



# PRESSURE AND PRESSURE DERIVATIVE INTERPRETATION FOR VERTICAL WELLS IN NATURALLY FRACTURED AND COMPRESSIBLE FORMATIONS

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## ABSTRACT

An improved methodology for well test interpretation homogeneous and naturally fractured and compressible reservoirs with a single fluid is presented. New expression to find the permeability, permeability modulus and geomechanical skin factor from slope during radial flow regime have been introduced and corrections of for the minimum point and the intercept between the unit-slope line taking place during the transition period and the radial flow regime are given so existing equations in the literature can be applied for the determination of the naturally fractured reservoir parameters. These new expressions were successfully applied to synthetic examples.

**Keywords:** stress sensitive formations, permeability modulus, radial flow regime.

## 1. INTRODUCTION

In compressible formations their permeability may depend upon formation effective stress. This effect cannot be negligible in unconsolidated formations since the permeability is a direct function of the pore pressure and can change during the reservoir productive life, Ostensen (1983).

From the well testing point of view a homogeneous variation of the permeability is accounted by the permeability modulus which was introduced by Pedrosa (1986) and defined by Equation (10). Since then, several researchers have been presented. Escobar, Urazan and Trujillo (2018) recent presented a methodology for well test pressure interpretation in stress sensitive formations drained by horizontal wells. They did a good review of existing literature. Escobar *et al* (1996) studied the effect of the poisson ratio and the young modulus on well pressure behavior. Duan *et al* (1998) performed a sensitive analysis by numerical simulation and lab tests to study the influence of stress on naturally fractured parameters. They concluded that the sensitivities are generated mainly by the network of fractures.

Escobar, Cantillo and Montealegre (2007) extended the *TDS* Technique, Tiab (1995), generating the pressure behavior using the model proposed by Celis *et al* (1994). They estimated the permeability using the pressure derivative slope during radial flow regime but the permeability is found by drawing a horizontal line throuout the falttest region. So they found and apparent permeability and depended on knowing a initial value. Since the pressure derivative minimum point taking place during the transition period is practically unaffected by stress sensitivity they do not provided corrections for such point.

In this work, the work presented by Escobar *et al* (2007) has been further improved. Permeability is estimated from the slope of the pressure derivative during radial flow regime. So is the permeability modulus. An equation for the total skin factor -geomechanical plus mechanical skin factors- is introduced. An advantage of

the *TDS* Technique is that in certain cases flow regimes can be artificially created without risking the interpretation. Then, from the permeability, the pressure derivative value for nonstress sensitive case is determined. This is represented by a horizontal line on the pressure derivative versus time log-log plot. Then, this value is used for the estimation of the mechanical skin factor; therefore, the geometrical skin factor is readily estimated by substrating the mechanical skin factor from the toal skin factor. Well developed correlations are provided for correcting the minimum point affected by the stress and the intercept between the unit slope and horizonatl radial flow regime lines. With that, the equations already presented by Engler and Tiab can be readily estimated. The expressions were successfully applied to simulated examples.

## 2. MATHEMATICAL BACKGROUND

Celis *et al* (1994) presented the mathematical model to account for the well pressure behavior of naturally fractured and stress sensitive reservoir:

$$P_D = -\frac{1}{\gamma_D} \ln \left[ 1 - \gamma_D I^{-1}(\bar{U}) \right] \quad (1)$$

where,

$$\bar{U} = \frac{K_0(u) + s u K_1(u)}{z \left[ u K_1(u) + z C_D K_0(u) + z s C_D u K_0(u) \right]} \quad (2)$$

$$u = \sqrt{z f(z)} \quad (3)$$

$$f(z) = \frac{\omega(1-\omega)z + \lambda}{(1-\omega)z + \lambda} \quad (4)$$

The dimensionless parameters used along this paper are given by:



