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STEP CHANGE REACTIVITY WITH TEMPERATURE FEEDBACK USING THE STOCHASTIC RK2-3ST METHOD

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ABSTRACT

In this article, we present the second order-third stage Runge-Kutta method (RK2-3st) as an alternative way to give solution to the stochastic point kinetics equations with Newtonian temperature feedback effects considering step change reactivity. We consider the Doppler effect to obtain an approximation to an adiabatic nuclear reactor. We realize diverse numerical experiments, which are obtained using study cases reported in the literature. We consider several initial values for the step reactivity to study the efficiency and precision of the method. We obtain several values for reactivity, mean values and standard deviation for the neutron population density and the concentration of delayed neutron precursors. The numerical results obtained with the RK2-3st method indicate that it is efficient, of low computational cost and easy to implement.

Keywords: Doppler effects, neutron density, Newtonian temperature feedback, Runge-Kutta stochastic method.

INTRODUCTION

One of the most important models in nuclear physics, that allow to know the dynamics of a nuclear reactor, are the point kinetic equations [1], which are a set of nonlinear ordinary differential equations, which are highly coupled. This set considers m groups of delayed neutron precursors which describe the behavior of the neutron density and the concentration density of the delayed neutron precursors [2]. The point kinetic equations contain time dependent parameters as reactivity; the neutron density and the concentration of delayed neutron precursors, with a random behavior along time [3-4].

Several investigators have focused their studies in modeling the nuclear reactor without considering stochastic effects, in which temperature feedback effects are considered. These models provide an estimation of the transitory behavior of the neutron density and the concentration of the delayed neutron precursors, which allows to have timely control of the nuclear reactor and at the same time guarantee the production of energy. Some work on this research are: Power series solutions (PWS) method [5], converged accelerated Taylor series (CATS) method [6], enhanced piecewise constant approximation (EPCA) method [7], ITS2 [8], the generalized Adams-Bashforth-Moulton Method [9] y the 8th-order Adams-Bashforth-Moulton (ABM8) method [10]. However, since the dynamics of a nuclear reactor are stochastic, the following methods give solutions to the point kinetics equation with temperature feedback equations: Backward Euler and Crank Nicholson approximations [11], Euler Maruyama method, Milstein free derivative method [12] and the strong Taylor 1.5 method [13].

Due to the point kinetic equations having no exact analytic solution, we propose the stochastic method of numerical derivation of the Runge-Kutta second order-third stage (RK2-3st) [14-15] as a method to solve numerically the point kinetic stochastic equations with temperature feedback effects when considering the step change reactivity.

In this article, we initially show the kinetic model of a point nuclear reactor with effects of temperature feedback, described by the stochastic point kinetic equations. Later, we propose the stochastic numerical derivation method RK2-3st to give solution to the dynamic system of a nuclear reactor with adiabatic approximation. In the result section, we consider several study cases which are available in the literature for several initial conditions of the step change reactivity. These numerical results are compared with other methods of the numerical solution so that we can study efficiency and precision of the proposed RK2-3st method.

THEORETICAL CONSIDERATIONS THE NONLINEAR STOCHASTIC POINT KINETIC EQUATIONS

The temporal behavior of the neutron flux in a nuclear reactor, as the neutron density and the delayed neutron precursor concentration, is described by the stochastic point kinetics equation [2-3]:

$$d\hat{x}(t) = \left[A\hat{x}(t) + F(t)\right]dt + B^{1/2}d\hat{W}(t) \quad (1)$$

where $\hat{x}(t)$ is the random variable vector given the following form:

$$\hat{x}(t) = \begin{bmatrix} n(t) \\ c_1(t) \\ c_2(t) \\ \vdots \\ c_m(t) \end{bmatrix} \quad (2)$$

