

RK2-2ST METHOD TO SOLVE THE STOCHASTIC POINT KINETIC EQUATIONS WITH NEWTONIAN TEMPERATURE FEEDBACK USING STEP EXTERNAL REACTIVITY

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ABSTRACT

This article presents a two-stage second-order stochastic Ruge-Kutta method (RK2-2st) to solve the stochastic equations of point kinetics for a nuclear reactor with Newtonian temperature feedback effects considering step external reactivities. The average values and standard deviations of the neutron density and the population of delayed neutron precursors with a fixed step size are calculated, the Doppler effects considered correspond to the typical adiabatic approximation. The proposed method has a great advantage with respect to other derivative numerical stochastic methods, because it requires fewer derivatives and has low computational cost. The numerical experiments developed indicate that the proposed method has good precision and is very efficient for the study of stochastic point kinetics with Newtonian temperature feedback. The numerical results obtained are compared with deterministic and stochastic methods reported in the literature.

Keywords: neutron density, concentration of delayed neutron precursors density, newtonian temperature feedback, runge-kutta stochastic method.

INTRODUCTION

The equations of the point kinetics allow to know the operation of a nuclear reactor, in these devices the production of energy is carried out through the process of nuclear fission which occurs due to the high probability of interaction between thermal neutrons and fissile material. In order to have a tight control of the distribution of neutrons as to guarantee energy production, the dynamics of the nuclear reactor must be precisely known [1].

The equations of point kinetics consist of a stiff system of nonlinear deterministic differential equations that describe the dynamics of the neutron density and the concentration of delayed neutron precursors in a tightly coupled reactor over time. However, the actual dynamical processes in a nuclear reactor are stochastic in nature [2-3]. The equations of the stochastic point kinetics allow us to estimate the mean value and the standard deviation of the random variables [4].

There are different methods reported in the literature that solve the stochastic point kinetic equations such as: The Partial Constant Approximation Method (PCA) and Monte Carlo Method [5], Euler-Maruyama Method (EM) [6], the Taylor Scheme (1.5) [6-7], the Simplified Stochastic Kinetic Method (SSPK) [8], the Analytical Exponential Model (AEM) [9], the Efficient Stochastic Model (ESM) [10], the Double Diagonalization and Decomposition Method (DDDM) [11] and the Implicit Euler-Maruyama Method [12] (implicit EM). At present, many investigations on models of nuclear reactors are being carried out, particularly of models that consider the effects of temperature feedback [13]. These models allow to estimate the transient behaviour of the neutron density and the concentration of delayed neutron precursors, which in turn allows the timely control of the nuclear

reactor. In these reactors, reactivity is affected by temperature feedback.

Some methods reported in the literature that solve the deterministic equations of point kinetics with temperature feedback effects are: The Power Series Solutions (PWS) Method [14], the Converged Accelerated Taylor Series (CATS) Method [15], the Enhanced Piecewise Constant Approximation (EPCA) Method [16], ITS2 [17], the Generalized Adams-Bashforth-Moulton Method [18] and the 8th-order Adams-Bashforth-Moulton (ABM8) Method [19]. The most recent methods in the case of stochastic equations of point kinetics with feedback effects are the Euler-Maruyama Method, the Milstein Free Derivative Method [13] and the Strong-Order Taylor scheme (1.5) [20].

Due to the nature of the problem, this paper proposes to solve the stochastic equations of point kinetics with feedback effects from the implementation of the twostage second-order stochastic Ruge-Kutta method, (RK2-2st) [21], this numerical method is advantageous because it requires the calculation of very few derivatives of the system functions.

In this work, the kinetic model of a nuclear power reactor with feedback effects is initially described. Next, the stochastic method of numerical derivation RK2-2st is proposed to solve the stochastic equations of point kinetics with feedback effects. Finally, the numerical results of the RK2-2st method are shown for the solution of several numerical experiments for different types of reactivity inputs [22].

