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# CLASSICAL AND NON-LINEAR PREDICTIVE CONTROL APPLIED TO A NON-LINEAR SYSTEM OF COUPLED TANKS

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**ABSTRACT**

This document presents the design, implementation and comparison of algorithms to control the liquid level in a non-linear system of two coupled tanks. First, a Proportional-Integral (PI) controller was designed; then, a Model-based Predictive Controller (MPC) using the Non-linear Extended Prediction Self-Adaptive Control (NEPSAC) algorithm was designed. The performance of the controllers was evaluated in two scenarios: Setpoint Tracking and Disturbance Rejection. It is concluded that the NEPSAC algorithm presents better performance with respect to the PI, since the PI works correctly in regions very close to the setpoint while the NEPSAC, not requiring linearization of the system, has a performance that does not depend on the setpoint, presenting excellent characteristics at any reference point. To validate the results obtained, the Root Mean Square Error (RMSE) was used, which resulted in better values for the NEPSAC than for the PI.

**Keywords:** level control, MBPC, NEPSAC, PI, RMSE.

## 1. INTRODUCTION

The control variables with the greatest relevance in industrial processes are level, pressure, temperature, and flow. The control of these variables is critical for production processes since they are essential to generate income or losses [1]. It is here that different control techniques play an important role in reducing costs and increasing profits. For example, in the paper-making process is important to control the temperature of the rollers, since if the temperature is kept high, this implies greater unnecessary energy consumption by increasing production costs and decreasing profits. Due to this, the control technique used must be able to respond to incidents or eventualities that occur in the system to be controlled.

Liquid level control is very important in industrial processes, the Surcolombiana University is not indifferent to this reality, and for this reason it acquired the CE105MV level plant shown in Figure-1. The plant consists of two tanks, each tank contains a discharge valve (valve B and C), and in addition, there is a valve that connects both tanks (valve A). This plant has the same characteristics of a real industrial system, allowing students to relate to these types of processes and train for the challenges proposed by the industry.

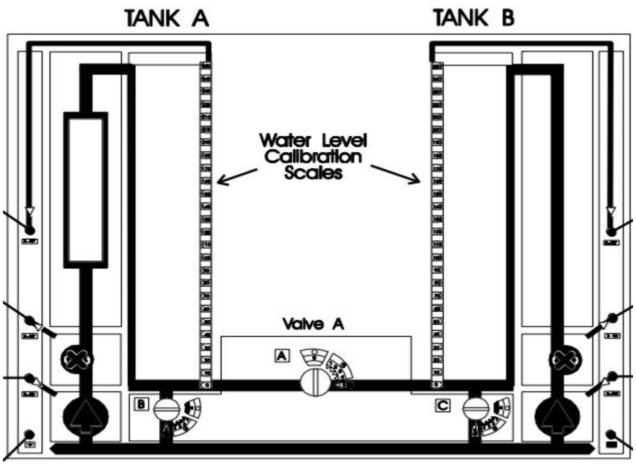


**Figure-1.** CE105MV system.

The most used type of control for liquid level systems is the classic or advanced control techniques [2] - [6], where linearization of the system to be controlled is necessary by choosing a reference point. However, these systems are usually non-linear, and because of this, control will be carried out in this document using NEPSAC, and the performance of the application of a non-linear control algorithm versus a classical technique such as PI will be analyzed.

## 2. MATERIALS AND METHODS

### 2.1 Coupled Tanks System

The system is composed of two tanks interconnected with each other by means of valve A. The fluid inlet to the system is provided by a pump as can be seen in Figure-2. In this article the control for the level of the tank in $H_2$ is developed, which depends on the amount of liquid supplied by the pump by means of the flow $Q_i$ to the tank 1 and the value of the discharge coefficients of the valves B and C, thus as of the restriction that valve A may cause.

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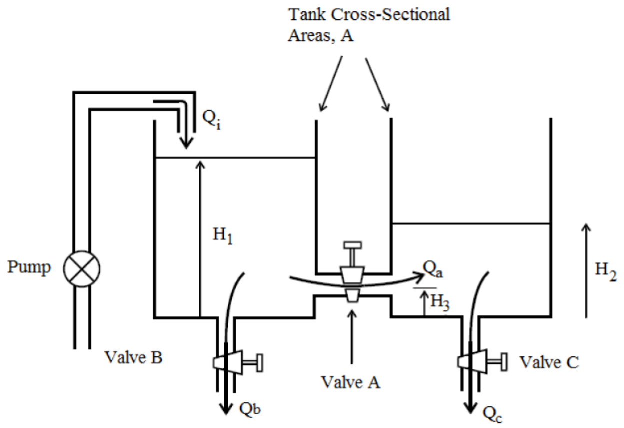