Given the initial conditions

$$\hat{x}(0)=\begin{bmatrix} n(0) \\ c_1(0) \\ c_2(0) \\ \vdots \\ c_m(0) \end{bmatrix} \tag{3}$$

where $n(t)$ is the neutron density, $c_i(t)$ delayed neutron precursor density of the i-th group of precursors, m is the number of delayed neutron precursor groups and t is the time. A the matrix of the coefficient described by,

$$A=\begin{bmatrix} \frac{\rho(t)-\beta}{\Lambda} & \lambda_1 & \lambda_2 & \cdots & \lambda_m \\ \frac{\beta_1}{\Lambda} & -\lambda_1 & 0 & \cdots & 0 \\ \frac{\beta_2}{\Lambda} & 0 & -\lambda_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ \frac{\beta_m}{\Lambda} & 0 & 0 & \cdots & -\lambda_m \end{bmatrix} \tag{4}$$

where $\rho(t)$ is the reactivity, β_i is the fraction of delayed neutrons of the i-th precursor group, β is the total fraction of delayed neutrons, λ_i is the decay constant of the i-th precursor group, $F(t)$ is the vector of neutron source defined by,

$$F(t)=\begin{bmatrix} q(t) \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{5}$$

where $q_{(t)}$ is the external neutron source.

B is the covariance matrix with pulsed neutron approximation [16], given by,

$$B=\begin{bmatrix} \zeta^* & a_{1^*} & a_{2^*} & \cdots & a_{1^*} \\ a_{1^*} & b_{1,1^*} & b_{1,2^*} & \cdots & b_{1,m^*} \\ a_{2^*} & b_{2,1^*} & b_{2,2^*} & \cdots & b_{2,m^*} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{m^*} & b_{m,1^*} & b_{m,2^*} & \cdots & b_{m,m^*} \end{bmatrix} \tag{6}$$

where,

$$\zeta^*=\left[\frac{\rho(t)-\beta}{\Lambda}\right]\left(-1-\rho(t)+2\beta+(1-\beta)^2\upsilon\right)n(t)+\sum_{i=1}^{m}\lambda_i c_i(t)+q(t) \tag{7}$$

$$a_{i^*}=\beta_i\left((1-\beta)\upsilon-1\right)\left[\frac{\rho(t)-\beta}{\Lambda}\right]n(t)-\lambda_i c_i(t) \tag{8}$$

$$b_{i,j^*}=\beta_i\beta_j\left[\frac{\rho(t)-\beta}{\Lambda}\right]n(t)+\delta_{i,j}\lambda_i c_j(t) \tag{9}$$

Given υ is the average number of neutrons emitted per fission and $\delta_{i,j}$ is the Kronecker delta. Finally on equation (1), $\hat{W}(t)$ is the vector of the Wiener processes or Brownian motion defined the following way,

$$\hat{W}(t)=\begin{bmatrix} W_1(t) \\ W_2(t) \\ W_3(t) \\ \vdots \\ W_m(t) \end{bmatrix} \tag{10}$$

For the case of thermal reactors, the Doppler feedback is due primarily to epithermal capture resonances in the non-fissionable fuel isotopes as well as Th^{232}, U^{238}, and Pu^{240}. Reactivity due to the Doppler effect for the equilibrium point (ρ_0 and T_0) is described by [17]:

$$\rho(t)=\rho_0-\alpha_{T_f}^{D}\left(T_f(t)-T_0\right) \tag{11}$$

where $\alpha_{T_f}^{D}=\frac{\partial\rho}{\partial T_f}$, is the Doppler temperature coefficient and T_f is the fuel temperature.

For the Doppler feedback, the adiabatic approximation represents the most limiting case in the nuclear reactor in relation with a positive reactivity insertion, since it is considered as a hypothetical scenario with an emission of power with no heat elimination, and the Doppler effects generate a self-control of the reactor that is satisfactory at maintaining safety margins. The equation represents the step change with temperature feedback feeding along the time, which is given by:

$$d\rho(t)=-\alpha_{T_f}^{D}H\,n(t)dt \tag{12}$$

where $H=P_0/(\rho_f Cp_f)$ given P_0 the nominal potential, ρ_f the density and Cp_f the specific heat of the fuel.

The initial condition on equation (12) is subject to:

$$\rho(t_0)=\rho_0 \tag{13}$$

SECOND-ORDER TWO-STAGE STOCHASTIC RUNGE-KUTTA SCHEME

The stochastic scheme of Runge-Kutta of the s stage [14], is given by

$$\bar{X}_{n+1}=\bar{X}_n+\Delta\sum_{j=1}^{s}\alpha_j a\left(t_n+\mu_j\Delta,\eta_j\right)+\sum_{k=1}^{m}\Delta\hat{W}_n^k\sum_{j=1}^{s}\beta_j^k b^k\left(t_n+\mu_j\Delta,\mu_j\right)+R \quad (14)$$

