VOL. 15, NO. 8, APRIL 2020 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

PRESSURE DERIVATIVE ANALYSIS FOR RADIAL OR LINEAR GEOMETRIES COMPOSITE RESERVOIRS

Freddy H. Escobar, Francisco José Amado and Faiber Robayo
Universidad Surcolombiana/CENIGAA, Avenida Pastrana - Cra 1, Neiva, Huila, Colombia
E-Mail: fescobar@usco.edu.co

ABSTRACT

A new methodology for pressure-transient well test interpretation in both linear or radial geometry composite reservoirs is presented here. For this, expressions of the unified behavior and trend correlations from the maximum and minimum points of the pressure derivative curve observed during the transition period between the two regions that compose the reservoir are developed for the determination of diffusivity ratio and mobility. The results obtained in both cases with the developed equations and correlations provided excellent results which match very closely the input ones, and, therefore; with a very low deviation error as illustrated by the synthetic examples.

Keywords: composite reservoir, diffusivity ratio, mobility ratio, storativity ratio, radial discontinuity.

1. INTRODUCTION

Nowadays, the analysis of pressure tests is the cheapest way for reservoir characterization. However, almost all of its methods are isotropic methods, that suppose a homogeneous reservoir behavior with a given average porosity and permeability and most of the times with only one fluid saturating the porous media, with constant fluids property along the reservoir, which due to either the natural heterogeneity inside the reservoir or multiphase fluid flow in porous media, those assumptions of homogeneity in some cases can induce significant error during the interpretation of the pressure transient data.

For that reason, there is a well-known model for representing reservoirs with these kinds of heterogeneities that allows performing a better representation of the inside reservoir phenomena and its physical features and therefore a better interpretation of pressure transient data for its characterization. That model is called as composite reservoir and currently is generalized for both radial and linear geometry, thanks of the several studies made by numerous authors for improving the understanding and then extend of its applicability, which currently is very diverse already.

Amongst the most notorious studies we may name: first of all, Loucks and Guerrero (1961) and Carter (1966) that were the first scholars to propose and use the radial composite model in which the well is in the center of the reservoir and it is located at a distance R_i from the internal boundary. Second, the researches performed by Kuchuk and Habashy (1997) and Bourgeois et al (1996) that extended the composite model for a linear reservoir with infinite y finite extension respectively, in which the well is located at a distances L_i from the internal boundary. Also, other mayor investigations that can be highlighted here are: Levitan and Crawford (2002) who made a study of the pressure behavior in a linear infinite reservoir with a given mobility and storativity lateral profile, Brown (1985) investigated the behavior of the pressure response in composite reservoir with values of mobility and storativity ratio in the order of 0 to 4 and 0 to 30 respectively. Boussalem, Tiab and Escobar (2002) presented a new methodology for the analysis of the pressure behavior due the variations of de mobility ratio in a closed composite reservoir,

Additionally, several researches have been made to extent and improve the applicability of the composite reservoir model in the study of complex process during the exploitation of petroleum reservoirs such as enhanced oil recovery processes, acid stimulation or formation damage modelling. Some of the most relevant publications that can be point out are: Wattenbarger and Ramey (1970) used the radial composite model for better modelling of the wellbore damage (skin region) of a given well and obtained the pressure transient behavior for such system using finite differences, Satman *et al* (1980) presented an analytical solution for a two-zone infinitely large composite reservoir undergoing thermal recovery process. Furthermore, Ambastha and Ramey (1989) studied the use of the radial composite reservoir of two-regions for the interpretation well test data from reservoirs with ongoing thermal projects and closed or constant-pressure outer border, Onyekonwu (1985) and Barua and Horner (1987) presented an analytical solution using type-curve matching for a three-region composite reservoir for later applications to reservoirs undergoing EOR thermal procedures, and later Ambastha and Ramey (1992) published a study in which presented for the first time the pressure derivative behavior for three-region composite reservoir with ongoing thermal EOR method. Lately, Escobar, Martinez and Bonilla (2011) presented a methodology to analyze the pressure and pressure derivate behavior for different mobility and diffusivity ratios without using type-curve matching to analyze well test under thermal recovery, later the same authors Escobar, Martinez, Bonilla (2012a) y Escobar, Martinez, Bonilla (2012b), presented new methodologies, numerical the first one and analytical the other, for modelling through a radial composite reservoir of three-regions well tests data of wells under ongoing thermal recovery process.

