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MATRIX FORMULATION WITH EXPONENTIAL FILTER FOR REACTIVITY CALCULATIONS

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ABSTRACT

In this work, we study the method of matrix formulation when the neutron density presents noise. An exponential filter to reduce fluctuations in the calculation of reactivity is proposed. Neutron density fluctuations are simulated by assuming that its values are around an average density value with a Gaussian distribution. The results are presented for an exponential form of the neutron density and several standard deviations, with two different sample sizes for filtering that produce delays in the calculation of reactivity and with a first order of delay low-pass filter.

Keywords: exponential filter, matrix formulation, reactivity, numerical simulation.

INTRODUCTION

Nuclear fission is the physical process in which the nucleus of a heavy atom such as ^{235}U or ^{239}Pu can capture a neutron (neutron capture) forming a composite nucleus, causing the atom to become unstable to the point that it exceeds the potential barrier that is needed to cause fission [1]. The heavy atom is divided into two or more atoms each with less mass. In addition, it can emit from zero to several neutrons, emit neutrinos and release about 200 MeV of energy [2]. The emitted neutrons can again divide the heavy nuclei causing a chain reaction [3].

The chain reaction in a nuclear reactor is controlled by means of the control rods, their function is to keep the density of the neutron population constant, operating the reactor with safety and stability [4]. The motion of these control rods is made according to the design of the reactor, this motion creates fluctuations in the density of neutrons causing fluctuations to occur also in the reactivity.

Several papers on the calculation of reactivity have been published [5-9]. A recent publication [10] uses the matrix formulation to a good approximation, however, it did not considered fluctuations in neutron density.

The present work considers fluctuations in neutron density and also proposes the use of an exponential filter to reduce fluctuations, the results are compared with those obtained without considering fluctuations and with a first order of delay low-pass filter.

The theoretical aspects are presented in the next section, followed by the proposed method using the matrix formulation and the exponential filter, then the results obtained by the different numerical experiments are presented. Finally, the conclusions are presented.

THEORETICAL ASPECTS

The equations of point kinetics are a set of seven coupled equations which describe the time dependence of the neutron population, as well as the decay of neutron precursors [2]

$$\frac{dn(t)}{dt} = \left[\frac{\rho(t)-\beta}{\Lambda}\right] n(t) + \sum_{i=1}^{m} \lambda_i\, C_i(t) \qquad (1)$$

$$\frac{dC_i(t)}{dt} = \frac{\beta_i}{\Lambda} n(t) - \lambda_i C_i \qquad (2)$$

With initial conditions

$$n(t=0) = n_0 \qquad (3)$$

$$C_i(t=0) = \frac{\beta_i}{\Lambda \lambda_i} n_0 \qquad (4)$$

Where $n(t)$ is the density of neutrons, $C_i(t)$ is the concentration of the *i-th* group of precursors of delayed neutrons, $\rho(t)$ is the nuclear reactivity, Λ is the instantaneous neutron generation time, β_i is the effective fraction of the *i-th* group of delayed neutrons, β is the total effective fraction of delayed neutrons, and λ_i is the decay constant of the *i-th* group of delayed neutron precursors

It is possible to obtain an expression for reactivity by solving equations (1) and (2) using the initial conditions represented by equations (3) and (4):

$$\rho(t) = \beta + \frac{\Lambda}{n(t)}\frac{dn(t)}{dt} - \frac{\Lambda}{n(t)} \sum_{i=1}^{m} \lambda_i C_i(t) \qquad (5)$$

Where the concentration of precursors is described as:

$$C_i(t) = \frac{\beta_i}{\Lambda}\left[\frac{e^{-\lambda_i t}}{\lambda_i}\langle n_0\rangle + \int_0^t e^{\lambda_i(t'-t)} n(t')dt'\right] = \frac{\beta_i}{\Lambda} H_i(t) \qquad (6)$$

Where $H_i(t)$ is the history of the neutron density, however, calculating the reactivity using equations (5) and (6) has a high computational cost due to the integral term in equation (6), it is better to solve the system given by equations (1) and (2) without using the historical. The following section explains how it can be done.

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PROPOSED METHOD

The matrix formulation method [10] proposes a matrix function to solve for the concentration of delayed neutron precursors using equation (2), as follows:

$$\frac{d\vec{x}(t)}{dt} = S(t)\vec{x}(t), \qquad \vec{x}(0) = \vec{x}_0 \tag{7}$$

Where $\frac{d\vec{x}(t)}{dt}$ and $\vec{x}(t)$ are vector functions of dimension $m+1$, and $S(t)$ of dimension $(m+1)\times(m+1)$.