$$P_D = \frac{k_{fb} h \Delta P}{141.2 q \mu B} \tag{5}$$

$$t_D * P_D' = \frac{k_{fb} h (t^* \Delta P')}{141.2 q \mu B} \tag{6}$$

$$t_D = \frac{0.0002637 k_{fb} t}{\phi \mu c_t r_w^2} \tag{7}$$

$$C_D = \frac{0.894 C}{\phi c_t h r_w^2} \tag{8}$$

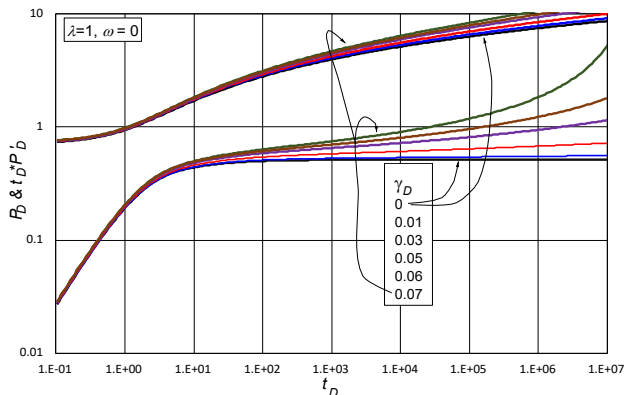
$$\gamma_D = \frac{141.2 q \mu B}{k_{fb} h} \gamma \tag{9}$$

being  $\gamma$  the permeability modulus introduced by Pedrosa (1986):

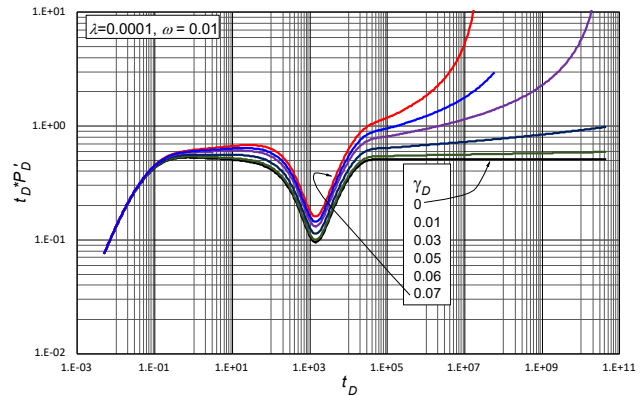
$$\gamma = \frac{1}{k} \frac{\partial k}{\partial P} \tag{10}$$

**2.1. Interpretation methodology**

Figure-1 presents the transient pressure behavior for the homogeneous reservoir case, that is, when  $\lambda=1$  and  $\omega = 0$ . As the dimensionless permeability modulus increases so does both pressure and pressure derivative. The pressure derivative deviates upward from the classic zero-slope line which is governed by:



**Figure-1.** Dimensionless pressure and pressure derivative versus time behavior for a homogeneous and stress sensitive reservoir.

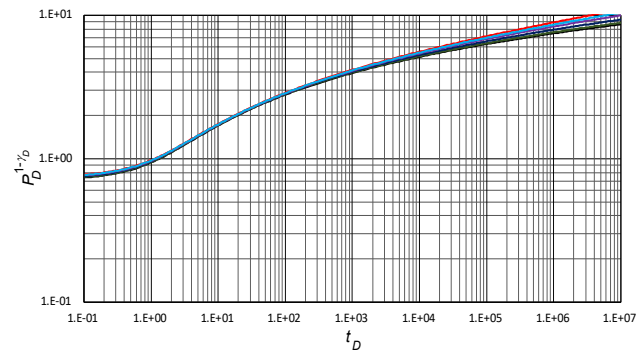


**Figure-2.** Dimensionless pressure derivative versus time behavior for a heterogeneous and stress sensitive reservoir when  $\lambda=0.0001$  and  $\omega=0.01$

$$t_D * P_D' = 0.5 \tag{11}$$

A similar behavior of the homogeneous reservoir occurs after the transition period of a naturally fractured reservoir as given in Figure-2. Both pressure and pressure derivative deviate upward from the classic homogenous behavior. There is a period which can be approached to a straight line of slope,  $m$ . The pressure derivative is then governed by:

$$\log(t_D * P_D')_{ss} = m \log(t_D)_{ss} + \log(0.5) \tag{12}$$



**Figure-3.** Unification of the dimensionless pressure during radial flow for a homogeneous and stress sensitive reservoir.

which becomes:

$$(t_D * P_D')_{ss} = 0.5 (t_D)_{ss}^m \tag{13}$$

Suffix  $ss$  stands for stress sensitive effects during radial flow regime,  $r$ . After replacing the dimensionless parameters into Equation (13) and solving for reservoir permeability it yields:

$$k_{fb} = \left[ \frac{70.6 q \mu B}{h (t^* \Delta P')_{ss}} \left( \frac{0.0002637 t_{ss}}{\phi \mu c_t r_w^2} \right)^m \right]^{\frac{1}{1-m}} \tag{14}$$