In this work, the model presented by Kuchuck and Habashy (1997) is used in a commercial software to generate all the pressure and pressure derivative response data of the behavior of the variation of mobility and diffusivity ratios in both composite reservoir geometries

VOL. 15, NO. 8, APRIL 2020 ISSN 1819-6608
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with the purpose to proceed to develop all the proposed methodology of composite reservoir pressure transient analysis without using curve-type matching and based only on the characteristics points observed in the shape of the pressure derivative in the log-log plot. This model does not consider the effects generated due the coefficient of leakage of the leaky faults, for the case in which a leaky fault is presented in the composite transition we assumed always a fully leakness-fault behavior.

2. MATHEMATICAL MODEL

In 1997, Kuchuck and Habashy presented in their paper publication a new method or mathematical model for solve the diffusion equation for composite reservoir in which the properties of the rock, fluids or both change laterally either gradually or sharp generating the composite behavior.

This model is based on the concept of reflection-transmission of physics electromagnetics applied to solve problems of fluid flow in porous media of heterogeneous reservoir in 3D where the heterogeneity changes along of one axis.

As main contribution of the model, we can highlight the solution founded can have any number of composite regions and different boundary conditions in the *x*, *y* and *z* directions, that allow the method can be use in wide variety of composite systems, such as fractured reservoir, faulted reservoir, deltaic reservoir, amongst others. Additionally, the methodology works for whatever type of well like vertical, deviated and horizontal well, also, allows finding the wellbore storage and wellbore damage effects of a given well that are easily to incorporate to the solution presented here because they are given in the Laplace transform domain.

Applying the mentioned model to observe the influence of the variation of mobility and diffusivity ratios on pressure derivative's shape on log-log plot in both geometries for a two-regions infinite composite reservoir, the following behaviors were obtained. See Figures 1-4.