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THEORETICAL CONSIDERATIONS

STOCHASTIC POINT KINETIC EQUATIONS

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The dynamic processes in a nuclear reactor are stochastic in nature [2-3]. The stochastic equations of point kinetics allow us to know the time dependence of the processes involved in the temporal behaviour of the neutron flux in the reactor, the neutron density and the concentration of delayed neutron precursors. These equations are given by:

$$d\hat{x}_{(t)} = \left[A\hat{x}_{(t)} + \hat{Q}_{(t)}\right]dt + B^{1/2}d\hat{W}_{(t)}$$
(1)

Where,

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$$\hat{x}_{(t)} = \begin{bmatrix} n_{(t)} \\ c_{1(t)} \\ c_{2(t)} \\ \vdots \\ c_{m(t)} \end{bmatrix}$$

$$(2)$$

$$A = \begin{bmatrix} \frac{\rho_{(t)} - \beta}{\Lambda} & \lambda_1 & \lambda_2 & \cdots & \lambda_m \\ \frac{\beta_1}{\Lambda} & -\lambda_1 & 0 & \cdots & 0 \\ \frac{\beta_2}{\Lambda} & 0 & -\lambda_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ \frac{\beta_m}{\Lambda} & 0 & 0 & \cdots & -\lambda_m \end{bmatrix}$$
(3)

$$\hat{Q}_{(t)} = \begin{bmatrix} q_{(t)} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$
(4)

$$\hat{W}_{(t)} = \begin{bmatrix} W_{1(t)} \\ W_{2(t)} \\ W_{3(t)} \\ \vdots \\ W_{1(t)} \end{bmatrix}$$
(5)

$$B = \begin{bmatrix} \zeta^{*} & a_{1}^{*} & a_{2}^{*} & \cdots & a_{1}^{*} \\ a_{1}^{*} & b_{1,1}^{*} & b_{1,2}^{*} & \cdots & b_{1,m}^{*} \\ a_{2}^{*} & b_{2,1}^{*} & b_{2,2}^{*} & \cdots & b_{2,m}^{*} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{m}^{*} & b_{m,1}^{*} & b_{m,2}^{*} & \cdots & b_{m,m}^{*} \end{bmatrix}$$
(6)

Where

$$\begin{split} \boldsymbol{\zeta}^{*} &= \left[\frac{\rho(t)-\beta}{\Lambda}\right] \left(-1-\rho(t)+2\beta+\left(1-\beta\right)^{2}\upsilon\right) \boldsymbol{n}(t) + \sum_{i=1}^{m} \lambda_{i}c_{i}(t)+\boldsymbol{q}(t) \\ \boldsymbol{a}_{i}^{*} &= \beta_{i}\left(\left(1-\beta\right)\upsilon-1\right) \left[\frac{\rho(t)-\beta}{\Lambda}\right] \boldsymbol{n}(t) - \lambda_{i}c_{i}(t) \\ \boldsymbol{b}_{i,j}^{*} &= \beta_{i}\beta_{j}\left[\frac{\rho(t)-\beta}{\Lambda}\right] \boldsymbol{n}(t) + \delta_{i,j}\lambda_{i}c_{j}(t) \end{split}$$

 $n_{(t)}$ is the density of neutrons, $c_{i(t)}$ is the precursor concentration of *i-th* group of delayed neutron, β_i is the fraction of delayed neutrons of the *i*-th group of precursors, β is the total fraction of delayed neutrons, λ_i is the decay constant of *i*-th group of precursors, v is the fission rate of the neutrons, $q_{(t)}$ is the source of neutrons and $\delta_{i,j}$ is the Kronecker's delta. *B* is the covariance matrix with approximation of pulsed neutrons [12] and in equation (5) $W_{(t)}$ are the Winner processes (5). In equation (1), when the covariance matrix is equal to zero, it corresponds to the deterministic equations of point kinetics.

In thermal reactors, Doppler feedback is mainly due to epithermal capture resonances in non-fissile fuel isotopes, as well as in Th^{232} , U^{238} , and Pu^{240} . The temperature and reactivity coefficient for the Doppler effect are given by [23]:

$$\alpha_{T_f}^D = \frac{\partial \rho}{\partial T_f} \tag{7}$$

$$\rho(t) = \rho_0 - \alpha_{T_f}^D \left(T_f(t) - T_0 \right) \tag{8}$$

Where T_f is the temperature of the fuel.

