USE OF PROBABILISTIC METHODS FOR DESIGN OF CLT PANELS

Pavel Dobeš and Antonín Lokaj
VŠB – TU Ostrava, Faculty of Civil Engineering, Ludvíka Podéště, Ostrava - Poruba, Czech Republic
E-Mail: pavel.dobes1@vsb.cz

ABSTRACT

The paper deals with the use of probabilistic methods for determination of the stiffness and load-carrying capacity of CLT panels. CLT panels are large-format building components composed of cross-oriented solid timber layers. The panels were experimentally tested using a four-point bending test, where a load-deformation curve was recorded. The results of the experimental testing entered into the probabilistic calculation as random variables and they were then compared with an analytical calculation a values given in Eurocode 5.

Keywords: Timber, CLT panel, modulus of elasticity, probabilistic calculation, random variable.

INTRODUCTION

The main objective of the study is to use probabilistic methods for determination of stiffness and strength parameters of CLT panels based on data obtained from four-point bending test. The aim is to compare the measured data of stiffness and bending strength of real size specimens with theoretically calculated values according to valid European standard for design of timber structures [1]. A CLT panel composed of 5 layers (3 longitudinal and 2 transverse layers) with a total thickness of 160 mm was chosen for the testing and subsequent probabilistic calculation.

CLT PROPERTIES

Cross-laminated timber (CLT) is a large-format building component composed of cross-oriented solid wood layers. CLT panels are usually composed of 3 to 7 layers. These layers are glued together in all directions, and polyurethane adhesives are mostly used for gluing them. They are usually made of spruce, but they can also be made of pine or other coniferous wood. Wood is dried at moisture content of around 8% at production, which ensures high resistance to atmospheric influences and prevents cracking. With respect to the drying and gluing technology, they are characterized by their dimensional stability even in the event of significant changes in the moisture of the environment.

CLT panels are popular for their excellent strength and stiffness characteristics and also offer excellent fire resistance. Relatively high thermal accumulation is another important feature. Cross-laminated timber structures are naturally diffused open, and that's why they can regulate moisture in the interior.

Thanks to a wide range of positive properties, a wide range of uses are available, both indoors and outdoors. CLT panels currently find use in civil and residential buildings for the construction of walls, ceilings and roofs. Last but not least, they can be combined with other structural systems (e.g. masonry structures) [5], [6], [7].

DETERMINATION OF STIFFNESS IN BENDING OF CLT PANELS USING LABORATORY TESTS

The stiffness parameters (global modulus of elasticity in bending) of the CLT panels were determined according to base principles given in standards EN 408+A1 [2] and EN 16351 [3].

The minimum length of the test specimen is 19 times the height. The specimen is simply supported with a span equal to 18 times the height. One of the supports must allow movement in the longitudinal direction. The method of loading, test arrangement and support of the specimen is illustrated in Figure-1 and Figure-2.

Figure-1. Tested specimen subjected to a four-point bending test.

Global stiffness in bending

The load speed must be constant up to the maximum applied load corresponding to 40% of the estimated maximum load capacity. Deformation w is measured in the middle of the span and from the center of the tensional or compressional edge. This deformation must include all local pushing that can occur in supports and load points as well as deformation of the supports themselves. A load-deformation curve is recorded. The part of the graph between 10% and 40% of the estimated maximum load capacity is used for regression analysis, with a correlation coefficient of 0.99 and better being required [2], [11].

The global stiffness in bending is given by the formula:

$$E_{m,g} = \frac{3 \cdot a \cdot l^2 - 4 \cdot a^3}{2 \cdot b \cdot h^3 \cdot \left(2 \cdot \frac{w_2 - w_1}{F_2 - F_1} - \frac{6 \cdot a}{5 \cdot G \cdot b \cdot h}\right)} \quad (1)$$

Where is

$F_2 - F_1$ increment of load [N] on the regression line with a correlation coefficient of 0.99 and better;

$w_2 - w_1$ increment of deformation [mm] corresponding to$F_2 - F_1$;

G ishear modulus (value 650 N/mm^2);

a distance from the support to the location of loading [mm].