Where,

$\mu_1=0$; $\eta_1=\bar{X}_n$;

$$\eta_j=\bar{X}_n+\Delta\sum_{i=1}^{j-1}\lambda_{ji}a\left(t_n+\mu_i\Delta,\eta_i\right)+\sum_{k=1}^{m}\Delta\hat{W}_n^k\sum_{i=1}^{j-1}\gamma_{ji}^k b^k\left(t_n+\mu_i\Delta,\eta_i\right);$$

the numerical constants α_j; μ_j; β_j^k; λ_{ji} and γ_{ji}^k with $j=2,...,s$; $\hat{W}_{(t)}$ are Brownian motions and R is an appropriate term, which is obtained when comparing the approximation of equation (14) with the schema β-equivalent of the simplified Taylor method of the β order.

The Runge-Kutta stochastic schema of order two-stage three on Eq. (14) for $s>2$, is obtained for the scalar case

$d=m=1$,

$$\bar{X}_{n+1}=\bar{X}_n+\left[\alpha_1 a+\alpha_2 a\left(t_n+\mu\Delta,\eta\right)\right]\Delta+\begin{bmatrix}\beta_1 b+\beta_2 b\left(t_n+\mu\Delta,\bar{\eta}\right)\\+\beta_3 b\left(t_n+\mu\Delta,\bar{\bar{\eta}}\right)\end{bmatrix}\Delta\hat{W}_n+R \quad (15)$$

where, $\eta=\bar{X}_n+\lambda a\Delta+\gamma b\Delta\hat{W}_n$; $\bar{\eta}=\bar{X}_n+\bar{\lambda}a\Delta+\bar{\gamma}b\Delta\hat{W}_n$; $\bar{\bar{\eta}}=\bar{X}_n+\bar{\bar{\lambda}}a\Delta+\bar{\bar{\gamma}}b\Delta\hat{W}_n$; $a=a\left(t_n,\bar{X}_n\right)$; $b=b\left(t_n,\bar{X}_n\right)$.

The functions $a\left(t_n+\mu_j\Delta,\eta_j\right)$ and $b\left(t_n+\mu_j\Delta,\eta_j\right)$ are truncated expansions of the second order, given by:

$$f\left(t+\Delta,X_t+\Delta X\right)\overset{(2)}{=}f+\frac{\partial f}{\partial t}\Delta+\sum_{i,j=1}^{d}\frac{\partial^2 f}{\partial x^i\partial x^j}\frac{\Delta X^i\Delta X^j}{2}+\sum_{i=1}^{d}\frac{\partial f}{\partial x^i}\Delta X+$$
$$\left[\frac{\partial^2 f}{\partial t^2}+\sum_{i,j=1}^{d}c^{ij}\frac{\partial^3 f}{\partial t\partial x^i\partial x^j}+\frac{1}{2}\sum_{i,j,k}^{d}\left(\sum_{l=1}^{d}c^{kl}\frac{\partial c^{ij}}{\partial x^l}\right)\frac{\partial^3 f}{\partial x^i\partial x^j\partial x^k}+\frac{1}{4}\sum_{i,j,k,l=1}^{d}c^{ij}c^{kl}\frac{\partial^4 f}{\partial x^i\partial x^j\partial x^k\partial x^l}\right]\frac{\Delta^2}{2}$$
$$+\sum_{i=1}^{d}\left(\frac{\partial^2 f}{\partial t\partial x^i}+\frac{1}{2}\sum_{j,k=1}^{d}c^{jk}\frac{\partial^3 f}{\partial x^i\partial x^j\partial x^k}\right)\Delta\left(\Delta X^i\right) \quad (16)$$

$\overset{(2)}{=}$ indicates a second order approximation on the functions. For the scalar case and considering equation (16) where $c=b^2$, we obtain

$$f\left(t+\Delta,X_t+\Delta X\right)\overset{(2)}{=}f+f_{10}\Delta+f_{01}\Delta X+\left(f_{20}+b^2f_{12}+b^3b_{01}f_{03}+\frac{b^4}{4}f_{04}\right)\frac{\Delta^2}{2}$$
$$+\left(f_{11}+\frac{b^2}{2}f_{03}\right)\Delta\left(\Delta X\right)+f_{02}\frac{\left(\Delta X\right)^2}{2} \quad (17)$$

In order to simplify the mathematical notation, we write $f_{ij}=\dfrac{\partial^{i+j}f}{\partial t^i\partial x^j}$.