Each of the variables presented in equation (7) can be written as:

$$\frac{d\vec{x}(t)}{dt} = \begin{bmatrix} n(t) \\ C_1(t) \\ C_2(t) \\ \vdots \\ C_{m(t)} \end{bmatrix}; \quad \vec{x}(t) = \begin{bmatrix} n(t) \\ C_1(t) \\ C_2(t) \\ \vdots \\ C_{m(t)} \end{bmatrix}; \quad \vec{x}(0) = n(0)\begin{bmatrix} 1 \\ \beta_1/\Lambda\lambda_1 \\ \beta_2/\Lambda\lambda_2 \\ \vdots \\ \beta_m/\Lambda\lambda_m \end{bmatrix} \tag{8}$$

Where n_0 and x_0 are the initial conditions given by equations (3) and (4).

The term $S(t)$ represents a matrix function that can be written as:

$$S(t) = \begin{bmatrix} \frac{n'(t)}{n(t)} & 0 & 0 & \dots & 0 \\ \frac{\beta_1}{\Lambda} & -\lambda_1 & 0 & \dots & 0 \\ \frac{\beta_2}{\Lambda} & 0 & -\lambda_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\beta_m}{\Lambda} & 0 & 0 & \dots & -\lambda_m \end{bmatrix} \tag{9}$$

It can be seen that the matrix $S(t)$, as given in equation (9), is independent of time if the neutron density $n(t)$ has an exponential form.

It is possible to integrate equation (7) to obtain:

$$\vec{x}_{n+1} = \exp\left(\int_{t_n}^{t_{n+1}} S(t)\right)\vec{x}_n \tag{10}$$

Where $\vec{x}_n$ is the vector value of the function at an instant time t_n and $\vec{x}_{n+1}$ at an instant time t_{n+1}.

We can write the exponential matrix solution represented in equation (10) in the following way:

$$\vec{x}_{n+1} = X_n e^{D_n} X_n^{-1}\vec{x}_n \tag{12}$$

where X_n is a matrix whose columns are the eigenvectors of $\int_{t_n}^{t_{n+1}} S(t)\,\mathrm{dt}$, and D_n is a diagonal matrix whose diagonal elements are the eigenvalues of $\int_{t_n}^{t_{n+1}} S(t)\,\mathrm{dt}$

Now, it is necessary to take into account fluctuations in neutron density due to the motion of the control rods. For this, it is necessary to generate some noise (with a Gaussian form) around the average neutron density [11]:

$$\overline{P}_i = \frac{1}{N}\sum_{j=1}^{N} P_j \tag{12}$$

The noise is associated with the standard deviation, and it is assumed in this work that it varies from σ = 0.001 to σ = 0.01 to analyse the resistance to noise as well as the accuracy of the method when it is subjected to such fluctuations in neutron density. Firstly, the first order delay low-pass filter [5] is used:

$$P_i = P_{i-1} + \frac{T}{T+\tau}(\overline{P}_i - P_{i-1}) \tag{13}$$

With a filter constant τ that varies from τ = 0.1 to τ = 0.5

Next, the reduction of fluctuations with the exponential filter is studied, this is a least squares adjustment [12]:

$$y(x) = De^{Ex} \tag{14}$$

where D and E have constant values.

In order to solve equation (14), a linearization of the data is made, from which we obtain:

$$Y = EX + C \tag{15}$$

Where

$$Y = \ln(y) \quad , \quad X = x \quad , \ C = \ln(D) \tag{16}$$

The constants E and C are calculated using the least squares method, taking into account the normal Gaussian equations:

$$\left(\sum_{k=1}^{N} x_k^2\right)E + \left(\sum_{k=1}^{N} x_k\right)C = \sum_{k=1}^{N} x_k y_k \tag{17}$$

$$\left(\sum_{k=1}^{N} x_k\right)E + NC = \sum_{k=1}^{N} y_k \tag{18}$$

And the constant D is calculated through equation (16):

$$D = e^C \tag{19}$$

RESULTS AND DISCUSSIONS

The results of the numerical simulations of the matrix formulation method in the presence of noise given by equation (11) are shown below, with a random number generating function of 2 x 10^{31}-1, the results are also presented when applying the exponential filter given by equation (14) for two different sample numbers. The constants used correspond to those of Uranium-235, the decay constant is λ_i= (0.0127, 0.0317, 0.115, 0.311, 1.4 and 3.87 s^{-1}), the fraction of delayed neutrons is β_i= (0.000266, 0.001491, 0.002849, 0.000896 and 0.000182), the instantaneous neutron generation time is $\Lambda = 2\times10^{-5}$ s. The time step is set at h = *0.01 s* and the number of samples is varied from a = *100* to a = *200.*

