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PID CONTROLLER APPLIED TO THE SYSTEM BALL AND PLATE CE 151

Faiber Robayo Betancourt[1], Freddy Humberto Escobar[2] and Fredy H. Martínez S.[3]
[1]Departamento de Ingeniería Electrónica, Facultad de Ingeniería, Universidad Surcolombiana, Neiva, Huila, Colombia
[2]CENIGAA, Facultad de Ingeniería, Universidad Surcolombiana, Neiva, Huila, Colombia
[3]Facultad Tecnológica, Universidad Francisco José de Caldas, Bogotá D. C., Colombia
E-Mail: faiber.robayo@usco.edu.co

ABSTRACT
This paper describes the design and implementation of a PID controller applied to the Ball and Plate CE 151 system. The design technique used is Root Locus Method in discrete time and the controller is implemented in MatLab. Because the system has an unstable dynamic, multiple PID controllers were designed for different operating points and using the Root Mean Square Error (RMSE), the best PID controller is selected. This controller is subjected to other performance tests such as, reference tracking, disturbance rejection and robustness test to observe the behavior of the controller.

Keywords: PID controller, MatLab, ball and plate, performance, robustness.

1. INTRODUCTION

Currently, control systems are part of the everyday life of human beings, the purpose of a control system is achieved by manipulating variables control a domain on the output variables so that these become desired values. There are a great variety of control systems applied to different processes such as temperature systems, level, pressure and position. In most cases, so the control system performs its function using a controller, usually a PID controller is necessary. It is well known that the PID control topologies are widely used due to its good performance and robustness, in fact, 95% of the control processes in the industry are PID. (Astrom & Hagglund, 2009).

The "Ball and Plate CE 151" system is characterized as inherently unstable and higher order which makes it attractive for the design of control systems. Many researches have been carried out with this system as described next. The nonlinear model mathematically plant is implemented in Simulink (Sink, 2014). Fuzzy rules MatLab model are implemented, likewise, tests are performed from the simulator and the actual plant where favorable results are obtained.

Control PID techniques and fuzzy logic is applied to a multivariable plant and ball platform, characterized in that nonlinear and inherently unstable (Penco & Modesti, 2016). Mathematical modeling of the plant is carried out; the programming language used is MatLab and Simulink. Algorithms are subjected to various tests as circular trajectory tracking; it is concluded that the fuzzy controller obtains superior results to those obtained with the classic PID controller.

The linear quadratic Control LQ for 2DOF (degrees of freedom) controller structure is applied, using spectral factorization. As a result, an almost optimal and relatively robust controller capable of providing a good reference tracking and disturbance rejection is implemented. In this paper it is shown that the LQ controller is very suitable for this model. (ŠPAČEK, 2016).

The stabilization problem of the Ball and Plate system is studied using different methods developed to control infra-driven Euler-Lagrange for (EL) systems. The model of the system approximately ball and plate is constructed first with two separate systems and then each system approaches mixing some systems. (Alpaslan & Goren-Sumer, 2017).

Building a platform Ball and low cost plate is developed (Polania, 2016). This platform allows you to experiment and see the response of the drivers currently implemented in industrial processes modeling gathering, analysis and control in real time and thus obtaining a validation of the drivers. Controlled PID (Proportional, Integral, Derivative) are performed.

The objective of this project is to design and implement a PID controller applied to the Ball and Plate CE 151 system in order to evaluate its performance. As a future work is proposed to investigate about both classical and modern control methods to compare the results obtained in this proposal.

2. MATERIALS AND METHODS

2.1 Ball and Plate System CE 151

Ball and Plate System CE 151 was developed and manufactured by a company dedicated to the research and building control systems, technical computing and model-based design as shown in Figure-1 (HUMUSOF, 2019).

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Figure-1. Ball and plate system CE 151.

The design consists of a plate of 40 cm x 40 cm on a pivot with two degrees of freedom, which are driven by two stepper motors and a series of pulleys; each engine has a resolution of 0.001 degrees to reach a smooth and precise movement.

The system has a direct connection to a CPU via 38 pin cable and an acquisition data card PCI MF624; the position of the ball is obtained by USB camera and image segmentation.