The above equation applies to the radial flow regime of a homogeneous reservoir and the second radial flow regime (after the "trough") of a naturally fractured reservoir. When  $m=0$ , non-stress sensitive case, Equation (14) will convert into:

$$k_{fb} = \frac{70.6q\mu B}{h(t^* \Delta P)_r} \quad (15)$$

Equation (15) was already developed by Tiab (1995). Tiab (1995) used the following pressure behavior during radial flow regime:

$$P_D = 0.5 \left\{ \ln \left( \frac{t_D}{C_D} \right)_r + 0.80907 + \ln [C_D e^{2s}] \right\} \quad (16)$$

Figure-3 shows a unified plot for homogeneous behavior. For information of the determination of unified behaviors, the reader is referred to Escobar, Bonilla and Hernandez (2018) and Escobar *et al.* (2018). In this plot can be seen that the dimensionless pressure to the power  $(1-g_D)$  are very close at early radial flow regime, then, Equation (16) is assumed for a stress sensitive reservoir to be:

$$P_D^{1-\gamma_D} = 0.5 \left\{ \ln \left( \frac{t_D}{C_D} \right)_r + 0.80907 + \ln [C_D e^{2s}] \right\} \quad (17)$$

Dividing Equation (17) by Equation (13) and solving for the geomechanics and mechanical skin factor,

$$s_{gm} = \frac{1}{2} \left[ \frac{\left( \frac{0.0002637 k_{fb} t_{ss}}{\phi \mu c_r r_w^2} \right)^m \Delta P_{ss}}{(t^* \Delta P')_{ss} \left( \frac{k_{fb} h \Delta P_{ss}}{141.2 q \mu B} \right)^{\gamma_D}} - \ln \left( \frac{k_{fb} t_{ss}}{\phi \mu c_r r_w^2} \right) + 7.43 \right] \quad (18)$$

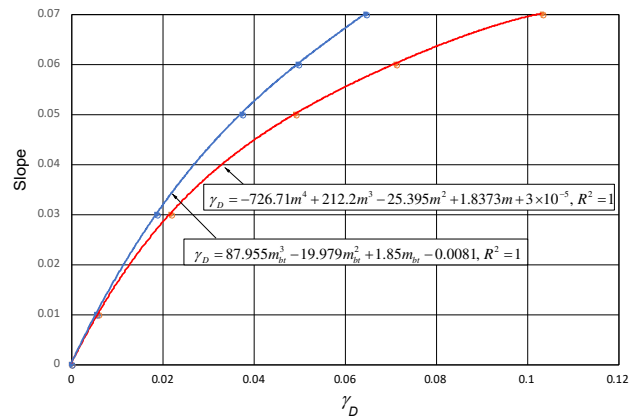
When  $m=0$  and  $\gamma_D = 0$ , then, the mechanical skin factor equation is obtained, Tiab (1995):

$$s_m = \frac{1}{2} \left[ \frac{\Delta P_r}{(t^* \Delta P')_r} - \ln \left( \frac{k_{fb} t_r}{\phi \mu c_r r_w^2} \right) + 7.43 \right] \quad (19)$$

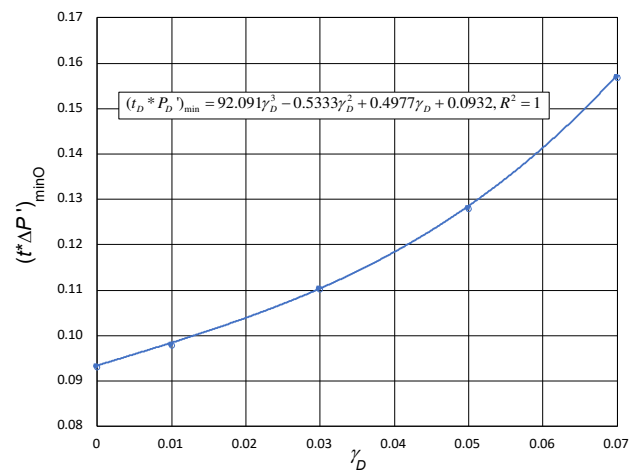
Since the pressure drop curve is also affected by stress, then, it is recommended to remove such effect by modifying Equation (19) as:

$$s_m = \frac{1}{2} \left[ \frac{\Delta P_{ss}^{1-\gamma_D}}{(t^* \Delta P')_r} - \ln \left( \frac{k_{fb} t_{ss}}{\phi \mu c_r r_w^2} \right) + 7.43 \right] \quad (20)$$

Equation (14), (15), (18) and (20) applies to a heterogenous reservoirs on the radial flow regime once the transition period has ceased.