The diameter of a rod (or rods) is much smaller than the length of the reactor core, which implies that heat transfer in the axial direction is insignificant with respect to the radial direction. The equations of heat transfer in cylindrical coordinates in the radial direction are given by [22]:

$$\rho_f C p_f \frac{\partial T_f}{\partial t} = k_f \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T_f}{\partial r} \right) + q_f'''(t)$$
(9)



where ρ_f and Cp_f are the specific density and heat of the fuel, respectively, k_f is the thermal conductivity of fuel, and q_f'' is the thermal power per unit volume, which is given by:

$$q_f''(t) = P_0 n(t)$$
 (10)

Here P_0 is the nominal potential. Now, the integration is performed in the cross-sectional area of the bar:

$$\int_{A} \left(\rho_{f} C p_{f} \right) \frac{\partial T_{f}}{\partial t} dA = \int_{A} \left[k_{f} \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T_{f}}{\partial r} \right) + P_{0} n(t) \right] dA (11)$$

The differential area element (*dA*) in cylindrical coordinates is $r dr d\theta$ for $0 \le \theta \le 2\pi$ and $0 \le r \le R_p$ where R_p is the radius of the bar. Then, the following result is obtained:

$$\rho_f C p_f \frac{dT_f}{dt} = \frac{2k_f}{R_p^2} \left(rk_f \frac{dT_f}{dr} \right)_{r=R_p} + P_0 n(t)$$
(12)

The first term on the right side has two possibilities:

$$\left(rk_{f}\frac{\partial T_{f}}{\partial r}\right)_{r=R_{p}} = \begin{cases} 0, & Adiabatic \\ -R_{p}G(T_{f}-T_{\infty}), & NonAdabatic \end{cases}$$
(13)

Where G is the heat transfer coefficient and T_{∞} is the coolant temperature away from the bar wall. In this work, the adiabatic case was studied:

$$\frac{dT_f}{dt} = \frac{P_0}{\rho_f \ Cp_f} n(t) \tag{14}$$

This fast presentation in the derivation of the point equation of the fuel temperature makes it possible to clearly establish the constant coefficients of the equation and the scope of the mathematical model. This equation can be rewritten as:

$$\frac{dT_f}{dt} = H n(t) \tag{15}$$

Where $H = P_0 / (\rho_f C p_f)$.

The Eq. (8) and (15) are widely applied to the dynamic analysis of nuclear reactors with Doppler feedback, this equation was presented more than 40 years ago by Hetrick (1971). The adiabatic approach for Doppler

feedback represents the most limiting case in the nuclear reactor in relation to the insertion of a positive reactivity, because it considers the hypothetical scenarios where in a power excursion without heat removal, the Doppler effects generate a reactor self-control that satisfactorily keeps the safety margins.

Now, equation (8) is derived with respect to time and equation (15) is applied, which leads to:

$$\frac{d\rho}{dt} = -\alpha_{T_f}^D H n(t) \tag{16}$$

The Eq. (16) represents the step change with temperature feedback.

SECOND-ORDER TWO-STAGE STOCHASTIC RUNGE-KUTTA SCHEME

The stochastic scheme of Runge-Kutta stage s [21], is given

$$\bar{X}_{n+1} = \bar{X}_n + \Delta \sum_{j=1}^s \alpha_j a(t_n + \mu_j \Delta, \eta_j) + \sum_{k=1}^m \Delta \hat{W}_n^k \sum_{j=1}^s \beta_j^k b^k (t_n + \mu_j \Delta, \mu_j) + R$$
(17)

Where, $\mu_1 = 0$, $\eta_1 = \overline{X}_n$,

$$\eta_j = \bar{X}_n + \Delta \sum_{i=1}^{j-1} \lambda_{ji} a(t_n + \mu_i \Delta, \eta_i) + \sum_{k=1}^m \Delta \hat{W}_n^k \sum_{i=1}^{j-1} \gamma_{ji}^k b^k (t_n + \mu_i \Delta, \eta_i) \quad \text{for}$$