Table-1 shows the values obtained for the maximum difference in reactivity. With a deviation of σ =

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0.001 it is possible to observe that when using the exponential filter with a sample number of a = *200* there is a significant reduction in the maximum difference between the reactivities for ω ≤ 1.00847. Figure-1 shows the reactivity curve that compares the reference method, which is obtained by analytically solving equation (5), with the matrix formulation method in the presence of noise, which has a deviation of σ = *0.001*. Table-2 shows that the average error decreases for any value of ω when the number of samples a increases, although the reduction is more noticeable for ω ≤ 0.12353. Figure-2 shows the reactivity curve when using the exponential filter to reduce fluctuations, simulations are made for a number of samples a = *100* and a standard deviation σ = *0.001*.

Table-1. Maximum difference for a neutron population density of the form $P(t) = exp(\omega t)$ with different values of ω and standard deviation σ = *0.001*.

P(t)=exp(ωt) , *h* =0.01 s , σ=0.001		**Max. Diff [pcm]**				
ω	**t [s]**	**Noise**	**LP Filter τ=0.1**	**LP Filter τ=0.5**	**Exponential Filter sample=100**	**Exponential Filter sample=200**
0.006881	500	3.3336	0.7949	1.9303	0.4221	0.2625
0.01046	800	3.2363	0.9795	2.9050	0.4184	0.2629
0.016957	300	2.6227	1.4559	4.6532	0.3730	0.2085
0.02817	600	2.9040	2.3449	7.6066	0.4157	0.2749
0.12353	300	1.9858	9.6991	30.0547	0.4782	0.3816
1.00847	100	3.8853	61.7343	146.3407	2.1409	2.1557
11.6442	10	7900.80	545.3547	545.3547	159.8753	676.6720

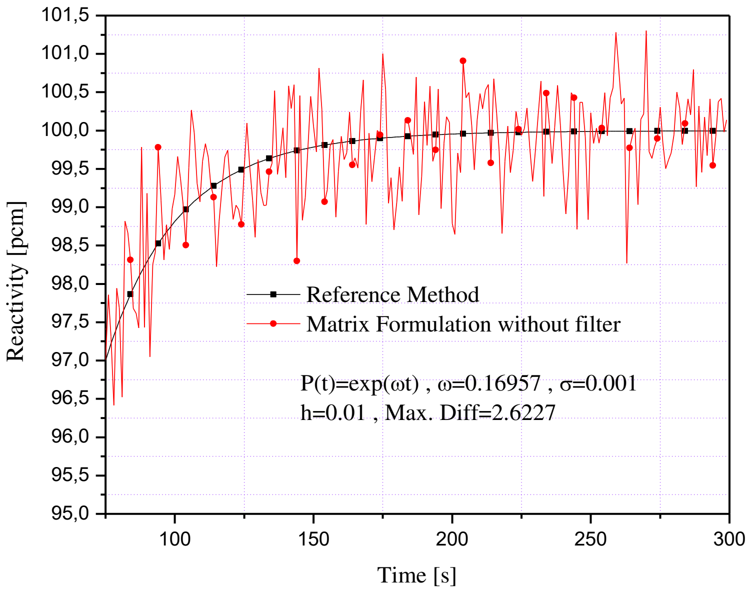

Figure-1. Reactivity curve for $P(t) = exp(\omega t)$ with ω = 0.016957 and σ =*0.001* without filter.

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Table-2. Mean error for a neutron population density of the form $P(t) = \exp(\omega t)$ with different values of ω and standard deviation $\sigma = 0.001$.