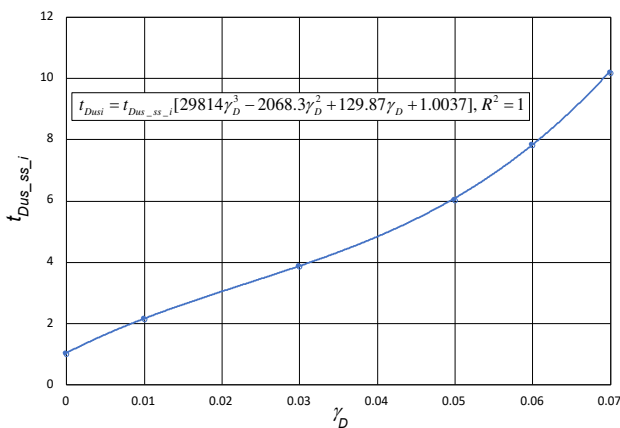
**Figure-4.** Correlation between the pressure derivative slope during radial flow regime and the dimensionless permeability modulus.



**Figure-5.** Correlation between the minimum point pressure derivative, at the "trough", and the dimensionless permeability modulus.

**2.2. Determination of the permeability modulus**

As observed in Figures -1 and -2, the pressure derivative is affected by the dimensionless permeability modulus. Although, at some time during radial flow regime the pressure derivative deviates upwards, a short straight line can be seen. The slope of such line can be correlated with the permeability modulus. The curve at the right of Figure-4 displays a perfect correlation between the slope and the minimum once the transition period vanishes. Then, a correlation is developed to obtain  $\gamma_D$  from the slope of the radial flow regime,



**Figure-6.** Correction factor for the unit-slope and radial flow intersection.

$$\gamma_D = -726.71m^4 + 212.2m^3 - 25.395m^2 + 1.8373m + 3 \times 10^{-5}, R^2 = 1 \quad (21)$$

The wellbore storage effects on pressure tests run in naturally fractured systems are normally much higher than those in homogenous reservoirs. This is because storage can also take place in the fractured network. In the hypothetical case that the radial flow regime before the “trough” is free of these effects another correlation -see left curve in Figure-4 is provided for the estimation of  $\gamma_D$  from such slope,  $m_{br}$ , such as:

$$\gamma_D = 87.955m_{br}^3 - 19.979m_{br}^2 + 1.85m_{br} - 0.0081, R^2 = 1 \quad (22)$$

### 2.3. Correction of the minimum pressure derivative and the intersect of the unit-slope line and the radial flow regime lines

As indicated by Engler and Tiab (1996) the minimum point during the pressure derivative is meant to be used for the estimation of both the interporosity flow parameter,  $\lambda$ , and the, especially, the dimensionless storativity coefficient,  $\omega$ . This minimum point is affected by the permeability modulus as indicated in Figure-2. Figure-5 displays a perfect correlation between these two key parameters. Then, the observed minimum point can be corrected from the following expression:

$$(t^* \Delta P')_{\min} = (t^* \Delta P')_{\min 0} [-16.836\gamma_D^2 - 4.6115\gamma_D + 0.995] \quad (23)$$

By the same token, the correction of the intersection formed between the unit-slope line occurring during the transition period and the radial flow regime line, Figure-6, allows finding the following fitting equation:

$$t_{usi} = t_{us_{ss_i}} [29814\gamma_D^3 - 2068.3\gamma_D^2 + 129.87\gamma_D + 1.0037] \quad (24)$$

As indicated by both Engler and Tiab (1996) and Tiab, Igbokeoyi and Restrepo (2007) the minimum point may be affected by wellbore storage. In such cases both

provided criteria for its quantification and their way of correction.

### 2.4. Determination of naturally fractured reservoir parameters

Once the minimum point is corrected, the naturally fractured reservoir parameters can be obtained from the expressions developed by Engler and Tiab (1996), Tiab *et al* (2007) and Tiab and Escobat (2003) presented in appendix A.