2.2 Mathematical Model

To simplify the mathematical model, following assumptions are made:

- The contact between the ball and the plate is not lost at any time.
- The ball rolls on the plate but does not slide.
- All friction forces between the ball and the plate are disregarded.

That will be the symbols used in the mathematical model in Table-1 are described.

Table-1. Variable notation.

Variable	Units	Description
x	m	Ball position in X axis
$\dot{x}$	m/s	Ball speed in X axis
$\ddot{x}$	m/s^2	Ball acceleration in X axis
y	m	Ball position in X axis
$\dot{y}$	m/s	Ball speed in X axis
$\ddot{y}$	m/s^2	Ball acceleration in X axis
θx	rad	Plate inclination in X axis
θy	rad	Plate inclination in Y axis
$\dot{\theta}$	rad/s	Angular velocity vector of rotating ball
m	kg	Ball mass
R	m	Ball radius
Ib	kgm^2	Moment of inertia of a ball, Ib=2/5 mR^2
$\dot{s}$	m/s	Linear velocity due to rotation
h	m	Ball height
g	m/s^2	Gravity force, g = 9.81m/s^2

For the mathematical model, Lagrangian mechanics is used, which provides a formulation of the dynamic equations of motion equivalent to those derived using Newton's Second Law method; however, the Lagrangian approach is favorable for more complex systems (Spong, *et al.,* 2009). The difference between the kinetic energy (T) and the potential energy (V) is known as Langrangian (equation 1).

$$L = T - V \tag{1}$$

The kinetic and potential energy are given by equation 2 and 3 respectively.

$$T = \frac{1}{2}\left[Ib\left(\frac{V}{R}\right)^2 + mv^2 + Ib\dot{\theta}^2 + m\dot{s}^2\right] \tag{2}$$

$$V = -mgh \tag{3}$$

Replacing equations 2 and 3 in equation 1 results in equation 4.

$$L = \frac{1}{2}Ib(\dot{\theta}^2{}_x + \dot{\theta}^2{}_y + \frac{\dot{x}^2}{R^2} + \frac{\dot{y}^2}{R^2} + \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 + \left(x\dot{\theta}_x + y\dot{\theta}_y\right)^2 + mg\left(x\sin(\theta_x) + y\sin(\theta_y)\right)\right) \tag{4}$$

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Including equation 5, known as the Euler-Lagrange equation:

$$\frac{d}{dx}\left(\frac{\partial L}{\partial qi}\right) - \frac{\partial L}{\partial qi} = Qi \tag{5}$$

Now, equation 4 can be substituted in equation 5 and developing the equation are obtained 6 and 7.

$$\ddot{x} = \frac{mx\dot{\theta}^2{}_x + my\dot{\theta}_x\dot{\theta}_y + mg\sin(\theta_x)}{\left(\frac{Ib}{R^2} + m\right)} \tag{6}$$

$$\ddot{y} = \frac{my\dot{\theta}^2{}_y + mx\dot{\theta}_x\dot{\theta}_y + mg\sin(\theta_y)}{\left(\frac{Ib}{R^2} + m\right)} \tag{7}$$

Equations 6 and 7 present a multivariable and non-linear dynamics; if the aim is obtaining a linear mathematical model it is always necessary to ignore certain non-linearities and distributed parameters that may be presented in the dynamic system (Ogata, 2010); taking into account that, the plate inclination is between ± 0.139626 radians and the plate movements are slow to stabilize the ball, the following approximations can be applied:

$$\sin(\theta) \cong \theta;\ \dot{\theta}_x \cong 0; \dot{\theta}_y \cong 0 \tag{8}$$

Applying 8 to 6 and 7, it is reduced to:

$$\ddot{x} = \frac{mg\theta_x}{\left(\frac{Ib}{R^2} + m\right)};\ \ddot{y} = \frac{mg\theta_y}{\left(\frac{Ib}{R^2} + m\right)} \tag{9}$$

Replacing the gravity (g) and the inertia of the ball (Ib) in equation 9:

$$\ddot{x} = 7\theta_x\ ;\ \ddot{y} = 7\theta_y \tag{10}$$

Finally, the Laplace transform is applied to 10.

$$Gs(x) = \frac{7}{s^2};\qquad Gs(y) = \frac{7}{s^2} \tag{11}$$

2.3 Mathematical Model Validation

The operating point chosen for the design of the PID controller is 0.06981 radians as the inclination angle of the plate; this operating point is in the middle of the inclination range that the plate can take. In order to have a closer mathematical model to the real dynamic of system, the transfer function is adjusted, for this, data of the ball position is taken with the plate inclined 0.06981 radians. Initially, the adjustment of the transfer function gain is made; with this the mathematical model gets nearer to the ball position data, however, there is still a difference between the dynamics, therefore, a filter is added. This filter provides the effects of friction that were previously ignored; Figure-2 summarizes this process.