 $j = 2, ..., s, \alpha_j, \mu_j, \beta_j^*, \lambda_{ji}$ and γ_{ji}^* are numerical constants, the $\hat{W}_{(i)}$ are Brownian motion and *R* is an appropriate

term. The scalar case for which d = m = 1 and s = 2 is considered in Eq. (17) to obtain the second-order two-

stage stochastic Runge-Kutta scheme, thus obtaining

$$\bar{x}_{n+1} = (18)$$

$$\bar{x}_n + \left[\alpha_1 a(t_n, \bar{x}_n) + \alpha_2 a(t_n + \mu \Delta, \bar{x}_n + \lambda a(t_n, \bar{x}_n) \Delta + \gamma b(t_n, \bar{x}_n) \Delta \hat{W}_n)\right] \Delta + \left[\beta_1 b(t_n, \bar{x}_n) + \beta_2 b(t_n + \mu \Delta, \bar{x}_n + \lambda a(t_n, \bar{x}_n) \Delta + \gamma b(t_n, \bar{x}_n) \Delta \hat{W}_n)\right] \Delta \hat{W}_n + R$$

function

The

$$a(t_n + \mu\Delta, \overline{X}_n + \lambda a(t_n, \overline{X}_n)\Delta + \gamma b(t_n, \overline{X}_n)\Delta \hat{W}_n)$$

expansion, which is defined in the following way

 $a(t_n + \mu\Delta, X_n + \lambda a(t_n, X_n)\Delta + \gamma b(t_n, X_n)\Delta W_n)$ is obtained from the truncated second-order Taylor

$$f(t + \Delta, X_{t} + \Delta X)^{(2)} =$$

$$f + f_{10}\Delta + f_{01}\Delta X + f_{02}\frac{(\Delta X)^{2}}{2} + \left[f_{11} + \frac{1}{2}b^{2}(t, X_{t})f_{03}\right]\Delta(\Delta X) +$$

$$\left[f_{20} + b^{2}(t, X_{t})f_{12} + b^{3}(t, X_{t})b_{01}(t, X_{t})f_{03} + \frac{1}{4}b^{4}(t, X_{t})f_{04}\right]\frac{\Delta}{2}$$
(19)

Where $\stackrel{(2)}{=}$ refers to the third order approximation of the function, and in order to simplify the writing $f_{ij} = \frac{\partial^{i+j} f(t,x)}{\partial t^i \partial x^j}$. For the second order approximation,





the combinations of Brownian products given by $\Delta^{i} \left(\Delta \hat{W}_{n} \right)^{j}$ with i = j = 1, 2, ..., n are given by:

$$\Delta \left(\Delta \hat{W}_{n}\right)^{i} \left(\Delta \hat{W}_{n}\right)^{j} \stackrel{(2)}{=} \begin{cases} \Delta^{2} & \text{if } i \neq j \\ 0 & \text{if } i = j \end{cases}$$

$$\left(\Delta \hat{W}_{n}\right)^{i} \left(\Delta \hat{W}_{n}\right)^{j} \left(\Delta \hat{W}_{n}\right)^{k} \stackrel{(2)}{=} \begin{cases} 3\Delta \left(\Delta \hat{W}_{n}\right)^{i} & \text{if } i = j = k \\ \Delta \left(\Delta \hat{W}_{n}\right)^{i} & \text{if } j = k \neq i \end{cases}$$

$$\Delta^{i} \left(\Delta \hat{W}_{n}^{1}\right)^{j_{1}} \left(\Delta \hat{W}_{n}^{2}\right)^{j_{2}} \dots \left(\Delta \hat{W}_{n}^{m}\right)^{j_{m}} \stackrel{(2)}{=} 0 & \text{if } i + \frac{j_{1} + \dots + j_{m}}{2} \geq \frac{5}{2}$$

$$(20)$$

The approximations of Eq. (20) are implemented in Eq. (19) to obtain

$$a\left(t_{n}+\mu\Delta, \overline{X}_{n}+\lambda a\left(t_{n}, \overline{X}_{n}\right)\Delta+\gamma b\left(t_{n}, \overline{X}_{n}\right)\Delta\hat{W}_{n}\right) \quad \text{and} \\ b\left(t+\mu\Delta, \overline{X}_{n}+\lambda a\left(t_{n}, \overline{X}_{n}\right)\Delta+\gamma b\left(t_{n}, \overline{X}_{n}\right)\Delta\hat{W}\right), \\ \text{which produces:}$$