The combination of the products of the Brownian motions with the $\beta=2$ approximation corresponds to:

$$\Delta\left(\Delta\hat{W}_n\right)^i\left(\Delta\hat{W}_n\right)^j\overset{(2)}{=}\begin{cases}\Delta^2 \text{ if } i\neq j\\0 \text{ if } i=j\end{cases} \quad (18\text{-a})$$

$$\left(\Delta\hat{W}_n\right)^i\left(\Delta\hat{W}_n\right)^j\left(\Delta\hat{W}_n\right)^k\overset{(2)}{=}\begin{cases}3\Delta\left(\Delta\hat{W}_n\right)^i \text{ if } i=j=k\\\Delta\left(\Delta\hat{W}_n\right)^i \text{ if } j=k\neq i\end{cases} \quad (18\text{-b})$$

$$\Delta^i\left(\Delta\hat{W}_n^1\right)^{j_1}\left(\Delta\hat{W}_n^2\right)^{j_2}\cdots\left(\Delta\hat{W}_n^m\right)^{j_m}\overset{(2)}{=}0 \text{ if } i+\frac{j_1+j_2+\ldots+j_m}{2}\geq\frac{5}{2} \quad (18\text{-c})$$

where, $i=j=1,2,...,n$.

Considering on equation (17), the approximations to equations (18a-18c) we obtain,

$$a\left(\tau_n,\eta\right)\Delta\overset{(2)}{=}a\Delta+a_{10}\mu\Delta^2+aa_{01}\gamma\Delta\left(\Delta\hat{W}_n\right)+\frac{1}{2}a_{02}b^2\gamma^2\Delta^2 \quad (19)$$

$$b\left(\tau_n,\bar{\eta}\right)\Delta\hat{W}_n\overset{(2)}{=}b\Delta\hat{W}_n+bb_{01}\bar{\gamma}\Delta\left(\Delta\hat{W}_n\right)^2+b_{10}\mu\Delta\left(\Delta\hat{W}_n\right)+ab_{01}\bar{\lambda}\Delta\left(\Delta\hat{W}_n\right)+$$
$$\frac{3}{2}b_{02}b^2\Delta\left(\Delta\hat{W}_n\right)+b\left(b_{11}+\frac{1}{2}b_{03}b^2\right)\mu\bar{\gamma}\Delta^2+abb_{02}\bar{\lambda}\bar{\gamma}\Delta^2 \quad (20)$$

$$b\left(\tau_n,\bar{\bar{\eta}}\right)\Delta\hat{W}_n\overset{(2)}{=}b\Delta\hat{W}_n+bb_{01}\bar{\bar{\gamma}}\Delta\left(\Delta\hat{W}_n\right)^2+b_{10}\mu\Delta\left(\Delta\hat{W}_n\right)+ab_{01}\bar{\bar{\lambda}}\Delta\left(\Delta\hat{W}_n\right)+$$
$$\frac{3}{2}b_{02}b^2\Delta\left(\Delta\hat{W}_n\right)+b\left(b_{11}+\frac{1}{2}b_{03}b^2\right)\mu\bar{\bar{\gamma}}\Delta^2+abb_{02}\bar{\bar{\lambda}}\bar{\bar{\gamma}}\Delta^2 \quad (21)$$

Replacing equations (19-21) on equation (15), we obtain

$$\bar{X}_{n+1}=\bar{X}_n+\left(\alpha_1+\alpha_2\right)a\Delta+\left(\beta_1+\beta_2+\beta_3\right)b\Delta\hat{W}_n+\left(\beta_2\bar{\gamma}+\beta_3\bar{\bar{\gamma}}\right)bb_{01}\left(\Delta\hat{W}_n\right)^2+$$
$$\left[\alpha_2\mu a_{10}+\alpha_2\lambda aa_{01}+\left(\beta_2\bar{\gamma}+\beta_3\bar{\bar{\gamma}}\right)\mu b\left(b_{11}+\frac{1}{2}b^2b_{03}\right)+\frac{1}{2}\alpha_2\gamma^2a_{02}b^2+\left(\beta_2\bar{\lambda}\bar{\gamma}+\beta_3\bar{\bar{\lambda}}\bar{\bar{\gamma}}\right)abb_{02}\right]\Delta^2+$$
$$\left[\alpha_2\gamma a_{01}b+\left(\beta_2+\beta_3\right)\mu b_{10}+\frac{3}{2}\left(\beta_2\bar{\gamma}^2+\beta_3\bar{\bar{\gamma}}^2\right)b^2b_{02}+\left(\beta_2\bar{\lambda}+\beta_3\bar{\bar{\lambda}}\right)ab_{01}\right]\Delta\left(\Delta\hat{W}_n\right)+R \quad (22)$$