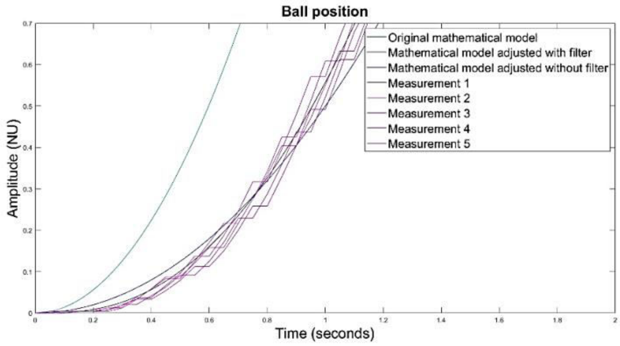

Figure-2. Mathematical model adjustment.

Thus, the transfer function with the respective adjustments is shown in equation 12.

$$Gp(s) = \frac{3.1}{0.2s^3 + s^2} \tag{12}$$

2.4 Design of the PID Controller

The PID controller is the sum of a proportional, integral and derivative action (Ogata, 2010); each of these actions play a different role in the control algorithm, therefore, has advantages and disadvantages, which are presented in Table-2.

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Table-2. Advantages and disadvantages of proportional, integral and derivative actions.

Action	Advantage	Disadvantage
Proportional	Speed increases and the system response reduces the error of the system steady.	Increasing instability of the system.
Integral	Decreases the steady-state error, plus It provides robustness and minimizes the presence of noise in the system response.	Increasing instability of the system and decreases the response speed system.
derivatively	Increases the stability of the system and the system response speed.	The steady-state error remains the same.

From the equation 12, the transfer function is given in continuous time, then it must be discretized for the design of the controller (Ogata, 1996). Because the technique chosen for the design is in discrete time, a time of 0.15 seconds is selected; this time show does not compromise the operation of the CPU and enables the operation of the algorithm. The transfer function in discrete time is shown in equation 13.

$$Gp(z) = \frac{0.007302z^2 + 0.02448z + 0.005063}{z^3 - 2.472z^2 + 1.945z - 0.4724} \quad (13)$$

The Root Locus diagram of the transfer function in discrete time is shown in Figure-3. This figure shows the position of the poles and zeros of the system.

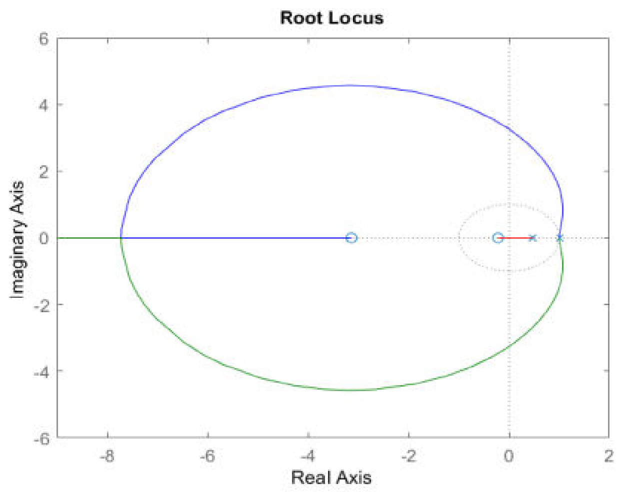

Figure-3. Root locus of the transfer function in discrete time.

Noting Figure-3, which contains the Root Locus of the transfer function in discrete time, the existence of any pole outside the unit circle is not appreciated; However, Table-3 shows the exact location of the poles and zeros. It is observed the existence of a pair of complex conjugate poles located outside the unit circle causing the instability of the system.