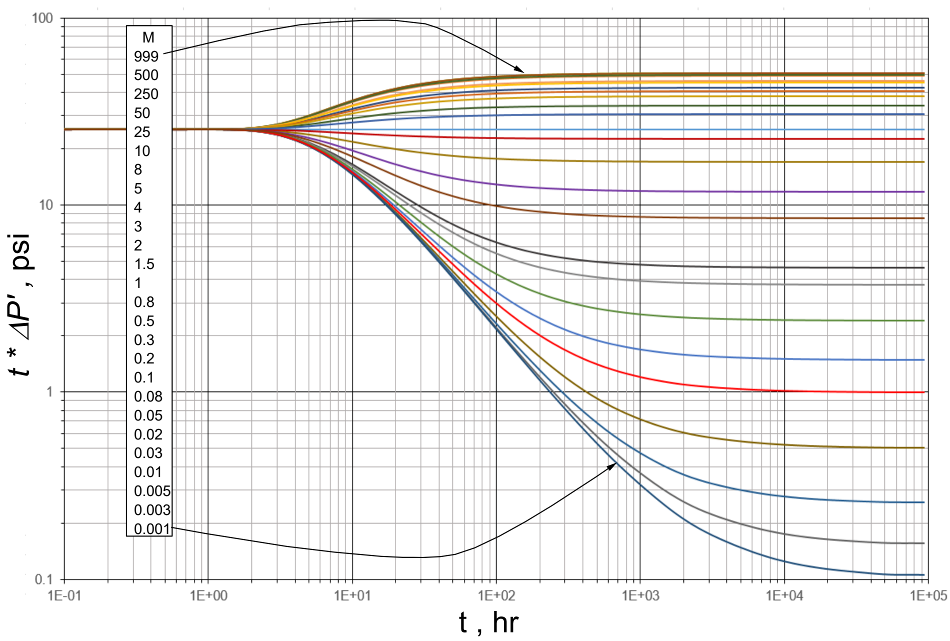

Figure-1. Behavior of pressure derivative in linear geometry varying the ratio *M* and leavingthe ratio D constant and equal to 1.

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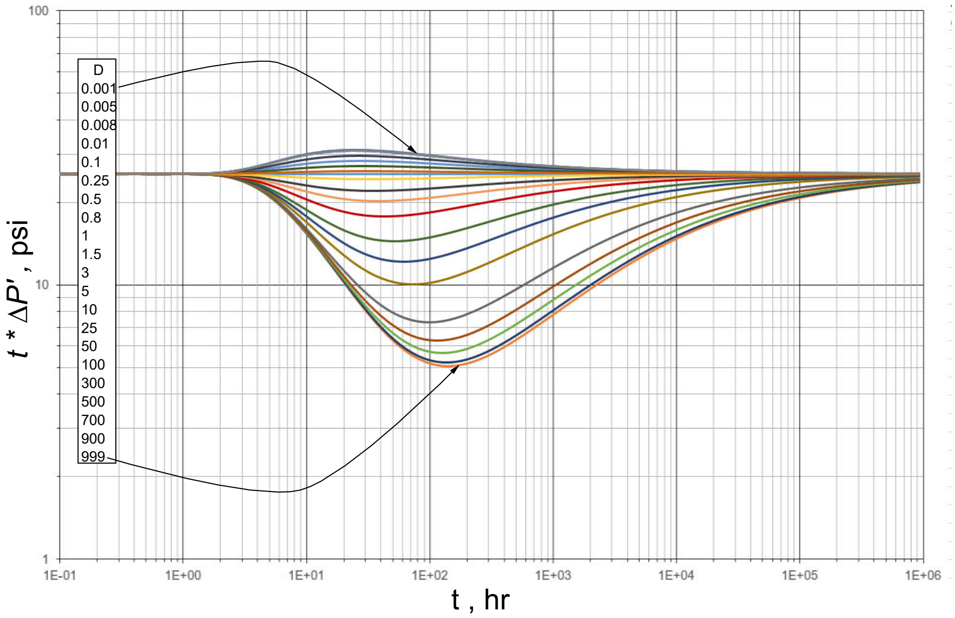

Figure-2. Behavior of pressure derivative in linear geometry varying the ratio *D* and leaving the ratio *M* constant and equal to 1.

In Figures 1 and -4, the variation of the mobility ratio is observed when the diffusivity ratio is constant and equal to 1 for the linear and radial geometries, respectively. It is observed that the variation of this parameter influences the position of the external radial flow with respect to the internal radial flow, that is, if the second radial flow is higher or lower with respect to the first, therefore, it can be concluded that the maximum or minimum point of the pressure derivative curve corresponds to the height of the external radial flow. Additionally, in Figures 2 and 3 the variation of the diffusivity ratio is observed when the mobility ratio is constant and equal to one for the linear and radial geometry, respectively. Observing that the variation of this parameter influences the shape of the transition zone between the two radial flows generating a hump upwards or downwards, therefore, it was identified that the maximum or minimum point is the highest point of the ridge or the lowest point of the hump valley, respectively.

