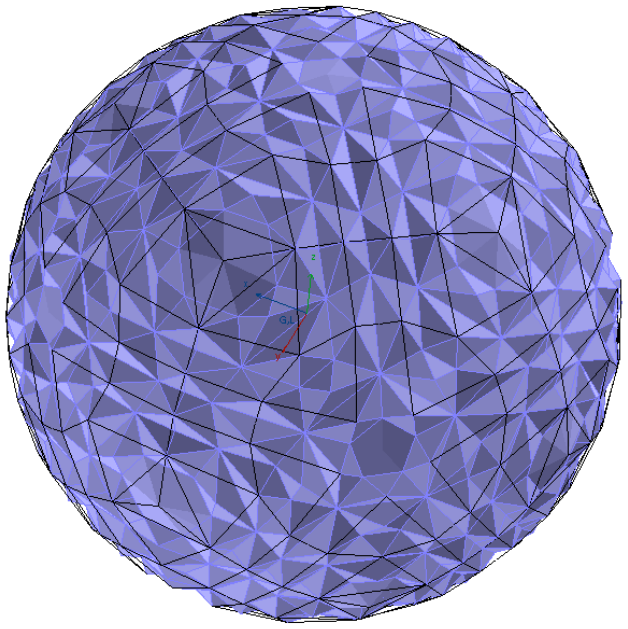

Figure-6. Geometric model of a class I2;P4 geodesic shell in the form of a sphere.

CONCLUSIONS

This study has investigated the research on the application of the duality principle in technical applications. It has been found that this principle can be used for the formation of double-contour geodesic shells. Programs for the geometrical modelling of double-contour geodesic shells have been considered. It is found, that there is no software implementation of geodesic double-contour shells with tetrahedral pyramids. As a result, a ArchiCAD parametric object of a dopuble-contour geodesic shell with tetrahedral pyramids was developed. This program can be used in the architectural and structural design of unique buildings.

REFERENCES

[1] Travush V.I., Antoshkin V.D., Erofeyeva I.V., Gudozhnikov S.S. 2014. Issledovanie konstruktivno-tekhnologicheskih vozmozhnostej sbornyh sfericheskih obolochek [Investigation of structural and technological capabilities of prefabricated spherical shells] Regional'naya arkhitektura i stroitel'stvo [Regional Architecture and Engineering]. - Penza, PGUAS, N 2, - pp.89-101.

[2] Pavlov G.N. 2007. Avtomatizatsiya arkhitekturnogo proyektirovaniya geodezicheskikh kupolov i obolochek [Automation of architectural design of geodesic domes and shells]. dis. ... d-ra tekhn. nauk [thesis]: 05.13.12. N. Novgorod. p. 245.

[3] G. Wei, .J. S. Dai 2012. Duality of the Platonic Polyhedrons and Isomorphism of the Regular Deployable Polyhedral Mechanisms (DPMs). J. S. Dai et al. (eds.) Advances in Reconfigurable Mechanisms and Robots I, DOI: 10.1007/978-1-4471-4141-9_68, Springer-Verlag London. pp. 759-771.

[4] B. Grunbaum, G.C. Shephard 2013. Duality of Polyhedra. M. Senechal (ed.), Shaping Space, DOI 10.1007/978-0-387-92714-5, 15, Springer. pp. 211-216.

[5] W. Achtziger. 2006. Simultaneous optimization of truss topology and geometry revisited Martin P. Bendsoe *et al*. (eds), IUTAM Symposium on Topological Design Optimization of Structures, Machines and Materials: Status and Perspectives, Springer. pp. 413-423.

[6] E. Wohlleben 2018: Duality in Non-polyhedral Bodies Part I: Polyliner. International Conference on Geometry and Graphics ICGG ICGG 2018 - Proceedings of the 18th International Conference on Geometry and Graphics. pp. 484-499.

[7] M. Desbrun, F. de Goes.2014. The Power of Orthogonal Duals. K. Anjyo (ed.), Mathematical Progress in Expressive Image Synthesis I, 3Mathematics for Industry 4, DOI: 10.1007/978-4-431-55007-5_1, Springer Japan. pp.3-6.

[8] H. Barki, F. Dupont, F. Denis, K. Benmahammed, H. Benhabiles 2013. Contributing Vertices-based Minkowski Difference (CVMD) of polyhedra and applications.3D Res. 04, 04(2013)1, 3D Research Center, Kwangwoon University and Springer. pp. 1-16.

[9] Yong-jin Liu, Kai Tang, Ajay Joenja.2006. A new representation of orientable 2-manifold polygonal surfaces for geometric modeling. Journal of Zhejiang University SCIENCE A. 7(9): 1578-1588.

[10] P.W. Christensen, A. Klarbring 2009. An Introduction to Structural Optimization,Chapter 9 Topology Optimization of Distributed Parameter Systems. Springer Science + Business Media B.V. pp. 179-201.

[11] Vygodskij M.Ya.1977. Spravochnik po vysshej matematike [Handbook of higher mathematics]. M.: Nauka [Science]. p. 871.

[12] Mal'cev A.I. 1975. Osnovy linejnoj algebry [Basics of linear algebra]. - Nauka [Science]. p. 400.

[13] Lakhov A.Ya. 2016. Double-contour geodesic domes with an internal second contour. ARPN Journal of Engineering and Applied Sciences. 11(13): 8150-8154.

[14] Lakhov A.Ya. 2019. Single-contour geodesic shells with quadrangle plates. ARPN Journal of Engineering and Applied Sciences. 14(8): 1578-1583.