$$a(t_{n} + \mu\Delta, \overline{X}_{n} + \lambda a(t_{n}, \overline{X}_{n})\Delta + \gamma b(t_{n}, \overline{X}_{n})\Delta \hat{W}_{n})\Delta \stackrel{(2)}{=} a(t_{n}, \overline{X}_{n})\Delta + a_{10}(t_{n}, \overline{X}_{n})\mu\Delta + a(t_{n}, \overline{X}_{n})a_{01}(t_{n}, \overline{X}_{n})\lambda\Delta^{2} + b(t_{n}, \overline{X}_{n})a_{01}(t_{n}, \overline{X}_{n})\gamma\Delta (\Delta \hat{W}_{n}) + \frac{1}{2}b^{2}(t_{n}, \overline{X}_{n})a_{02}(t_{n}, \overline{X}_{n})\gamma^{2}\Delta^{2}$$

$$(21)$$

$$b(t_{n} + \mu\Delta, \bar{X}_{n} + \lambda a(t_{n}, \bar{X}_{n})\Delta + \gamma b(t_{n}, \bar{X}_{n})\Delta \hat{W}_{n})\Delta \stackrel{(2)}{=} \\ b(t_{n}, \bar{X}_{n})\Delta \hat{W}_{n} + b(t_{n}, \bar{X}_{n})b_{01}(t_{n}, \bar{X}_{n})\Delta (\Delta \hat{W}_{n})^{2} + \\ b_{10}(t_{n}, \bar{X}_{n})\mu\Delta (\Delta \hat{W}_{n}) + a(t_{n}, \bar{X}_{n})b_{01}(t_{n}, \bar{X}_{n})\lambda\Delta^{2}(\Delta \hat{W}_{n}) + \\ \frac{3}{2}b_{02}(t_{n}, \bar{X}_{n})b^{2}(t_{n}, \bar{X}_{n})\gamma^{2}\Delta (\Delta \hat{W}_{n}) + \\ b(t_{n}, \bar{X}_{n}) \Big\{ b_{11}(t_{n}, \bar{X}_{n}) + \frac{1}{2}b_{03}(t_{n}, \bar{X}_{n})b^{2}(t_{n}, \bar{X}_{n}) \Big\} \mu\gamma\Delta^{2} + \\ a(t_{n}, \bar{X}_{n})b(t_{n}, \bar{X}_{n})b_{02}(t_{n}, \bar{X}_{n})\lambda\gamma\Delta^{2}$$

The functions given in Eq. (21) and Eq. (22) are replaced into equation (18) to obtain

$$\begin{split} \bar{x}_{n+1} &= \bar{x}_n + (\alpha_1 + \alpha_2) a(t, \bar{x}_n) \Delta + (\beta_1 + \beta_2) b(t, \bar{x}_n) \Delta \hat{w}_n + \mu \alpha_2 a_{10}(t, \bar{x}_n) \Delta^2 + \\ & \left[\lambda \alpha_2 a(t, \bar{x}_n) a_{01}(t, \bar{x}_n) + \gamma \lambda \beta_2 a(t, \bar{x}_n) b(t, \bar{x}_n) b_{02}(t, \bar{x}_n) \right] \Delta^2 + \\ & \left[\frac{1}{2} \gamma^2 \alpha_2 b^2(t, \bar{x}_n) a_{02}(t, \bar{x}_n) + \gamma \mu \beta_2 b(t, \bar{x}_n) \left(b_{11}(t, \bar{x}_n) + \frac{1}{2} b^2(t, \bar{x}_n) b_{03}(t, \bar{x}_n) \right) \right] \Delta^2 \\ & + \beta_2 \gamma b(t, \bar{x}_n) b_{01}(t, \bar{x}_n) (\Delta \hat{w}_n)^2 + R \end{split}$$

$$\begin{aligned} (23)$$

The scheme found in Eq. (23) must be equivalent to a simplified order-two Taylor scheme [23]