**Figure-2.** Schematic of coupled tanks [7].

## 2.2 Mathematical Model

The equations that describe the mathematical behavior of the system are:

$$A\dot{h_1}(t) + C_{12}a\sqrt{2g[h_1(t) - h_2(t)]} + C_1 a\sqrt{2gh_1(t)} = K_b V_i(t)$$

$$A\dot{h_2}(t) + C_2 a\sqrt{2gh_2(t)} = C_{12}a\sqrt{2g[h_1(t) - h_2(t)]}$$

The first equation describes the relationship between the input voltage $V_i(t)$ and the liquid level in the tank $h_1(t)$, taking into account the flow that passes through the valve A together with the difference in level between $h_1(t)$ and $h_2(t)$, in addition to valve B.

The behavior described in the second equation is due to the relationship between the level of the tank $h_2(t)$ and the flow that passes through the valve A together with the difference in level between $h_1(t)$ and $h_2(t)$, in addition to the discharge coefficient of valve C, which causes the level to be reduced due to the effect of gravity.

The nonlinearity of the process can be appreciated since in both equations there are relationships that involve the square root.

The values of the system parameters are shown in Table-1.

**Table-1.** System parameters [8].

| Symbol | Description | Value |
|---|---|---|
| $A$ | Cross-sectional area of the tanks | $9350\text{x}10^{-6}\text{m}^2$ |
| $a$ | Cross-sectional area of the valve orifice | $78.50\text{x}10^{-6}\text{m}^2$ |
| $C_{12}$ | Discharge coefficient of valve A | 0.5 |
| $C_1$ | Discharge coefficient of valve B | 0.2 |
| $C_2$ | Discharge coefficient of valve C | 0.2 |
| $h_{max}$ | Maximum liquid level | 0.25 m |
| $v_{max}$ | Maximum input voltage | 10 V |
| $K_b$ | Pump gain | $6.66\text{x}10^{-6}\text{m}^3/\text{sV}$ |
| $g$ | Gravity constant | $9.8\ \text{m/s}^2$ |

Although the NEPSAC algorithm does not need a linearization of the system, it is required to implement the PI control. Because of this, Taylor series are used to obtain a linear model. Taylor argues that it is possible to linearize an equation if it is derived with respect to the variable and analyzed at a setpoint. Performing this procedure with the previous equations and analyzing at the setpoint $H_2 = 0.1$ meters, the following transfer function is obtained:

$$G(s) = \frac{\alpha}{s^2 + \beta s + \gamma}$$

where $\alpha$, $\beta$, and $\gamma$ are respectively:

$$\alpha = \frac{K_b C_{12} a\sqrt{2g}}{4A^2\sqrt{H_1 - H_2}}$$

$$\beta = \frac{C_1 a\sqrt{2g}}{2A\sqrt{H_1}} + \frac{C_{12}a\sqrt{2g}}{A\sqrt{H_1 - H_2}} + \frac{C_2 a\sqrt{2g}}{2A\sqrt{H_2}}$$

$$\gamma = \frac{C_1 C_{12} a^2\, 2g}{4A^2\sqrt{H_1 - H_2}\sqrt{H_1}} + \frac{C_1 C_2 a^2\, 2g}{4A^2\sqrt{H_2}\sqrt{H_1}} + \frac{C_2 C_{12} a^2\, 2g}{4A^2\sqrt{H_1 - H_2}\sqrt{H_2}}$$

By replacing the values in Table-1, the following transfer function is obtained:

$$G(s) = \frac{2.619 * 10^{-5}}{s^2 + 0.1696s + 0.001793}$$

The NEPSAC algorithm does not require a linearization of the system but if its discretization is necessary, for this reason the non-linear equations are discretized, obtaining:

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$$h_1(k) = h_1(k-1) - \frac{aC_{12}}{A}\varphi - \frac{aC_1}{A}Ts\theta_1 + \theta$$

$$h_2(k) = h_2(k-1) + \frac{aC_{12}}{A}\varphi - \frac{aC_2}{A}Ts\theta_2$$

where $\varphi$, $\theta$, $\theta_1$, and $\theta_2$ are respectively:

$$\varphi = T_s\sqrt{2g[h_1(k-1) - h_2(k-1)]}$$

$$\theta = \frac{K_b T_s Vi(k-1)}{A}$$

$$\theta_1 = \sqrt{2g[h_1(k-1)]}$$

$$\theta_2 = \sqrt{2g[h_2(k-1)]}$$

In this case the sampling period $T_s = 1$ second.