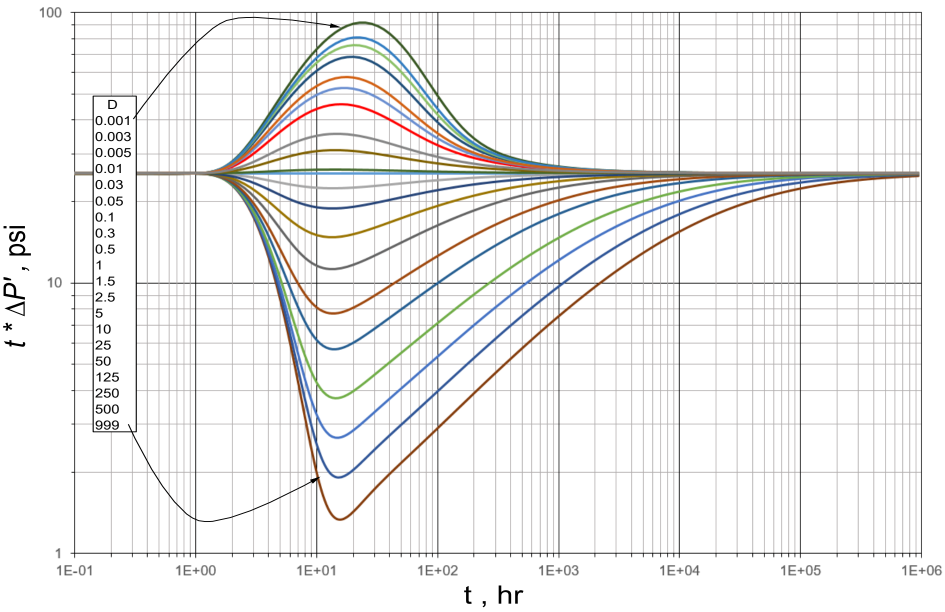

Figure-3. Behavior of pressure derivative in radial geometry varying the ratio *D* and leaving the ratio *M* constant and equal to 1.

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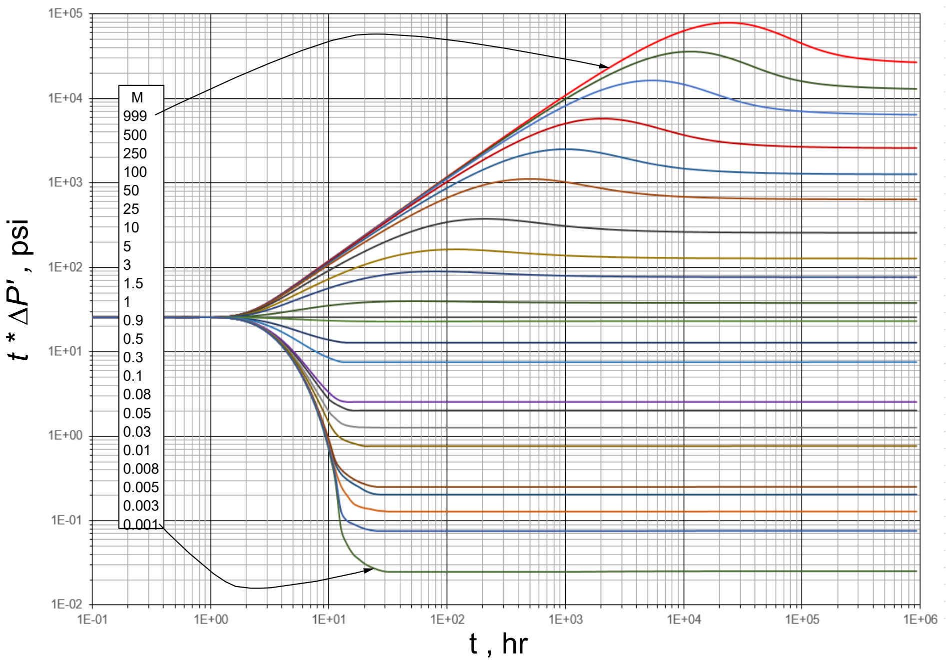

Figure-4. Behavior of pressure derivative in radial geometry varying the ratio *M* and leaving the ratio *D* constant and equal to 1.