$$\begin{split} \bar{x}_{n+1} &= \bar{x}_n + a(t,\bar{x}_n)\Delta + b(t,\bar{x}_n)\Delta \hat{w}_n + \frac{1}{2}b(t,\bar{x}_n)\frac{\partial b(t,\bar{x}_n)}{\partial x} \Big[\left(\Delta \hat{w}_n\right)^2 - \Delta \Big] + \\ \frac{1}{2} \Big[\frac{\partial b(t,\bar{x}_n)}{\partial t} + a(t,\bar{x}_n)\frac{\partial b(t,\bar{x}_n)}{\partial x} + b(t,\bar{x}_n)\frac{\partial a(t,\bar{x}_n)}{\partial x} + \frac{1}{2}b^2(t,\bar{x}_n)\frac{\partial^2 b(t,\bar{x}_n)}{\partial x^2}\Delta \left(\Delta \hat{w}_n\right) \Big] \Delta \left(\Delta \hat{w}_n\right) \\ &+ \Big[\frac{\partial a(t,\bar{x}_n)}{\partial t}\Delta^2 + a(t,\bar{x}_n)\frac{\partial a(t,\bar{x}_n)}{\partial x} + \frac{1}{2}b^2(t,\bar{x}_n)\frac{\partial^2 a(t,\bar{x}_n)}{\partial x^2} \Big] \Delta^2 \end{split}$$

Then, in order to equate the scheme found in Eq. (23) to the scheme given in Eq. (24) the following conditions must be met

$$\alpha_{1} + \alpha_{2} = 1 \qquad \lambda \alpha_{1} = \frac{1}{2} \qquad \mu \alpha_{2} = \frac{1}{2} \qquad \lambda \beta_{2} = \frac{1}{2} \qquad \gamma^{2} \alpha_{2} = \frac{1}{2}$$

$$\beta_{1} + \beta_{2} = 1 \qquad \gamma \alpha_{2} = \frac{1}{2} \qquad \mu \beta_{2} = \frac{1}{2} \qquad \gamma \beta_{2} = \frac{1}{2} \qquad R = \frac{1}{2} b(t, \bar{X}_{n}) b_{01}(t, \bar{X}_{n}) \Delta$$

$$(25)$$

The system has a unique solution when $\alpha = \alpha = \frac{1}{2}$, $\gamma = \mu = \lambda = 1$ and $\beta_1 = \beta_2 = \frac{1}{2}$. The parameters in Eq. (25) are replaced into Eq. (24) to obtain the two-stage second-order stochastic Ruge-Kutta method (RK2-2st)

$$\begin{split} \bar{X}_{n+1} &= \\ \bar{X}_n + \frac{1}{2} \Big[a(t_n, \bar{X}_n) + a(t_n + \Delta, \bar{X}_n + a(t_n, \bar{X}_n) \Delta + b(t_n, \bar{X}_n) \Delta \hat{W}_n) \Big] \Delta + \quad (26) \\ \frac{1}{2} \Big[b(t_n, \bar{X}_n) + b(t_n + \Delta, \bar{X}_n + a(t_n, \bar{X}_n) \Delta + b(t_n, \bar{X}_n) \Delta \hat{W}_n) \Big] \Delta \hat{W}_n - \\ \frac{1}{2} b(t_n, \bar{X}_n) \frac{\partial b(t_n, \bar{X}_n)}{\partial x} \Delta \end{split}$$

The Taylor scheme of order two presented by Eq. (24) has ten derivatives while the two-stage second-order stochastic Ruge-Kutta method given in Eq. (26) presents only one derivative, making the RK2-2st method an easy method to implement.

RESULTS

In this section, the accuracy of the proposed method (RK2-2st) is analysed to study the dynamics of nuclear reactors with temperature feedback effects. Solutions will be given for the stochastic equations of point kinetics -given by Eq. (1)- and the reactivity temperature for the step change with temperature feedback given by Eq. (16).

All results started from the initial conditions (at

 $t_0 = 0$) and with $n_0 = 1 n/cm^3$ and $C_{i(0)} = \frac{\beta_i n_{(0)}}{\lambda_i \Lambda}$. The parameters of the delayed neutron precursor groups correspond to a graphite reactor U^{235} , which are as follows: $\lambda_1 = 0.0124 s^{-1}$, $\lambda_2 = 0.0305 s^{-1}$, $\lambda_3 = 0.111 s^{-1}$, $\lambda_4 = 0.301 s^{-1}$, $\lambda_5 = 1.13 s^{-1}$, $\lambda_6 = 3.0 s^{-1}$, $\beta_1 = 0.00021$, $\beta_2 = 0.00141$, $\beta_3 = 0.00127$, $\beta_4 = 0.00255$, $\beta_5 = 0.00074$, $\beta_6 = 0.00027$, $\beta = 0.00645$, $\Lambda = 5.0 \times 10^{-5} s$ for a fixed step size $h = 10^{-3}$.