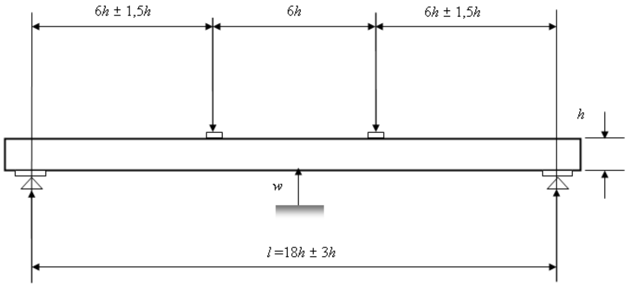

Figure-2. Test arrangement for measuring global modulus of elasticity in bending [2].

ANALYTICAL CALCULATION OF BENDING STIFFNESS ACCORDING TO EUROCODE 5

The calculation of bending stiffness (2) of cross-laminated timber is based on the theory of linear elasticity. It is only considered with layers in the direction of loading (longitudinal layers), the stiffness of the transverse layers is neglected in the calculation. Effective bending stiffness can be obtained on the basis of the formulas from Annex B of standard EN 1995-1-1 [1], which describes the procedure for stiffness calculation of mechanically connected beams. Steiner's theorem is applied in the calculation, considering a certain degree of compliance between the glued layers (coefficients of shear compliance γ).

$$(EI)_{ef} = \sum_{i=1}^{n} (E_i \cdot I_i + \gamma_i \cdot E_i \cdot A_i \cdot a_i^2) \quad (2)$$

Where is

E_i moduli of elasticity of the individual layers (determined according [4]) [N/mm^2];

I_i moments of inertia of the individual layers [mm^4];

A_i cross-sectional areas of the individual layers [mm^2];

a_i distances between the centre of mass of the individual layers and the centre of mass of the panel [mm].

However, the calculation of coefficients of shear compliance must be adjusted for glued beams according to the formulas given in [8] and [9].

$$\gamma_1 = \frac{1}{1 + \frac{\pi^2 \cdot E_1 \cdot A_1 \cdot \bar{h}_1}{L^2 \cdot G_R \cdot b}} \quad (3)$$

$$\gamma_2 = 1 \quad (4)$$

$$\gamma_3 = \frac{1}{1 + \frac{\pi^2 \cdot E_3 \cdot A_3 \cdot \bar{h}_2}{L^2 \cdot G_R \cdot b}} \quad (5)$$

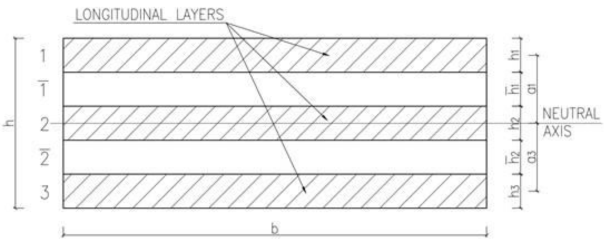

Figure-3. Designation of CLT panel layers for bending stiffness calculation.

EXPERIMENTAL DATA FOR THE PROBABILISTIC CALCULATION

CLT 160-5 consists of 5 layers - the outer layers (40 mm thick) and the middle layer are laid in the longitudinal direction, the intermediate layers (20 mm thick) are laid in the transverse direction. The geometrical specification of the panel and the test arrangement are shown in Figure-4.

Twenty-four specimens of the CLT 160-5 panel were tested. Table 1 shows the calculation of the global modulus of elasticity based on the results of the experiment according to the above mentioned formulas, as well as the bulk density, and measured maximum force, maximum bending moment and maximum deflection in the middle of the span caused by load corresponding to 40% of the estimated maximum force. Selected statistical characteristics of individual random variables (arithmetic mean, standard deviation, coefficient of variation, minimum and maximum value, range of values) are also given.