3. INTERPRETATION METHODOLOGY

Tiab (1993) proposed the *TDS* technique for the interpretation of pressure tests and is based on the use of straight lines and other observed and found characteristics, such as maximum, minimum and inflection points, on the behavior of the pressure derivative curve in a log-log plot. This technique has proven throughout diverse investigations made by several authors to be very powerful, precise and practical for the interpretation of pressure-transient data. In this work, it is based on the use of this technique as a basis to develop the proposed methodology. To begin, let's define the following expressions which will be of considerable use later:

Dimensionless time:

$$t_D = \frac{0.0002637kt}{\phi\mu c_t r_w^2} \quad (1)$$

Dimensionless pressure and pressure derivative:

$$P_D = \frac{\bar{k}h\Delta P}{141.2q\mu B} \quad (2)$$

$$t_D * P_D' = \frac{\bar{k}h(t*\Delta P')}{141.2q\mu B} \quad (3)$$

Additionally, by convention the mobility and diffusivity ratio are defined as:

$$M = \frac{(k/\mu)_1}{(k/\mu)_2} \quad (4)$$

$$D = \frac{(k/\mu\phi c_t)_1}{(k/\mu\phi c_t)_2} \quad (5)$$

Where sub-index 1 represents region 1, which by convention is the region closest to the well, and therefore sub-index 2 represents region 2, which is the farthest from the well.

In order to extend the *TDS* technique, for the case of study, the procedure to generate equations that describe the behavior of the composite system is based on making manipulation in the values of the pressure derivative, pressure and time dimensionless, looking for this obtain the equations that represent the maximum and minimum, as the case may be, in order to calculate the value of the mobility ratio or the diffusivity ratio and thus obtain the value of mobility or diffusivity of the outer zone, and thereby, characterize the whole reservoir.

One of the bases of the *TDS* technique is to determine unified equations for each characteristic point in the case under study, maximum and minimum points. Therefore, it allows finding combinations of certain variables, such as the ratio *M* or *D*, given together with the derivative of pressure or time (or both) to find mathematical equations that allow the reservoir to be characterized. To do this, one must to determine the variables that affect the behavior of the pressure and find the quantized value of each parameter. A tedious way is finding the parameters using the trial-and-error procedure, which was not used here. Another simpler way, that was

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used in this work, was the one used by Escobar *et al* (2017).

Applying the procedure mentioned above for all cases of variation, the following unification equations were obtained for each case:

- For linear geometry:

$$D = \sqrt[0.04]{1.825 - (t_D * P_D')_{\min}^{0.3}}, \text{ for } D>1 \quad (6)$$

$$D = \sqrt[0.0065]{1.845 - (t_D * P_D')_{\max}^{0.28}}, \text{ for } D<1 \quad (7)$$

$$M = \sqrt[0.0058]{\frac{1}{1.9692 - (t_D * P_D')_{r\_exter}^{0.075}}}, \text{ for } M>1 \quad (8)$$

$$M = \sqrt[1.02]{\frac{(t_D * P_D')_{r\_exter}}{3.87 - P_{D_{r\_exter}}^{0.5}}}, \text{ for } M<1 \quad (9)$$

For radial geometry:

$$D = \sqrt[0.045]{1.875 - \left(t_D * P_D^{'}\right)_{\min}^{0.18}}, \text{ for } D>1 \quad (10)$$

$$D = \sqrt[0.05]{1.862 - \left(t_D * P_D^{'}\right)_{\max}^{0.22}}, \text{ for } D<1 \quad (11)$$

$$M = \sqrt[0.807]{1.78 \times (t_D * P_D')_{r\_exter}^{0.8}}, \text{ for } M>1 \quad (12)$$

$$M = \sqrt[0.05]{\frac{1}{1.95 - (t_D * P_D')_{r\_exter}^{0.065}}}, \text{ for } M<1 \quad (13)$$