### 3. STEP-BY-STEP PROCEDURE

- Step 1.** Once the pressure and pressure derivative versus time log-log plot is obtained and a naturally fractured and stress sensitive reservoir is identified, find the radial flow regime and draw the most straight line on it. Determine the slope,  $m$ , of such line. Read also an arbitrary point on such line:  $t_{ss}$  and  $(t^* \Delta P')_{ss}$ . Read also the corresponding pressure drop,  $DP_{ss}$ , at the same arbitrary time.
- Step 2.** In the remote case that wellbore storage allows developing the first radial flow regime (before the transition period), do as in step 1 on this line and find the slope,  $m_{br}$ ,
- Step 3.** Use Equations (20) and (21) to estimate the dimensionless permeability modulus. Find an average is given the case.
- Step 4.** Use Equation (14) to find the fracture-bulk permeability.
- Step 5.** Use Equation (15) to find the pressure derivative during radial flow regime,  $(t^* \Delta P')$ , free of stress effects. It is recommended to draw a horizontal line through it on the pressure and pressure derivative plot.
- Step 6.** Estimate the skin factors with Equations (18) and (20).
- Step 7.** Read the observed minimum pressure derivative,  $(t^* \Delta P')_{\min 0}$ , and corrected by stress effects using Equation (23). Also, correct this point if wellbore storage affects such point as indicated by Engler and Tiab (1996) or Tiab *et al* (2007).
- Step 8.** Read the intercept between the unit-slope line, if observed during the transition period, and the horizontal line,  $t_{us_{ss_i}}$ , drawn in step 5 and estimate the interporosity flow parameter using Equation (26).
- Step 9.** Read the maximum point pressure derivative during wellbore storage,  $t_x$  and  $(t^* \Delta P')_x$ , the end time of the first radial flow regime,  $t_{e1}$ , and the



start time of the second radial flow regime,  $t_{b2}$ , and estimate the naturally fracture reservoir parameters with the equations given in appendix A and the directions of Engler and Tiab (1996) and/or Tiab *et al* (2007) which are compiled by Escobar (2019).

4. EXAMPLES

Table-1. Reservoir and fluid data for examples.

Parameter	Example 1	Example 2
$r_w$ , ft	0.35	0.35
$h$ , ft	72	129
$\phi$ , %	15	6
$k_{fb}$ , md	120	315
$q$ , Bbl/D	300	110
$B$ , rb/STB	1.2	1.35
$c_t$ , 1/psi	$1 \times 10^{-6}$	$2 \times 10^{-5}$
$s_m$	0	0
$C$ , bbl/psi	$1 \times 10^{-7}$	0.0001
$\mu$ , cp	3.5	7
$\omega$	0.005	0.01
$\lambda$	$1 \times 10^{-5}$	$41 \times 10^{-6}$
$\gamma_D$	0.03	0.05

4.1. Synthetic example 1

Figure-7 contains pressure and pressure derivative versus time data for a synthetic example generated with data of the second column of table 1. It is required to characterize this pressure test.

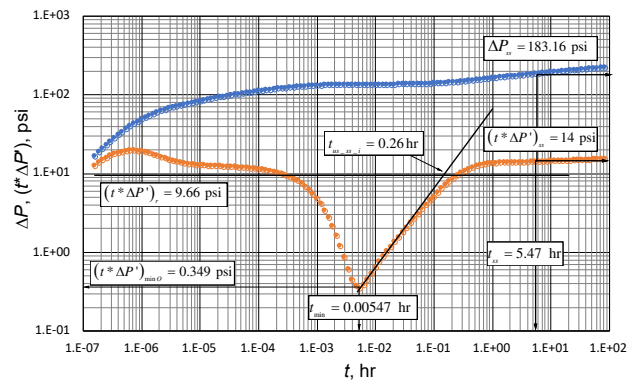


Figure-7. Pressure and pressure derivative log-log plot for synthetic example 1.

Solution

The following information is read from Figure-7.

$$\begin{aligned}
 t_F = t_{min} &= 0.00547 \text{ hr} & (t^* \Delta P')_{min0} &= 0.349 \text{ psi} \\
 t_{us\_t} &= 0.26 \text{ hr} & t_{ss} &= 5.47 \text{ hr} \\
 \Delta P_{ss} &= 183.16 \text{ psi} & (t^* \Delta P')_{ss} &= 14 \text{ psi}
 \end{aligned}$$

Additionally, the slope during radial flow regime was estimated to be 0.025. Use of Equation (21) allows finding the dimensionless permeability modulus.

$$\begin{aligned}
 \gamma_D &= -726.71(0.025)^4 + 212.2(0.025)^3 - 25.395(0.025)^2 \\
 &+ 1.8373(0.025) + 3 \times 10^{-5} = 0.034
 \end{aligned}$$

Use Equation (14) to determine reservoir permeability,

$$k_{fb} = \left[ \frac{70.6(300)(3.5)(1.2)}{(72)(14)} \right]^{\frac{1}{1-0.025}} \left( \frac{0.0002637(5.47)}{(0.15)(3.5)(1 \times 10^{-6})(0.35^2)} \right)^{0.025} = 128 \text{ md}$$

Use Equation (15) to find the pressure derivative during radial flow free of stress effects:

$$(t^* \Delta P')_r = \frac{70.6(300)(3.5)(1.2)}{(72)(128)} = 9.65 \text{ psi}$$

A horizontal line throughout this value is drawn on Figure-7.