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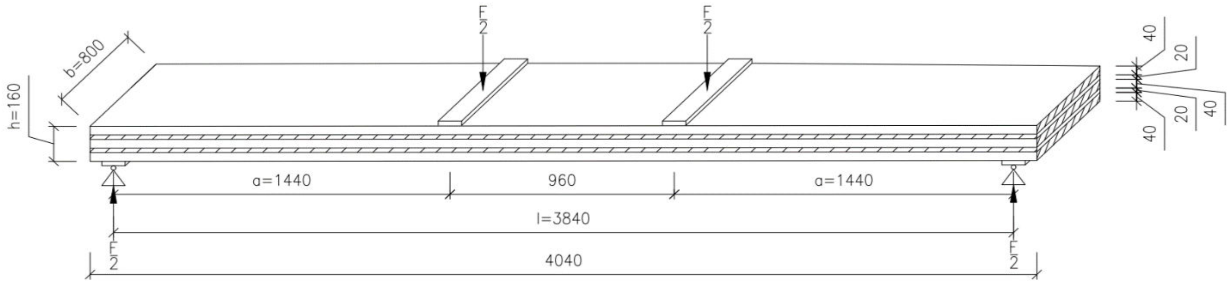

Figure-4. Geometrical specification and test arrangement of CLT 160-5 panel.

Table-1. Experimental data of CLT 160-5 used for the probabilistic calculation.

Specimen	$E_{m.g}$ [MPa]	ρ [kg/m³]	F_{MAX} [kN]	M_{MAX} [kNm]	w_{MAX} [mm]
	Global MOE	Bulk density	Max force	Max bending moment	Max deflection
1	9240	470	258.2	185.9	584.8
2	9524	469	243.8	175.5	619.3
3	9490	454	253.0	182.2	596.7
4	8988	471	235.9	169.8	639.9
5	9388	459	254.7	183.4	592.7
6	9314	456	254.3	183.1	593.6
7	9611	453	260.3	187.4	580.1
8	9128	452	246.8	177.7	611.8
9	9128	469	200.5	144.3	753.1
10	8536	465	251.4	181.0	600.6
11	8859	471	257.7	185.5	585.8
12	9691	468	263.0	189.4	574.0
13	8995	457	267.6	192.7	564.1
14	9682	462	235.8	169.7	640.3
15	9018	459	255.4	183.9	591.2
16	8942	457	245.1	176.4	616.0
17	9144	454	263.7	189.9	572.5
18	8881	459	238.5	171.7	632.9
19	8807	456	267.4	192.5	564.5
20	10176	460	267.6	192.7	564.1
21	10433	464	237.9	171.3	634.5
22	9917	462	232.8	167.6	648.4
23	9725	451	192.0	138.2	786.2
24	9880	457	204.3	147.1	738.9
AVG	9354	461	245.3	176.6	620.3
SD	458.7	6.2	20.4	14.71	58.62
COV	4.90%	1.36%	8.33%	8.33%	9.45%
Min	8536	451	192.0	138.2	564.1
Max	10433	471	267.6	192.7	786.2
Range	1897	20	75.6	54.4	222.1

Due to smaller number of specimens, for the purposes of probabilistic calculation, the measured values (random variables) were approximated by a suitable parametric distribution (the effort for the best fitting of the

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measured data with the parametric distribution). Selected parametric distributions and corresponding fitting coefficients are given in Table-2. One example for values of the bulk density is shown in Figure-5 (Raised-cosine distribution of 0.958 fitting coefficient) [12], [13].

Table-2. Parametric distributions used for measured data.