**2.3 PI Controller Design**

In order to design the PI controller, it is necessary to have the plant's linearized model around a setpoint. For this, the value $H_2 = 0.1$ meters is established as a reference and thus the PI controller is designed.

Using the Matlab® software, the PI is adjusted so that the closed-loop system presents a response with a shorter settling time and without overshoot. The PI that presented these characteristics has the following parameters:

$$K_p = 97.7385$$

$$K_i = 0.9023$$

It is important to note that the PI controller is represented by:

$$u(t) = K_p e(t) + K_i \int e(t)d\tau$$

$u(t)$ is the output signal of the PI controller, which in this case corresponds to the voltage applied to the pump. $e(t)$ is the input signal of the PI controller, which is defined as $e(t) = r(t) - y(t)$, where $r(t)$ is the setpoint and $y(t)$ is the output of the process, that is, the level of liquid in the tank. $K_p$ is the proportional gain and $K_i$ is the integral gain.

Applying the Laplace transform, the transfer function of the PI controller is found:

$$G_c(s) = 97.7385 + \frac{0.9023}{s}$$

**2.4 NEPSAC**

The NEPSAC algorithm is based on the following model for its implementation:

$$y(t) = x(t) + n(t)$$

Where $y(t)$ is the measurement of the process output, $x(t)$ is the output of the process model, and $n(t)$ is the disturbance.

This algorithm requires a prediction of future results to perform the control action, for this the term of prediction horizon $N_2$ is introduced, which can be defined as [9]:

$$y(t+k|t) = x(t+k|t) + n(t+k|t)$$

In this way the future output can be described as the contribution of two important parts, a base component and an optimal component [10].

$$y(t+k|t) = y_{base}(t+k|t) + y_{opt}(t+k|t)$$

However, for nonlinear systems, the term $y_{opt}(t+k|t)$ can gradually be made equal to zero in an iterative way by selecting the base control strategy $u_{base}(t+k|t)$ appropriately. The superposition principle is then no longer involved, and the algorithm leads to the optimal solution, even for non-linear systems.

At each sampling instant an initial $u_{base}(t+k|t)$ is selected. It should be the objective to obtain finally (in an iterativeway) a control policy $u_{base}(t+k|t)$which is as close as possible to the optimal strategy $u(t+k|t)$. In order to minimize the number of iterations, it is thus wise to make a good initial guess for $u_{base}(t+k|t)$. A simple but effective choice is to start with $u_{base}(t+k|t) = u(t+k|t-1)$, i.e. the optimal control policy derived at the previous sampling instant. Once a $u_{base}(t+k|t)$ has been chosen, $\delta u(t+k|t)$ is calculated. For a non-linear model, this is not the optimal control because the principle of superposition does not hold. However, it can be expected that the resulting $u$ signal is closer to the optimal control than the previous guess $u_{base}$. So, for a non-linear model, it is suggested to continue the procedure - at the same sampling instant - by taking $u(t+k|t)$ as a new set $u_{base}(t+k|t)$. Continuing this iterative procedure, it can be expected that $u_{base}$ will converge to the optimal $u$. Indeed, each time that $u_{base}$ is closer to $u$, it means that the $\delta u$ is smaller; and thus, also the term $y_{opt}$ becomes smaller. The superposition principle has less impact. Finally, when the $\delta u$ is practically zero, the superposition principle is no longer involved and the calculated control signal will thus be optimal, also for the non-linear system. The whole procedure is thus certainly not based on any local linearizationof the non-linear model.

The step response, which is constant for linear (or linearized) models differs in each point for non-linear models. Hence, the step response must be recalculated for each iteration. Therefore, the model is linearized around the point of interest, and consequently the step response is calculated. Then the step response of the discretized model can be calculated and used in the calculation of $U^*$.

An appropriate (i.e. close to the optimal value) initial guess for $u_{base}(t|t)$ lowers the number of iterations to get $\delta u(t|t) = 0$. Therefore, the initial guess is set equal to the actual control action at the previous time sample, $u(t-1)$. Since the control horizon $N_u = 1$, $u_{base}(t+k|t) = u_{base}(t|t)$ for $0 < k < N_2$. Iterations are stopped once $\delta u(t|t)$ is lower than a threshold ($10^{-6}$). It can be said that $u(t) = u_{base}(t|t)$.

At the end, clipping is applied to the control action $u(t)$, so that it is in the range $0 \leq u(t) < 10$ V.

## 3. RESULTS AND DISCUSSIONS

In order to verify the behavior of the two controllers and to achieve a comparison of the performance of the methods used in this article, the analysis was performed with the presentation of two scenarios, the first showing the response to disturbances rejection and the second where the setpoint tracking is observed.