Find the skin factors with Equations (20) and (18):

$$s_m = \frac{1}{2} \left[ \frac{183.16^{1-0.034}}{9.65} - \ln \left( \frac{128(5.47)}{(0.15)(3.5)(1 \times 10^{-6})(0.35^2)} \right) + 7.43 \right] = 0.1$$

$$s_{gm} = \frac{1}{2} \left[ \frac{\left( \frac{0.0002637(128)(5.47)}{(0.15)(3.5)(1 \times 10^{-6})(0.35^2)} \right)^{0.025} 183.16}{14 \left( \frac{128(72)(183.16)}{141.2(300)(0.75)(1.2)} \right)^{0.034}} - \ln \left( \frac{128(5.47)}{(0.15)(3.5)(1 \times 10^{-6})(0.35^2)} \right) + 7.43 \right] = 0.73$$

Then, the Geomechanical skin factor is found by the difference between the above. Then,  $s_g = s_{gm} - s_m = 0.73 - 0.1 = 0.63$ .

The minimum pressure derivative and the intersect between radial flow and unit-slope line are corrected with Equations (23) and (24), respectively.

$$\begin{aligned}
 (t^* \Delta P')_{min} &= 0.349[-16.836(0.034)^2 \\
 &- 4.6115(0.034) + 0.995] = 0.286 \text{ psi}
 \end{aligned}$$

$$\begin{aligned}
 t_{usi} &= 0.26[29814(0.034)^3 - 2068.3(0.034)^2 + \\
 &129.87(0.034) + 1.0037] = 1.092 \text{ hr}
 \end{aligned}$$

The dimensionless storativity coefficient is estimated with Equation (A.7)



$$\omega = 0.15866 \left\{ \frac{0.286}{9.65} \right\} + 0.54653 \left\{ \frac{0.286}{9.65} \right\}^2 = 0.0051$$

The interporosity flow parameter is estimated using Equations (A.1) and (A.4)

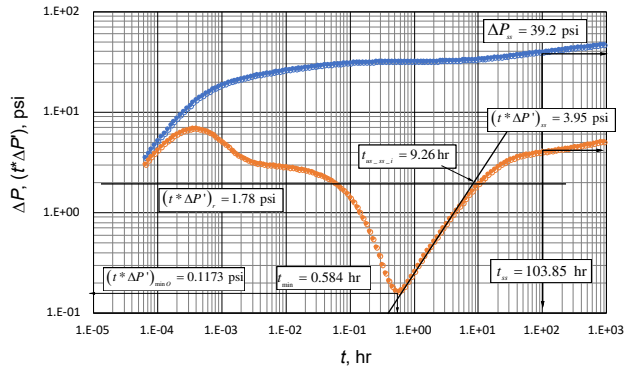


Figure-8. Pressure and pressure derivative log-log plot for synthetic example 2.

$$\lambda = \frac{3792(0.15)(1 \times 10^{-6})(3.5)(0.35)^2}{(128)(0.00547)}$$

$$\left[ 0.0051 \ln \left( \frac{1}{0.0051} \right) \right] = 9.5 \times 10^{-6}$$

$$\lambda = \left[ \frac{42.5(71)(0.15)(1 \times 10^{-6})(3.5)(0.35)^2}{(300)(1.2)} \right]$$

$$\frac{0.286}{0.00547} = 8.2 \times 10^{-6}$$

**Synthetic example 2**

Figure-8 contains pressure and pressure derivative versus time data for a synthetic example generated with data of the third column of Table-1. It is required to characterize this pressure test.

**Solution.**

The following information is read from Figure-8.

$$t_F = t_{min} = 0.584 \text{ hr} \quad (t^* \Delta P')_{min0} = 0.1623 \text{ psi}$$

$$t_{us\_ss\_i} = 0.26 \text{ hr} \quad t_{ss} = 103.85 \text{ hr}$$

$$\Delta P_{ss} = 39.2 \text{ psi} \quad (t^* \Delta P')_{ss} = 3.95 \text{ psi}$$

A procedure like example 1 was performed. Results are reported in Table-2 along with some other data.