Random variable	Parametric distribution	Fitting coefficient
Bulk density	Raised-cosine	0.958
Global modulus of elasticity	Laplace	0.917
Maximum deflection	Laplace	0.926
Maximum force	Laplace	0.945

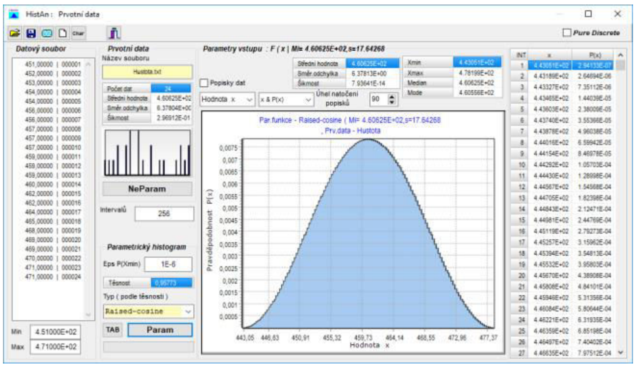

Figure-5. Parametric distribution used as approximation for measured values of bulk density.

Note: The median of the probability distribution obtained from the measured bulk density values approximately corresponds to the standard mean value for structural timber of the C30 strength class, ie $\rho_{mean} = 460\ kg/m^3$. Nevertheless, the C24 strength class is used in the analytical calculation according to Eurocode 5.

PROCEDURE OF PROBABILISTIC CALCULATION

Random variables of dimensions and cross-sectional characteristics

Technical report for experimental testing of CLT panels shows maximum permissible manufacturing tolerances of panel dimensions (layer thickness, panel width). The manufacturing tolerance for the layer thickness is ± 1 mm. The manufacturing tolerance for panel width is ± 5 mm. On the basis of these data, the mean, minimum and maximum dimensions of the panel and the respective cross-sectional characteristics were determined. The Gaussian normal distribution is used for the dimensions and cross-sectional characteristics in the probabilistic calculation, where the minimum and maximum values are to the left and right of the mean within 4 standard deviations. Parametric distribution of section modulus (in mm^3) is shown in Figure-6.

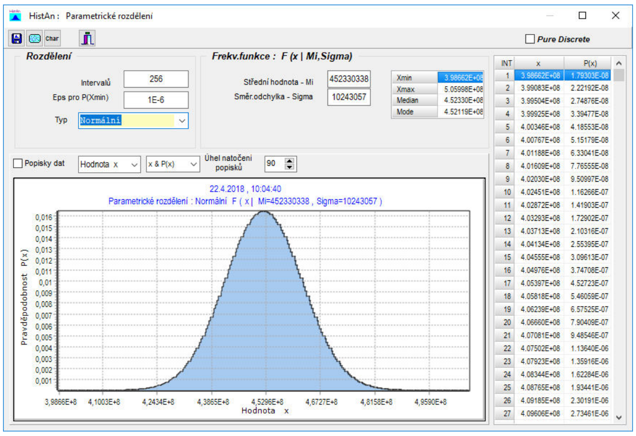

Figure-6. Parametric distribution (Gaussian) used for section modulus.

Load-carrying capacity in bending

The characteristic (standard) value of bending strength for structural timber of C24 strength class (determined according to [4]) is compared with the actual normal stress due to bending, which was determined by the experiment on real-size specimens. The load-carrying capacity was determined for short-term effects, without the influence of moisture and load duration (no modification factor for duration of load and moisture content). The free body diagram shown in Figure-4 applies for the calculation of the actual normal stress.

The characteristic (standard) strength values of timber materials are defined as the 5% quantile of the probability distribution. The reliability condition requires: The probability that the standard strength exceeds the stress (strength) determined by the experiment (ie failure probability) is less than 0.05. This assumption indicates a conservative (safe) design of the mentioned CLT panel for the ultimate limit state when using European standards for design of timber structures.