In parallel to the previous equations generated by the unification procedure, correlations were generated based on the behavior of the maxima and minima according to the case of the same data generated to obtain the unification equations. To do this, the value of the derivative was read in all the maximums and minimums and correlated with the value of the M and D ratio according to the case, so with the help of statistical commercial software, we can find the mathematically simplest equation that meets the trend of the data. The correlations obtained through the procedure mentioned are shown below:

- For radial geometry:

$$D = e^{2.703487 - 5.5699131*\left(t_D * P_D^{'}\right)_{\max}}, \text{ for } D<1 \quad (14)$$

$$D = -2.8284572 + \frac{0.69740513}{\left(t_D * P_D^{'}\right)_{\min}^{2}}, \text{ for } D>1 \quad (15)$$

$$M = \frac{1}{8.1973755*10^{-5} + \frac{0.49967937}{\left(t_D * P_D^{'}\right)_{rexter}}}, \text{ for } M<1 \quad (16)$$

$$M = \frac{1}{8.1973761*10^{-5} + \frac{0.49967937}{\left(t_D * P_D^{'}\right)_{rexter}}}, \text{ for } M>1 \quad (17)$$

- For linear geometry:

$$D = \left(9.6536555 - 12.249197*\left(t_D * P_D^{'}\right)_{\max}^{0.5}\right)^2, \text{ for } D<1 \quad (18)$$

$$D = \left(-1.9532788 + \frac{1.050968}{(t_D * P_D)^{1.5}}\right)^2, \text{ for } D>1 \quad (19)$$

$$M = \frac{1}{-1.0051894 + \frac{1.0022772}{(t_D * P_D')_{rexter}}} \approx \frac{1}{-1 + \frac{1}{(t_D * P_D')_{rexter}}}, \text{ for } M<1 \quad (20)$$

$$M = \frac{1}{-1.0000372 + \frac{0.99986572}{\left(t_D * P_D^{'}\right)_{rexter}}} \approx \frac{1}{-1 + \frac{1}{\left(t_D * P_D^{'}\right)_{rexter}}}, \text{ for } M>1 \quad (21)$$

4. EXAMPLES

4.1 Example 1: Radial Composite Reservoir

A synthetic pressure test for a vertical well in a radial composite reservoir of infinite extension with a mobility contrast M = 0.8 and diffusivity contrast of D=1 was simulated. The following fluid and rock properties data are assumed:

q= 320 STBD	k= 60 md	r_w= 0.35 ft
μ= 8 cp	ϕ= 12 %	h= 40 ft
β= 1.2 rb/STB	c_t= 3x10^{-6} psi^{-1}	Ri= 350 ft
M= 0.8	D= 1	Pi= 5000 psi