The schema obtained on equation (22) has to be β-equivalent to the Taylor schema of order 2, which is given by [18]:

$$\bar{X}_{n+1}=\bar{X}_n+a\Delta+b\Delta\hat{W}_n+\frac{1}{2}bb_{01}\left(\left(\Delta\hat{W}_n\right)^2-\Delta\right)+$$
$$\left[a_{10}+aa_{01}+\frac{1}{2}a_{02}\gamma^2b^2\right]\Delta^2+\frac{1}{2}\left[b_{10}+ab_{01}+\frac{1}{2}b^2b_{02}+ba_{01}\right]\Delta\left(\Delta\hat{W}_n\right) \quad (23)$$

Such that equation (22) and equation (23) be β-equivalent, we need to fulfil the equalities:

$$\alpha_1+\alpha_2=1; \qquad \beta_2\bar{\gamma}^2+\beta_3\bar{\bar{\gamma}}^2=\frac{1}{6}; \qquad \alpha_2\gamma=\frac{1}{2}$$
$$\beta_1+\beta_2+\beta_3=1; \qquad \beta_2\bar{\lambda}+\beta_3\bar{\bar{\lambda}}=\frac{1}{2}; \qquad \alpha_2\mu=\frac{1}{2} \quad (24\text{-a})$$

$$(\beta_2+\beta_3)\mu=\frac{1}{2};\qquad \beta_2\bar{\lambda}\bar{\gamma}+\beta_3\bar{\bar{\lambda}}\bar{\bar{\gamma}}=\frac{1}{2};\qquad \alpha_2\lambda=\frac{1}{2}$$
$$\left(\beta_2\bar{\gamma}+\beta_3\bar{\bar{\gamma}}\right)\mu=\frac{1}{2};\qquad \alpha_2\gamma^2=\frac{1}{2};\qquad R=\frac{1}{2}bb_{01}\left(\left(\Delta\hat{W}_n\right)^2-\Delta\right) \quad (24\text{-b})$$

We observe on equations (24a-24b) has a unique solution when $\gamma=\mu=\lambda=1$ and $\alpha_1=\alpha_2=1/2$. Replacing these equalities on equation (24a-24b), we obtain:

$$\beta_1=\frac{1}{2};\qquad \beta_2=\frac{1}{2+6\bar{\gamma}^2};\qquad \beta_3=\frac{3\bar{\gamma}^2}{2+6\bar{\gamma}^2}$$
$$\bar{\bar{\gamma}}=-\frac{1}{3\bar{\gamma}};\qquad \bar{\lambda}=\bar{\bar{\lambda}}=1;\qquad \bar{\gamma}\neq 0 \quad (25)$$

Replacing the equivalences from equation (25) on equation (23), we obtain Runge-Kutta stochastic schema of order two-third stage (RK2-3st):

$$\bar{X}_{n+1}=\bar{X}_n+\frac{1}{2}a\Delta+\frac{1}{2}b\Delta\hat{W}_n+\frac{1}{2}a\left(t_n+\mu,\bar{X}_n+a\Delta+b\Delta\hat{W}_n\right)\Delta+$$
$$\frac{1}{2}bb_{01}\left(\left(\Delta\hat{W}_n\right)-\Delta\right)+\frac{1}{2+6\gamma^2}b\left(t_n+\mu,\bar{X}_n+a\Delta+b\Delta\hat{W}_n\right)\Delta\hat{W}_n+$$
$$\frac{3}{2+6\gamma^2}b\left(t_n+\mu,\bar{X}_n+a\Delta-\frac{1}{3\bar{\gamma}}b\Delta\hat{W}_n\right)\Delta\hat{W}_n \quad (26)$$

However, RK2-3st given by equation (26) is 2-equivalent to the Taylor schema of order two, given by equation (23). We observe that the Taylor schema requires calculating more derivatives than the RK2-3st schema. The RK2-3st method is implemented to solve stochastic point kinetic equations with temperature feedback effects. We propose different numerical experiments to study, the step change reactivity temperature feedback in the nuclear reactors given on equation (12).