In order to solve the equations of point kinetics, initial reactivity steps of 0.5 \$, 0.75 \$, 1.0 \$ and 1.5 \$ were considered. It is assumed in Eq. (16) that the Doppler temperature reactivity coefficient is $\alpha_{T_f}^D = 5*10^{-5} K^{-1}$ and $H = 0.05 \ K \ cm^3 \ s^{-1}$. The peak of the neutron density

and H = 0.05 K cm s. The peak of the neutron density and the time in which it occurs, calculated by the RK2-2st method, are presented in Table-1. The values obtained



with the proposed method for the initial steps of reactivity of 0.5 \$, 0.75 \$ and 1.0 \$ are compared to the Split-Step Forward Method (EMM), the Derivate-Free Milstein Method (DFMM) [13] and the Strong-Order Taylor scheme (1.5) [20]. It is observed that the results obtained for these three reactivities are in accordance with the methods reported in the literature. Additionally, for the stochastic case, for initial step reactivities 1.5 \$ and 2.0 \$, the neutron density peaks and the times in which they occur are presented for the first time. It is confirmed that the RK2-2st method is of good precision and therefore allows the study of neutron densities for different initial step reactivities.

Table-1. Maximum peak of neutron density and its corresponding time for initial reactivity.

00	Split-Step Forward EMM		DFMM		Strong-Order Taylor (1.5)		RK2-2st	
P_0	Maximum	Time to peak	Maximum	Time to peak	Maximum	Time to peak	Maximum	Time to peak
0.5\$	46.4939	28.3400	46.2606	27.8400	55.6596	22.41	46.3310	28.42
0.75\$	163.707	8.795	164.22	8.75	185.44	7.1725	163.8654	8.757
1.0\$	760.589	1.065	769.238	1.0575	1192.48	0.725	808.9194	0.956
1.5\$	-	-	-	-	-	-	43921.23688	0.169

Figures 1 a) to d) show the variation of the neutron density as a function of time for five initial step reactivities, two different sample densities (blue and purple lines) and the average density (red line) are considered. Figures 2 a) to d) show the reactivity as a function of time for different values of initial step

reactivities. The fluctuations in density are close to the average values, indicating that the RK2-2st method has good precision. Tables 2-5 show the neutron density as a function of time for initial step reactivities of 0.5 \$, \$ 0.75 \$ 1.0 \$ and 1.5 \$, respectively.



Figure-1. Variation of neutron density with time for several initial step reactivities ρ_0

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Figure-2. Reactivity as a function of time for several initial step reactivities ρ_0

Table-2. Mean values and standard deviation of neutron density, the sum of the precursor group	oups and
reactivity for initial step reactivity $\rho_0 = 0.5$ \$.	

t(s)	$E[n_{(t)}]$	$\sigma[n_{(t)}]$	$E[c_{i(t)}]$	$\sigma [c_{i(t)}]$	$ ho_{\scriptscriptstyle (t)}$
0	1.0000E+00	-	1.6765E+03	-	5.0000E-01
10	1.3144E+01	3.1857E+00	5.7413E+03	5.5270E+02	4.7300E-01
20	3.6586E+01	3.6976E+00	1.9282E+04	1.3526E+03	3.7275E-01
30	4.5607E+01	3.6158E+00	3.5762E+04	1.0390E+03	2.0482E-01
40	3.6807E+01	4.0463E+00	4.2825E+04	4.1272E+02	4.5751E-02
50	2.7220E+01	3.7539E+00	4.2337E+04	3.2112E+02	-7.5248E-02
60	1.9668E+01	3.9698E+00	3.8773E+04	3.7454E+02	-1.6348E-01
70	1.5056E+01	3.9740E+00	3.4373E+04	3.8386E+02	-2.2818E-01
80	1.1688E+01	4.3015E+00	3.0026E+04	3.7656E+02	-2.7845E-01