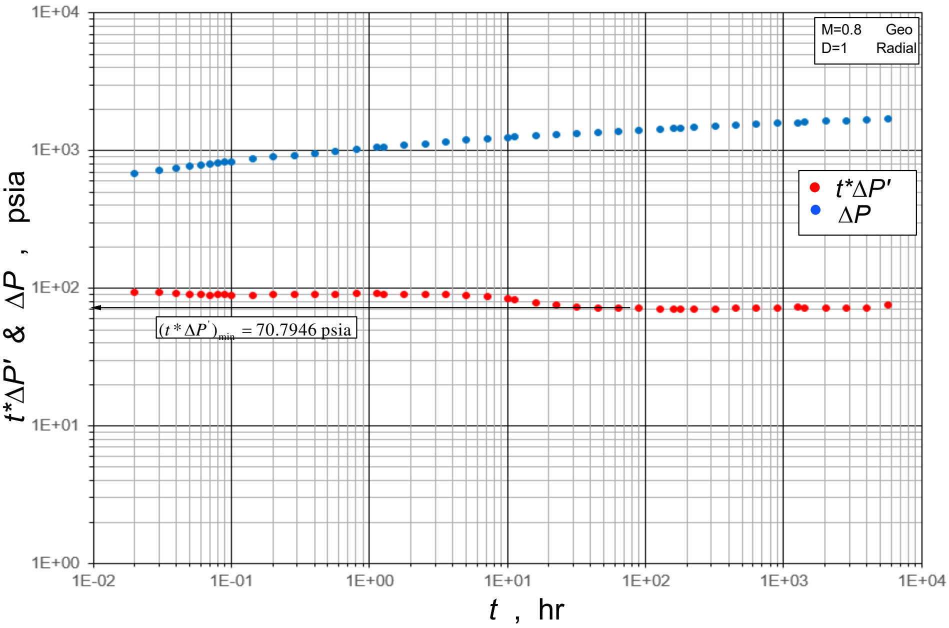

Figure-5. Behavior log-log pressure and pressure derivative data example 1.

Figure-5 is the log-log plot of the data obtained from the simulation, from this graph the minimum value of the pressure derivative is read, which is $(t^*\Delta P\ ')_{\min}$ = 70.7946 psia, and it is replaced in equation 13 obtaining as a result:

$$M = \sqrt[0.05]{\frac{1}{1.95 - \left(\frac{kh(t*dP')_{\min}}{141.2\mu\beta q}\right)^{0.065}}}$$

$$M = \sqrt[0.05]{\frac{1}{1.95 - \left(\frac{(60)(40)(70.7946)}{141.2(8)(1.2)(320)}\right)^{0.065}}} = 0.834$$

On the other hand, by using correlation 16 we obtain as a result:

$$M = \frac{1}{8.1973755\times10^{-5} + \dfrac{0.49967937}{\left(\dfrac{kh(t*\Delta P')_{\min}}{141.2q\mu\beta}\right)}}$$

$$M = \frac{1}{8.1973755*10^{-5} + \dfrac{0.49967937}{(0.3917)}} = 0.784$$

Table-1. Comparison of input and calculated parameters for example 1.

	Input	This study	Equation	Error
M	0.8	0.834	13	4.25%
		0.784	16	2%

4.2 Example 2: Linear Composite Reservoir

A synthetic pressure test for a vertical well in a linear composite reservoir of infinite extension with a diffusivity contrast D = 0.05 and mobility contrast of M=1 was simulated. The following fluid and rock properties are assumed:

q= 250 STBD	k= 35 md	r_w= 0.3 ft
μ= 3 cp	ϕ= 18 %	h= 25 ft
β= 1.15 rb/STB	c_t= 2x10^{-6} psi^{-1}	Li= 300 ft
M= 1	D= 0.05	Pi= 5000 psia

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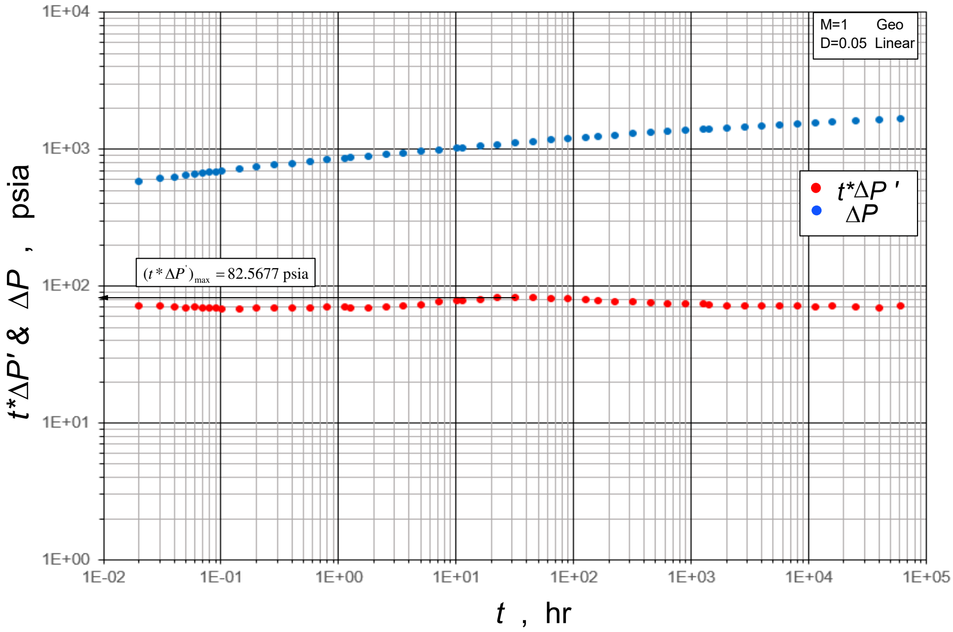