In Figure-3, it can be remarkably observed that the NEPSAC algorithm action is much faster when setting the setpoint than the PI control action.

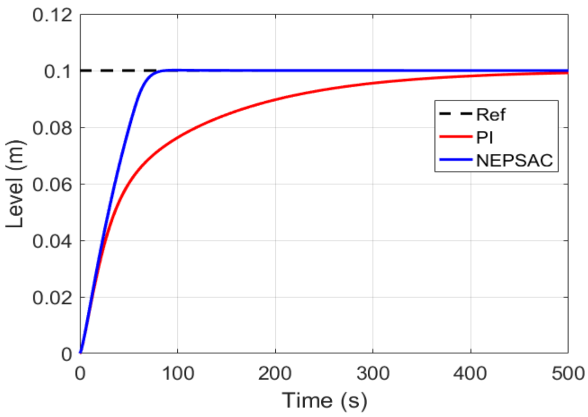


**Figure-3.** System response with setpoint 0.1 m.

The settling time obtained with the PI control was 356 seconds and with the NEPSAC algorithm it was approximately 73 seconds, thus being 4.8 times faster.

### 3.1 Scenario 1: Disturbance Rejection

Once the response stabilizes at 0.1 m, disturbances are applied to the system at 500 and 1000 seconds, changing the discharge coefficient of valve C in Figure-2, going from 0.2 to 0.3 and finally returning to 0.2. It is observed that two controllers present good robustness, always managing to return to the established operating point.

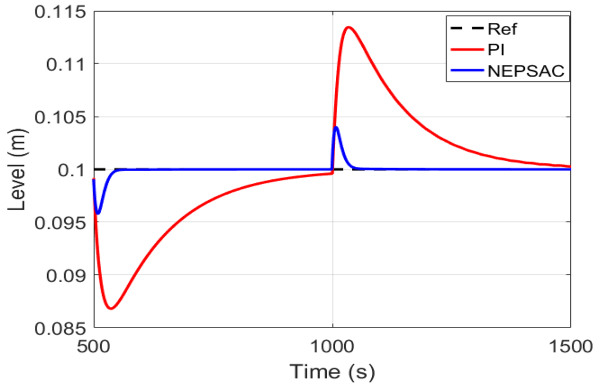


**Figure-4.** Disturbance rejection.

As shown in Figure-4, the NEPSAC algorithm with a prediction horizon $N_2 = 20$ is the one that presents the best behavior in this action, this value was chosen based on trial and error, since it was the one that allowed a smaller number of iterations in less simulation time.

After 500 seconds, the discharge coefficient value was increased from 0.2 to 0.3, increasing the outflow. For this reason, the level $H_2$ decreases reaching almost 0.005 m below the setpoint in the NEPSAC algorithm. Otherwise it occurs at 1000 seconds when $C_2$ is reset to 0.2, causing the $H_2$ to change 0.005 m above the reference and returns to its setpoint much faster.

The PI control action shows values of 0.015 m above and below the setpoint: both for the instant of 500 seconds and 1000 seconds. But this time the reaction is more abrupt, that is, the system is less effective to disturbances using this control technique, requiring more time to establish at the reference level.

It is important to note that control by NEPSAC makes more control effort than PI, this is reflected in Figure-5, since NEPSAC reacts more robustly to not allow disturbances to influence the system.

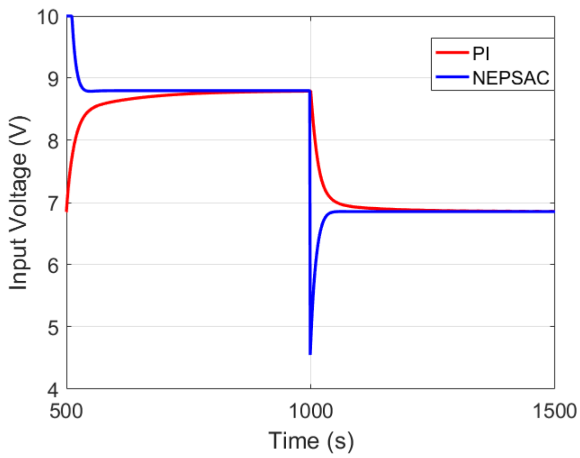


**Figure-5.** Control input for disturbance rejection.

### 3.2 Scenario 2: Setpoint Tracking

In order to evaluate how the system behaves at different operating points, the simulation is carried out by

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establishing setpoints of 0.04, 0.08, 0.12 and 0.16 m, and keeping the prediction horizon at $N_2 = 20$.