t(s)	$E[n_{(t)}]$	$\sigma[n_{(t)}]$	$E[c_{i(t)}]$	$\sigma [c_{i(t)}]$	$ ho_{\scriptscriptstyle (t)}$
0	1.0000E+00	-	1.6765E+03	-	7.5000E-01
10	1.5968E+02	4.8257E+00	6.8714E+04	1.0692E+03	3.9552E-01
20	7.9184E+01	4.6208E+00	9.4653E+04	3.2421E+02	-5.8085E-02
30	4.1640E+01	4.3205E+00	8.2153E+04	3.8215E+02	-2.7914E-01
40	2.5114E+01	4.4947E+00	6.7817E+04	3.2899E+02	-4.0355E-01
50	1.7243E+01	4.3066E+00	5.5800E+04	2.8667E+02	-4.8363E-01
60	1.2248E+01	4.4664E+00	4.6076E+04	2.5049E+02	-5.3971E-01
70	9.4991E+00	4.5487E+00	3.8183E+04	2.2982E+02	-5.8070E-01
80	7.4642E+00	4.9200E+00	3.1745E+04	2.1533E+02	-6.1274E-01

Table-3. Mean values and standard deviation of neutron density, the sum of the precursor groups and reactivity for initial step reactivity $\rho_0 = 0.75$ \$.

Table-4. Mean values and standard deviation of neutron density, the sum of the precursor groups and reactivity for initial step reactivity $\rho_0 = 1.0$ \$.

t(s)	$E[n_{(t)}]$	$\sigma[n_{(t)}]$	$E[c_{i(t)}]$	$\sigma ig[c_{i(t)} ig]$	$ ho_{\scriptscriptstyle (t)}$
0	1.0000E+00	-	1.6765E+03	-	1.0000E+00
10	1.3205E+02	5.5307E+00	1.6222E+05	3.7012E+02	-1.8570E-01
20	5.1283E+01	4.9185E+00	1.2539E+05	3.2699E+02	-5.0618E-01
30	2.8325E+01	4.7620E+00	9.6985E+04	3.2079E+02	-6.5286E-01
40	1.7920E+01	4.9321E+00	7.6680E+04	2.7424E+02	-7.3990E-01
50	1.2810E+01	4.8197E+00	6.1536E+04	2.4849E+02	-7.9830E-01
60	9.2661E+00	4.9347E+00	4.9870E+04	2.2075E+02	-8.4047E-01
70	7.3139E+00	5.0888E+00	4.0681E+04	2.0693E+02	-8.7185E-01
80	5.8433E+00	5.4162E+00	3.3355E+04	1.9305E+02	-8.9661E-01

Table-5. Mean values and standard deviation of neutron density, the sum of the precursor groups and reactivity for initial step reactivity $\rho_0 = 1.5$ \$.

t(s)	$E[n_{(t)}]$	$\sigma[n_{(t)}]$	$E[c_{i(t)}]$	$\sigma [c_{i(t)}]$	$ ho_{\scriptscriptstyle (t)}$
0	1.0000E+00	-	1.6765E+03	-	1.5000E+00
10	1.0725E+02	6.1961E+00	2.6170E+05	4.3232E+02	-8.4482E-01
20	4.0841E+01	5.7079E+00	1.7906E+05	3.4750E+02	-1.1008E+00
30	2.3362E+01	5.5885E+00	1.3305E+05	3.3948E+02	-1.2196E+00
40	1.5004E+01	5.7620E+00	1.0269E+05	2.9237E+02	-1.2920E+00
50	1.0838E+01	5.7493E+00	8.0861E+04	2.6619E+02	-1.3412E+00
60	7.8400E+00	5.7767E+00	6.4455E+04	2.3608E+02	-1.3770E+00
70	6.1917E+00	6.0754E+00	5.1798E+04	2.1949E+02	-1.4037E+00
80	5.0417E+00	6.2708E+00	4.1894E+04	2.0111E+02	-1.4247E+00

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CONCLUSIONS

The Second-Order Two-Stage Stochastic Runge-Kutta Method is a method with high precision which is adequate to solve the stochastic equations of point kinetics of a nuclear reactor with temperature feedback effects. The temperature feedback is included in the reactivity as a function of time for different types of inputs, in this work a step change with adiabatic temperature for the reactivity inputs was implemented. The RK2-2st method is found to be a good alternative to study nuclear reactors with Newtonian temperature feedback effects. In addition, the RK2-2st method is easy to implement and has a low computational cost because it requires the calculation of very few derivatives in the random functions.

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