The characteristic value of bending strength for C24 according to Eurocode 5:

$$f_{m,k,EC5} = 24\ \text{MPa} \tag{6}$$

Actual normal stress due to bending based on the experiment data:

$$f_{m,k,Test(var)} = \frac{F_{max(var)}}{2} \cdot a \cdot \frac{1}{W_{y(var)}} \cdot 10^6 \tag{7}$$

Where is

$F_{max(var)}$ random variable of the maximum force [kN];
$W_{y,(var)}$ random variable of the section modulus [mm^3].

Reliability condition RF1:

$$f_{m,k,EC5} \le f_{m,k,Test(var)} \tag{8}$$

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Stiffness in bending

The effective bending stiffness according to the analytical formula in Eurocode 5 is compared with the actual bending stiffness, which was determined by the experiment on real-size specimens. The theoretical bending stiffness was determined using the mean value of the modulus of elasticity in tension and compression $E_{0,mean}$ for the individual layers of the panel. The values of actual bending stiffness are determined for loads corresponding to approximately 10% to 40% of the estimated maximum load-carrying capacity, where the linear elastic behavior of the material can be assumed and thus the formulas based on linear elasticity can be used.

The mean (standard) stiffness values of timber materials are defined as the 50% quantile of the probability distribution. The reliability condition requires: The probability that the bending stiffness according to the standard exceeds the bending stiffness determined by the experiment (ie failure probability) is less than 0.5. This assumption indicates a conservative (safe) design of the mentioned CLT panel for the serviceability limit state when using European standards for design of timber structures.

The effective bending stiffness for CLT panel made of C24 according to Eurocode 5 (see analytical calculation of bending stiffness mentioned above):

$$\mathrm{EI_{ef,EC5}} = 4704235511932\,\mathrm{Nmm^2} \qquad (9)$$

Actual bending stiffness based on the experiment data:

$$\mathrm{EI_{ef,Test(var)}} = \mathrm{E_{m,g(var)}} \cdot \frac{1}{12} \cdot \mathrm{b_{(var)}} \cdot \mathrm{h^3_{(var)}} \qquad (10)$$

Where is

- $E_{m,g(var)}$ random variable of the global modulus of elasticity in bending [N/mm^2];
- $b_{(var)}$ random variable of the panel width [mm];
- $h_{(var)}$ random variable of the panel height [mm].

Reliability condition RF2:

$$\mathrm{EI_{ef,EC5}} \leq \mathrm{EI_{ef,Test(var)}} \qquad (11)$$

RESULTS OF THE PROBABILISTIC CALCULATION AND THEIR DISCUSSION

The 5% quantile of the actual stresses based on the experiment is equal to 26.86 N/mm^2, which is more than the characteristic (standard) strength value for timber C24, ie 24 N/mm^2. This value represents 1.09% quantile for the actual stresses (see Figure-7). The failure probability is 1.09%, which is less than 5% (see Figure-8). It can be assumed that the analytical calculation of load-carrying capacity in bending according to the standard for design of timber structures EN 1995-1-1 is safe for the mentioned CLT panel and can be relied upon that in practical application.

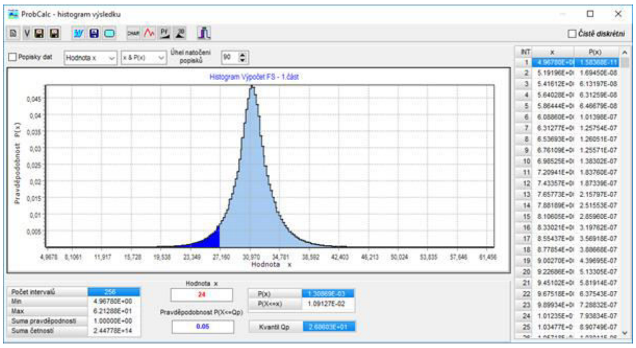

Figure-7. Histogram of actual stresses and 5% quantile.