P(t)=exp(ωt) , *h* =0.01 s , σ=0.001		**Mean Absolute Error [pcm]**				
ω	**t [s]**	**Noise**	**LP Filter τ=0.1**	**LP Filter τ=0.5**	**Exponential Filter sample=100**	**Exponential Filter sample=200**
0.006881	500	0.5238	0.1083	0.1100	0.0723	0.0527
0.01046	800	0.5087	0.1088	0.1081	0.0735	0.0468
0.016957	300	0.4863	0.1377	0.2520	0.0775	0.0397
0.02817	600	0.4560	0.1389	0.2140	0.0864	0.0414
0.12353	300	0.3909	0.3900	0.7979	0.2620	0.2232
1.00847	100	2.0289	2.6267	4.1575	2.0318	2.0150
11.6442	10	87.2737	44.4427	49.3786	17.9788	5.9841

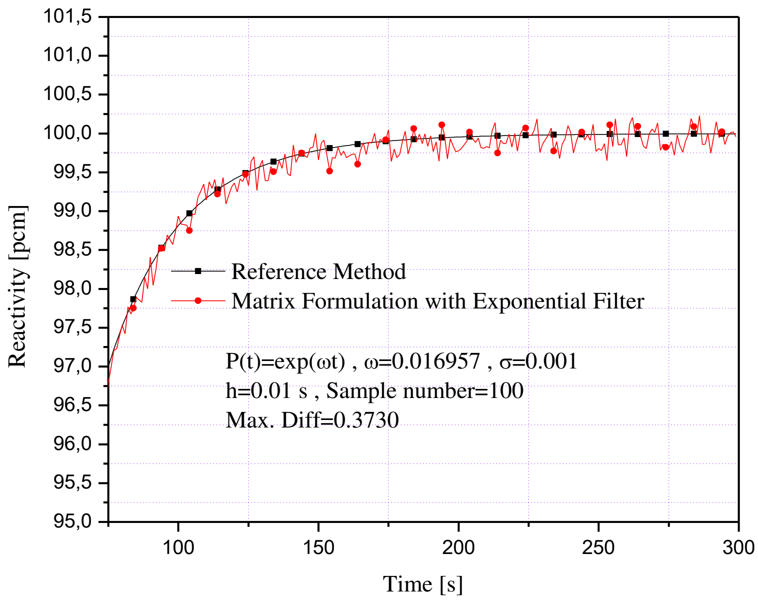

Figure-2. Reactivity curve for $P(t) = exp(\omega t)$ with ω = 0.016957 and $\sigma = 0.001$ with exponential filter.

In Table-3 the standard deviation is increased by $\sigma = 0.01$ and it is possible to see that for $\omega = 11.6442$ the maximum difference in reactivity increases as the number of samples increases, however, for small values of ω better values are obtained when considering samples of $a = 200$, this number of samples multiplied by the time step $h = 0.01$ s produces a delay in the calculation of the reactivity of about two seconds. Figure-3 shows the reactivity for ω = 0.12353 from which the exponential filter has not been applied. Table-4 shows the values corresponding to the average errors for different values of ω. It is observed that for ω = 11.6442 the mean error decreases significantly from 335.5513 pcm to 7.7319 pcm with $a = 100$ and to 5.9831 pcm with $a = 200$; likewise, it is noted that when applying exponential filter fluctuations are reduced for $\omega \leq 11.6442$. Figure-4 shows the reactivity curve where the exponential filter is applied to reduce fluctuations.

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Table-3. Maximum difference for a neutron population density of the form $P(t) = \exp(\omega t)$ with different values of ω and standard deviation $\sigma = 0.01$.

P(t)=exp(ωt) , *h* =0.01 s , σ=0.01		Max. Diff [pcm]				
Ω	**t [s]**	**Noise**	**LP Filter τ=0.1**	**LP Filter τ=0.5**	**Exponential Filter sample=100**	**Exponential Filter sample=200**
0.006881	500	34.7972	4.8499	3.3549	4.1474	2.6609
0.01046	800	33.7090	5.2399	4.1550	4.0447	2.5629
0.016957	300	26.9443	5.0602	5.5667	3.4736	2.3663
0.02817	600	29.8868	5.5087	7.9089	3.6948	2.2895
0.12353	300	21.0159	10.6556	30.1068	2.5946	1.7130
1.00847	100	21.2133	62.7170	146.7852	3.2914	3.4793
11.6442	10	$2.9638*10^{05}$	516.8443	462.7749	604.8399	676.7096