Table-3. Location of poles and zeros.

Roots	Number of items	Location
Ceros	1	-3.1311
	1	-0.2214
Polos	1	0.4717
	2	$1.001 \pm 0.337i$

The structure of the PID controller is shown in Figure-4. Because the dynamics of the system is unstable is not available with design parameters such as settling time or overshoot, therefore, the design is made of several controllers and determine constants Kp, Ki and Kd for each design, as shown in Table-4.

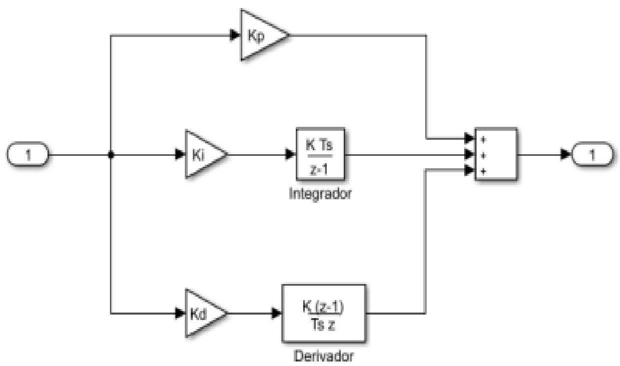

Figure-4. Structure of the PID controller.

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Table-4. Values constants Kp, Ki and Kd controllers designed.

PID	*Kp*	*Ki*	*Kd*
1	0.1809	0.0026	1.5349
2	0.3243	0.0065	1.9571
3	0.7520	0.0244	2.7112
4	3.2002	0.2632	4.096
5	0.1656	0.0023	1.4364
6	0.2937	0.0056	1.8452
7	0.6595	0.0202	2.5268
8	2.6242	0.1943	3.7993
9	0.1477	0.0020	1.3504
10	0.2708	0.005	1.7606
11	0.5964	0.0174	2.4034
12	2.2352	0.1519	3.5716

3. RESULTS AND DISCUSSIONS

3.1 Selecting the Best PID Controller

PID controllers designed are tested for reference tracking to determine the best performing; to achieve this, the Root Mean Square Error (RMSE) is performed and the results are shown in Table-5.

Table-5. RMSE values of the response of the twelve controllers designed.

PID	Eje X	Eje Y
1	0.2431	0.2445
2	0.0858	0.0813
3	0.0753	0.0639
4	0.2130	0.2297
5	0.2978	0.2505
6	0.1636	0.1260
7	0.0593	0.0615
8	0.345	0.334
9	0.3063	0.1547
10	0.1583	0.1767
11	0.0616	0.0711
12	0.3040	0.3120

According to the results of Table-5, it is possible to determine the PID controller 7 is the best controller designed, as has lower RMSE in its response.

3.2 Performance Controller

In order to determine the overall performance of the PID controller, it is subjected to various tests as reference track, disturbance rejection and robustness tests. To test track reference, a random signal is inserted to the input of the system and the response of the system in an operating point is recorded as shown in Figure-5.

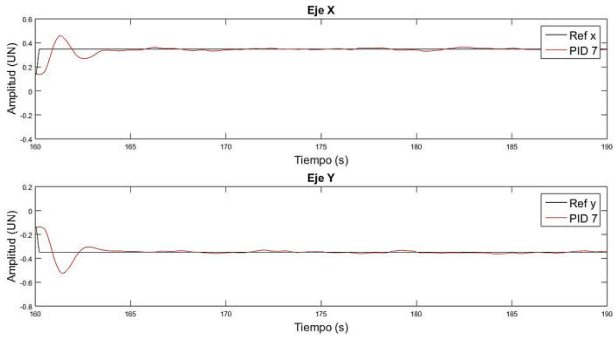

Figure-5. Monitoring the PID reference 7.

Figure-5 it is seen that the controller has 42,045% 54,796% and overshoot in X and Y axis respectively; On the other hand, the settling time is 3.85 and 2.75 seconds respectively. It is noted that the controller fulfills its main objective to stabilize the ball in the least possible time using less overshoot.

Test disturbance rejection involves introducing a known signal between the controller and the system to observe the behavior of the PID controller and the time it takes the controller to reject the disturbance.