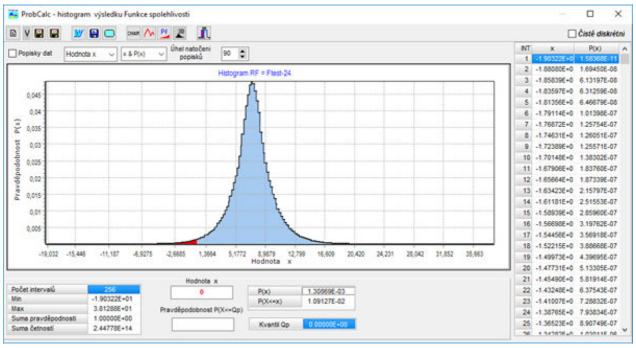

Figure-8. Histogram of reliability function RF1.

The 50% quantile (mean) of the actual global stiffness in bending based on the experiment is equal to 4.78872×10^{12} Nmm^2, which is more than the theoretical stiffness in bending for the mean values of the modulus of elasticity of the individual layers for timber C24, ie 4.70424×10^{12} Nmm^2. This value represents 34.85% quantile for the actual stiffnesses (see Figure-9). The failure probability is 34.85%, which is less than 50% (see Figure-10). It can be assumed that the analytical calculation of bending stiffness according to the standard for design of timber structures EN 1995-1-1 is safe for the mentioned CLT panel and can be relied upon that in practical application.

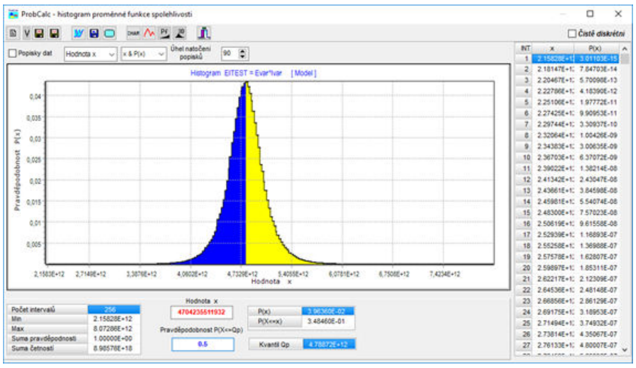

Figure-9. Histogram of actual stiffnesses and 50% quantile.

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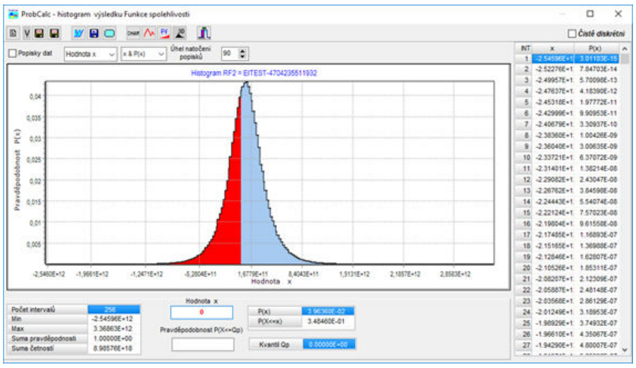

Figure-10. Histogram of reliability function RF2.

CONCLUSIONS

The paper outlines use of probabilistic methods for determination of stiffness and strength parameters of the CLT panels based on test data.

When calculating the load-carrying capacity and stiffness in bending according to Eurocode 5 in the practice, it is not possible to accurately determine strength class of timber of the real specimens and to implement their real material inhomogeneities and other uncertainties into the calculation, thus we can commit negligible inaccuracies that may not be on the safe side. This led to verification using probabilistic methods, where the calculations according to the standard were compared with the measured data, which entered into the probabilistic calculation as random variables. It has been verified that the standard procedures for design of timber structures according to Eurocode 5 are safe for the 160-5 CLT panel, both for the ultimate limit state and the serviceability limit state.

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