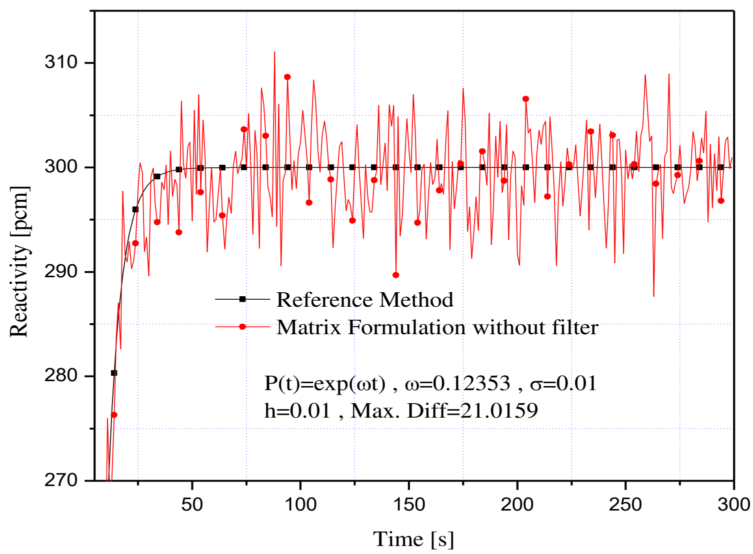

Figure-3. Reactivity curve for $P(t) = exp(\omega t)$ with ω = 0.12353 and $\sigma = 0.01$ without filter.

Table-4. Mean error for a neutron population density of the form $P(t) = \exp(\omega t)$ with different values of ω and standard deviation $\sigma = 0.01$.

P(t)=exp(ωt) , *h* =0.01 s , σ=0.01		Mean Absolute Error [pcm]				
ω	**t [s]**	**Noise**	**LP Filter τ=0.1**	**LP Filter τ=0.5**	**Exponential Filter sample=100**	**Exponential Filter sample=200**
0.006881	500	5.2360	0.9784	0.5440	0.6989	0.5723
0.01046	800	5.0804	0.9519	0.5362	0.6847	0.5416
0.016957	300	4.8502	0.9287	0.6302	0.6640	0.4855
0.02817	600	4.5236	0.8610	0.5723	0.6115	0.4554
0.12353	300	3.2583	0.8019	1.0015	0.5548	0.2480
1.00847	100	2.3700	2.7355	4.2659	2.1846	2.0323
11.6442	10	335.5513	14.8483	13.7032	7.7319	5.9831

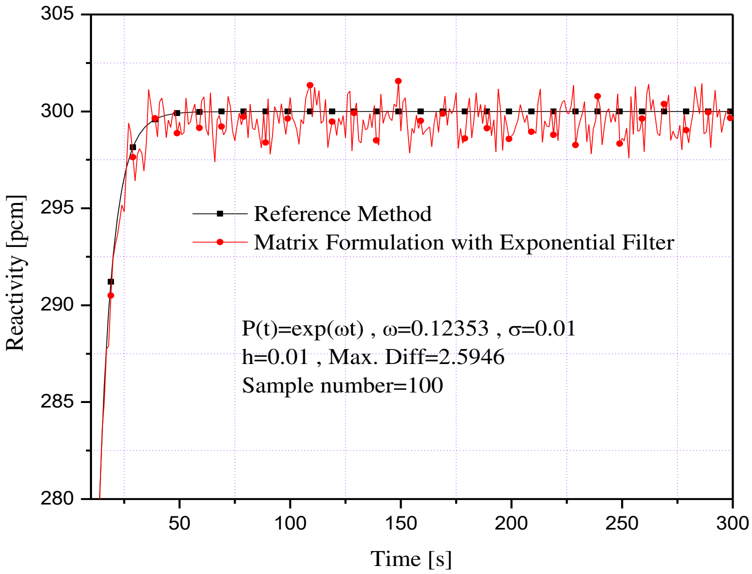

Figure-4. Reactivity curve for $P(t) = exp(\omega t)$ with ω = 0.12353 and σ =*0.01* with exponential filter.

CONCLUSIONS

In this work, numerical simulations were made with the matrix formulation method for the exponential form of the neutron density, considering that fluctuations occur with a Gaussian distribution around an average value. The exponential filter turns out to be very useful to reduce the fluctuations in the calculation of reactivity when considering standard deviations from $\sigma = 0.001$ to $\sigma = 0.01$ obtaining better reductions in fluctuations if a greater number of samples is considered, although this causes a delay between one and two seconds.

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