Figure-6. Behavior log-log pressure and pressure derivative data example 2.

Figure-6 is the log-log plot of the data obtained from the simulation of the test, from this graph the value of the maximum of the pressure derivative was read, which is $(t^*\Delta P')_{max} = 82.5677$ psia, and it is replaced in equation 7 and 18 as was done in the same way of the former example.

Notice that more values can be estimated in both exercises, but it is enough for practical purposes. Also, in both examples the ratio between the maximum time during wellbore storage and the minimum point during the transition period is higher than 10 and, according to Engler and Tiab (1996) there is no need of correction due to wellbore storage effects.

Table-2. Comparison of input and calculated parameters for example 2.

	Input	This study	Equation	Error
D	0.05	0.0525	7	5%
		0.048	18	4%

Although the estimation of naturally fractured reservoir parameters accepts one order of magnitude, Table-2 reports a good agreement between the simulated and the estimated parameters.

The input mechanical skin factors were taken as zero. The estimated values are close to these values. No errors were estimated for this case since this parameter allows a unit of difference.

5. DISCUSSIONS

As can be seen in the examples, the results obtained by this new methodology present an average error percentage for the equations of less than 8%, which makes it possible to indicate and highlight that the proposed equations and correlations have a good precision for the analysis of pressure tests of composite reservoirs for the purpose of reservoir characterization. Additionally, it is very important to point out that a large extent of the accuracy depend on the reading of the characteristic points in the log-log plot, thereby, it is worth note that the interpreter's expertise plays a great role in the quality of the interpretation.

6. CONCLUSIONS

The *TDS* technique was developed for the interpretation of pressure tests in a more efficient and practical way for linear and radial composite systems based on the characteristics of the shape of the pressure derivative log-log plot. The obtained methodology, which uses the maximum and minimum points observed on the pressure derivative curve, turns out to be of great applicability to calculate the contrast of the mobility or diffusivity of the regions that compose the two-region composite reservoirs in both geometries, in order to be able to characterize in a more efficient, practical and accurate way the farther region from the well. The numerous expressions developed were successfully verified with synthetic examples, obtaining an average error less than 8%, which is very reasonable in the field of pressure test analysis.

Nomenclature

Pi	Reservoir initial Pressure, psi
Ri	Radial internal boundary distance, ft
Li	Linear internal boundary distance, ft
M	Mobility Ratio
D	Diffusivity Ratio
c_t	Total Compressibility, 1/psi
h	Reservoirthickness, ft
P	Pressure, psi
P_D	Dimensionless Pressure
q	Flow Rate, STBD
s	Skin Factor
t	Time, hr
$t*\Delta P'$	Pressure Derivative, psi
t_D	Dimensionless Time
t_D*P_D'	Dimensionless Pressure Derivative

Greek

ϕ	Porosity, fraction
Δ	Change, drop
β	Formation Volumetric Factor, rb/STB
μ	Viscosity, cp

Suffices

D	Dimensionless
1	Internal Region
2	External Region
min	Minimum Point
max	Maximum Point
r_exter	External Radial

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