The behavior of the ball to introduce a disturbance in the system is shown in Figure-6. It is appreciated that the maximum amplitude attained by the ball in the X and Y axes is 0.4312 and 0.4632 units also the time it takes for the controller rejecting the disturbance is 4.1 and 3.9 seconds.

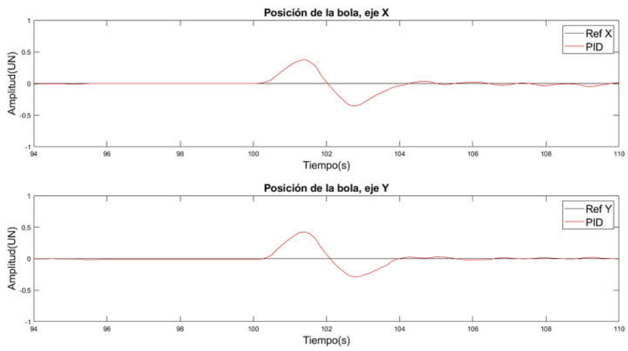

Figure-6. Disturbance rejection PID.

Finally, the robustness test is performed; for this test the ball change is performed by a ball with greater mass, this experiment represents a change in the dynamics of the system and provides an estimate of the behavior of the controller to a future change in the system. In Table-6, the RMSE values of the system response in the X and Y axis are displayed using the original ball and the heaviest ball.

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Table-6. RMSE values of the system response using the original ball and the heaviest ball.

	Original ball	Ball greater weight
X axis	.0551	.0645
Axis y	.0569	.0705

With the change of ball RMSE values of the response of the system suffered an increase of 17.05% and 23.90% in the X and Y axes respectively, despite the increased robustness of the controller, given that the second ball is much heavier than the original ball.

4. CONCLUSIONS

The results supplied by the PID controller show the good performance of this. The controller proved its ability to stabilize the position of the ball to follow a random reference point using overshoot and settling time within normal ranges for such systems. Furthermore, the controller showed to be able to reject the disturbance introduced using a time considerably low.

One of the characteristics of PID controllers is their high degree of robustness; the PID controller designed proved to be quite robust to the increments shown considerably lowered in the RMSE values of the response of the system when the change was made from the original ball to one of greater mass. Due to these results it is concluded that the technique of PID control is useful to use in systems with such dynamics, this confirms the great reception that the PID controller in control systems presents.

REFERENCES

Alpaslan Y. and Goren-Sumer L. 2017. Stabilizing of Ball and Plate System Using an Approximate Model. pp. 9601-9606.

Åström K., y Hägglund T. 2009. Control PID Avanzado. Pearson educación, vol. 1. Madrid, España. p. 488.

HUMUSOFT. Consultado el 9 de abril de 2019. [Online]. Available: https://www.humusoft.cz.

Hunde A. 2013. Design of Fuzzy Sliding Mode Controller for The Ball and Plate System. Trabajo de investigación (Magister de ciencias en Ingeniería Electrónica). Etiopía. p. 105.

MATHWORKS. Consultado el 8 de abril de 2019. [Online]. Available: https://la.mathworks.com.

Ogata K. 1996. Sistemas de control en tiempo discreto. 2ª edición. México: Prentice Hall Hispanoamérica. p. 757. ISBN 968-880-539-4.

Ogata K. 2010. Ingeniería de Control Moderna. 5th ed. Madrid: Pearson Education. p. 11.

Penco J. and Modesti M. 2018. Control Difuso vs PID para un Sistema de Bola y Plataforma. Consultado el 29 de enero de 2019. Available: http://ria.utn.edu.ar/bitstream/handle/123456789/2562/Control%20difuso%20versus%20PID%20para%20un%20sistema%20de%20bola%20y%20plataforma.pdf?sequence=1&isAllowed=y.

Polania E. 2016. Diseño, Implementación y Control de Sistema de Balance Ball and Plate. SENNOVA. 2(2): 135-149.

Spaček L. 2016. Digital Control of CE 151 Ball and Plate Model. Zlin, Trabajo de investigación (Magister en control automático e informática). p. 71.

M. Spong, S. Hutchison and M. Vidyasagar. 2009. Robot Modeling and Control. 5th ed. Toronto. p. 200.