Table-2. Results and comparison.

Parameter	Equation used	Input Example 1	Input Example 2	Obtained Example 1	Obtained Example 2	Abs. error Example 1	Abs. error Example 2
$k_{fb}$ , md	14	120	315	128	319.8	6.7	1.52
$\omega$	A.7	0.005	0.01	0.0051	0.012	2	20
$\lambda$	A.1	$1 \times 10^{-5}$	$1 \times 10^{-6}$	$9.5 \times 10^{-6}$	$1.09 \times 10^{-6}$	5	9
	A.4			$8.2 \times 10^{-6}$	$1.17 \times 10^{-6}$	18	17
$\gamma_D$	21	0.03	0.05	0.034	0.05	13	0
$s_m$	20	0	0	0.1	0.79	-	-
$s_{gm}$	18	-	-	0.73	1.44	-	-
$s_g$	-	-	-	0.63	0.65	-	-

**5. COMMENTS ON THE RESULTS**

Notice that more values can be estimated in both exercises, but it is enough for practical purposes. Also, in both examples the ratio between the maximum time during wellbore storage and the minimum point during the transition period is higher than 10 and, according to Engler and Tiab (1996) there is no need of correction due to wellbore storage effects.

Although the estimation of naturally fractured reservoir parameters accepts one order of magnitude, Table-2 reports a good agreement between the simulated and the estimated parameters.

The input mechanical skin factors were taken as zero. The estimated values are close to these values. No

errors were estimated for this case since this parameter allows a unit of difference.

**6. CONCLUSIONS**

- New expressions for the determination of the reservoir permeability and total skin factor (geomechanical and geometric) using a point during the inclined radial flow regime are successfully developed.
- Both dimensionless permeability modulus and permeability depends upon the estimation of the slope of the best straight line found during radial flow regime.





- c) A procedure for the determination of the geomechanical and mechanical skin factor is outlined.
- d) Correction ways for the pressure derivative minimum point and the intercept formed by transition unit slope and the radial flow regime under normal conditions are presented. This guarantees the application of existing equations for characterization of the naturally fractured reservoir parameters.

$B$	Oil volume factor, bbl/STB
$b$	Fraction of penetration/completion
$c$	Compressibility, 1/psia
$C$	Wellbore storage coefficient, bbl/psia
$D1$	Minimum point correction parameter
$D2$	Minimum point correction parameter
$c_t$	Total or system compressibility, 1/psia
$f(z)$	Matrix-fracture transfer function
$h$	Formation thickness, ft
$k$	Permeability, md
$k_{fb}$	Bulk fractured network permeability, md
$K_0$	First class modified Bessel function, zero order
$K_1$	Second class modified Bessel function, one order
$P$	Pressure, psia
$P_D'$	Dimensionless pressure derivative
$P_D$	Dimensionless pressure
$P_i$	Initial reservoir pressure, psia
$q$	Liquid flow rate, BPD
$r$	Radius, ft
$r_w$	Well radius, ft
$s_g$	Geomechanical skin factor
$s_m$	Mechanical skin factor
$s_{gm}$	Geomechanical plus mechanical skin factor
$t$	Time, hr
$t_p$	Production (Horner) time before shutting-in a well, hr
$t_D$	Dimensionless time based on well radius
$t^*\Delta P'$	Pressure derivative, psia
$z$	Laplace parameter

### Greek

$\Delta$	Change, drop
$\phi$	Porosity, fraction
$\gamma$	Permeability modulus, $\text{psi}^{-1}$
$\lambda$	Interporosity flow coefficient
$\mu$	Viscosity, cp
$\omega$	Dimensionless storativity coefficient

### Suffices

$b2$	Start of second radial flow regime
$D$	Dimensionless
$e1$	End of first radial flow regime
$F$	Inflection point
$fb$	Fracture network or fracture bulk
$i$	Intersection or initial conditions
$m$	Matrix or slope
$max$	Maximum point
$min$	Minimum point
$minO$	Observed minimum point
$r$	radial flow
$r_1$	Radial flow before transition period
$r_2$	Radial flow after transition period
$ss$	Stress sensitive
$usi$	Intersect of the pressure derivative lines of the unit-slope line during the transition and nonstress sensitive radial flow regime pressure derivative
$us\_ss\_i$	Intersect of the pressure derivative lines of the unit-slope line during the transition and stress sensitive radial flow regime pressure derivative
$x$	Maximum point of the pressure derivative