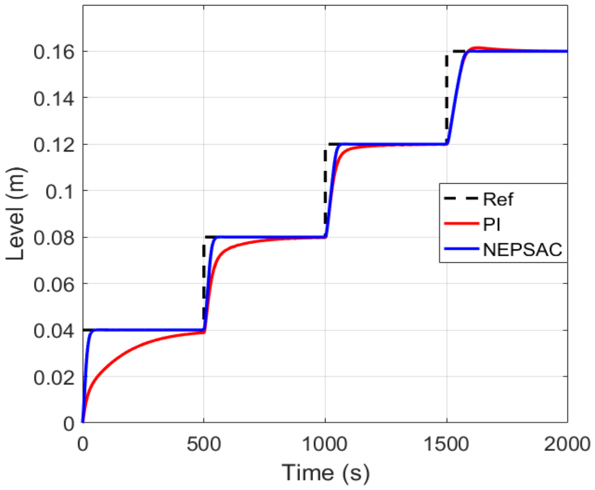


**Figure-6.** Setpoint tracking.

In the case of the PI control, the best responses were obtained in the references 0.12 m and 0.16 m, this because the design was carried out with the linearized system in 0.1 meters. The response at the 0.12 m operating point does not present an overshoot and is faster compared to the reference of 0.04 m and 0.08 m.

For the 0.16 m reference, the PI control obtains a slightly faster response than the previous ones, but because of this, the system presents a small overshoot as shown in Figure-6.

In the case of the NEPSAC algorithm, the system presents a good response in all cases, having minimal variations in the settling time, being always faster compared to the PI control and without presenting an impulse at any of the setpoints.

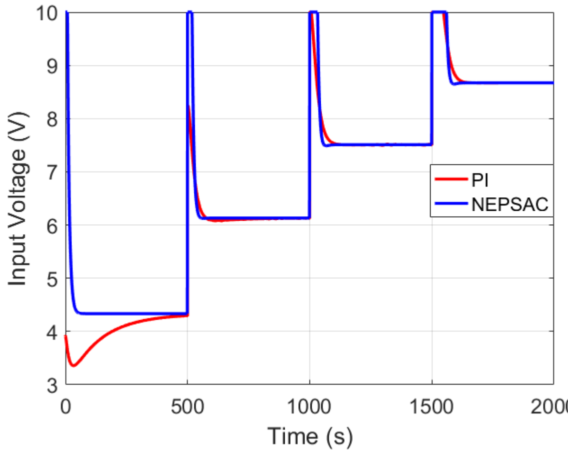


**Figure-7.** Control input for setpoint tracking.

The control effort can be identified in Figure-7. In the case of PI control, the system requires little control effort in the first two levels, without the need to cut the input signal voltage since it does not exceed 10V. For these two setpoints the system is very slow and the level in $H_2$ is enough to work within the maximum voltage range. In the case of steps 3 and 4, the established level requires a higher supply voltage, therefore it requires a cut-off of the input signal for 10V for both the PI control and the NEPSAC.

In order to obtain a detailed results analysis of the comparative study between the EPSAC and NEPSAC algorithms, the Root Mean Square Error (RMSE) variations were used.

$$RMSE = \sqrt{\frac{\sum_{t=1}^{N}[r(t) - y(t)]^2}{N}}$$

where $r(t)$ is the setpoint signal, $y(t)$ is the output signal, and $N$ is the number of samples. Table-2 shows the RMSE variations for both the PI and the NEPSAC algorithm when the two simulation scenarios are applied.

**Table-2.** RMSE Variations for EPSAC and NEPSAC Algorithms.

| Scenario | PI | NEPSAC |
|---|---|---|
| 1 | 0.59% | 0.07% |
| 2 | 0.98% | 0.73% |

According to Table-2, the NEPSAC algorithm presents a better performance according to the RMSE, being very low compared to the PI for the two test scenarios. However, the PI control technique is still a good alternative for plant control.

## 4. CONCLUSIONS

According to the results obtained, the NEPSAC algorithm presents better performance in the control of the CE105MV plant compared to that presented by the PI control technique.

In scenario 1, the NEPSAC algorithm is more effective in the presence of disturbances in the system, because it presents a greater control effort than the PI controller. This makes the $H_2$ changes not very significant. The same happens in scenario 2 for setpoint tracking.

Even though the NEPSAC algorithm is 0.52% more effective than the PI in the disturbance rejection scenario, it is much more difficult to implement than the latter, because it requires more computational cost for the iterations necessary for predictions. But this is not an impediment because today's computers have good features at a relatively low cost. Therefore, the NEPSAC control is ideal for control loops in non-linear systems.

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