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**APPENDIX - Expression for the Determination of naturally fractured reservoir parameter**

**Determination of λ,**

$$\lambda = \frac{3792(\phi c_i)_{f+m} \mu r_w^2}{k_{fb} t_F} \left[ \omega \ln \left( \frac{1}{\omega} \right) \right] \tag{A.1}$$

$$\lambda = \frac{\omega(1-\omega)}{50\beta t_{e1}} = \frac{\omega \ln(1/\omega)}{\beta t_{\min}} = \frac{1}{\beta t_{usi}} = \frac{5(1-\omega)}{\beta t_{b2}} \tag{A.2}$$

$$\beta = \frac{0.0002637k_{fb}}{(\phi c_i)_{f+m} \mu r_w^2} \tag{A.3}$$

$$\lambda = \left[ \frac{42.5h(\phi c_i)_{f+m} r_w^2}{qB} \right] \frac{(t^* \Delta P')_{\min}}{t_{\min}} \tag{A.4}$$

$$\lambda = \left( \frac{(\phi c_i)_{f+m} \mu r_w^2}{0.0002637k_{fb}} \right) \frac{1}{t_{usi}} \tag{A.5}$$

$$\lambda = \frac{1}{10C_D} \frac{(t^* \Delta P')_{\min}}{(t^* \Delta P')_x} \tag{A.6}$$

**Determination of ω,**

$$\omega = 0.15866 \left\{ \frac{(t^* \Delta P')_{\min}}{(t^* \Delta P')_r} \right\} + 0.54653 \left\{ \frac{(t^* \Delta P')_{\min}}{(t^* \Delta P')_r} \right\}^2 \tag{A.7}$$

$$\omega = \exp \left[ -\frac{1}{0.9232} \left( \frac{t_{\min}}{50t_{e1}} - 0.4386 \right) \right] \tag{A.8}$$

$$\omega = 0.19211 \left\{ \frac{5t_{\min}}{t_{b2}} \right\} + 0.80678 \left\{ \frac{5t_{\min}}{t_{b2}} \right\}^2 \tag{A.9}$$

$$\omega = \left( 2.9114 + 4.5104 \frac{(t^* \Delta P')_r}{(t^* \Delta P')_{\min}} - 6.5452 e^{0.7912 \frac{(t^* \Delta P')_r}{(t^* \Delta P')_{\min}}} \right)^{-1} \tag{A.10}$$

$$\omega = \left( 2.9114 + \frac{3.5688}{\lambda t_{D\min}} + \frac{6.5452}{\lambda t_{D\min}} \right)^{-1} \tag{A.11}$$

$$t_{D\min} = \frac{0.0002637kt_{\min}}{(\phi c_i)_{f+m} \mu r_w^2} \tag{A.12}$$

**Table-A.1.** Conditions for the minimum pressure derivative being affected by wellbore storage, after Tiab *et al* (2007).

λ	C <sub>D</sub>
10 <sup>-4</sup>	C <sub>D</sub> > 10
10 <sup>-5</sup>	C <sub>D</sub> > 100
10 <sup>-6</sup>	C <sub>D</sub> > 10 <sup>3</sup>
10 <sup>-7</sup>	C <sub>D</sub> > 10 <sup>4</sup>
10 <sup>-8</sup>	C <sub>D</sub> > 10 <sup>5</sup>

**Correction for wellbore storage effect of the minimum point:**

$$(t^* \Delta P')_{\min} = (t^* \Delta P')_r + \frac{(t^* \Delta P')_{\min 0} - (t^* \Delta P')_r [1 + 2D_1 D_2]}{1 + D_2 \left[ \ln \left( \frac{C}{(\phi c_i)_{f+m} h r_w^2} \right) + 2s - 0.8801 \right]} \tag{A.13}$$





where;

$$D_1 = \left[ \ln \left( \frac{qBt_{\min o}}{(t^* \Delta P')_r (\phi c_t)_{f+m} h r_w^2} \right) + 2s - 4.17 \right] \quad (\text{A.14})$$

and;

$$D_2 = \frac{48.02C}{qB} \left( \frac{(t^* \Delta P')_r}{t_{\min o}} \right) \quad (\text{A.15})$$

Equation A.1 was presented by Tiab and Escobar (2003). Equation A.2 through A.12 were presented by Engler and Tiab (1996) and the others by Tiab *et al* (2007). All those are compiled by Escobar (2019).