RESULTS

In this section we present several numerical experiments, on which we give numerical solutions to the stochastic point kinetics equations given by equation (1) with temperature feedback effects considering step change reactivity using equation (12), with the objective of analyzing efficiency and precision of the proposed RK2-3st method.

For the numerical experiments, the system starts with the following initial conditions $t_0=0$, for the neutron density $n_0=1\,n/cm^3$ and the delayed neutron precursor density $C_{i(0)}=\frac{\beta_i n_{(0)}}{\lambda_i\Lambda}$. For the U^{235} graphite nuclear reactor, the parameters of the delayed neutron precursor groups corresponding to: $\lambda_1=0.0124\,s^{-1}$, $\lambda_2=0.0305\,s^{-1}$, $\lambda_3=0.111\,s^{-1}$, $\lambda_4=0.301\,s^{-1}$, $\lambda_5=1.13\,s^{-1}$, $\lambda_6=3.0\,s^{-1}$, $\beta_1=0.00021$, $\beta_2=0.00141$, $\beta_3=0.00127$, $\beta_4=0.00255$, $\beta_5=0.00074$, $\beta_6=0.00027$, $\beta=0.00645$, $\Lambda=5.0\times10^{-5}s$, for a fixed step size $h=10^{-3}$ and using 500 trials. We study the point kinetics equations for the following steps of the initial reactivity: $\rho_0=0.5\$$, $\rho_0=0.75\$$, $\rho_0=1.0\$$ y $\rho_0=1.5\$$. We consider that the coefficient for the reactivity of Doppler temperature has a value of $\alpha_{T_f}^D=5*10^{-5}K^{-1}$ and $H=0.05\ K\,cm^3s^{-1}$.

On Table-1 we present the peak values of the mean population of the neutron density and the time in which this occurs, as calculated by the RK2-3st method for the initial steps of the reactivity : $\rho_0=0.5\$$, $\rho_0=0.75\$$, $\rho_0=1.0\$$ and $\rho_0=1.5\$$. The values obtained with the proposed RK2-3st method for $\rho_0=0.5\$$, $\rho_0=0.75\$$, $\rho_0=1.0\$$ and $\rho_0=1.5\$$ are compared with the split-step forward EMM and the Derivate-free Milstein method [13], Taylor 1.5 strong order [20] and the Converged Accelerated Taylor Series (CATS) [**]. For $\rho_0=0.5\$$, we find that the values obtained with RK2-3st are very close to the split-step forward EMM and Derivate-free Milstein methods. The values obtained for $\rho_0=0.75\$$ are similar to the one reported with the Split-step forward EMM method. For $\rho_0=1.0\$$ and $\rho_0=1.5\$$, the values with the RK2-3st method are close to the ones reported on the CATS method. Finding that the proposed RK2-3st method is an efficient method that allows to study the stochastic point kinetic equations with temperature feedback effects considering the step change reactivity.

On Table-2 we tabulated the values of the reactivity given on ($), the mean values and the standard deviation of the neutron density and the sum of the delayed neutron precursor concentration.

Table-1. Peak of the neutron density and respective time to peak in compensated step change reactivity.

Method	$\rho_0 = 0.5\$$		$\rho_0 = 0.75\$$		$\rho_0 = 1.0\$$		$\rho_0 = 1.5\$$	
	Peak	**Time(s)**	**Peak**	**Time(s)**	**Peak**	**Time(s)**	**Peak**	**Time(s)**
Split-step forward EMM	46.4939	28.3400	163.707	8.795	760.589	1.065	-	-
Derivative-free Milstein method	46.2606	27.8400	164.22	8.75	769.238	1.0575	-	-
Taylor 1.5 strong order	55.6596	22.41	185.44	7.1725	1192.48	0.725	-	-
CATS	-	-	-	-	807.8681	0.953	43024.61	0.168
RK2-3st	45.8202	27.8	163.4927	8.9	805.4465	1.0	39472.3478	0.169

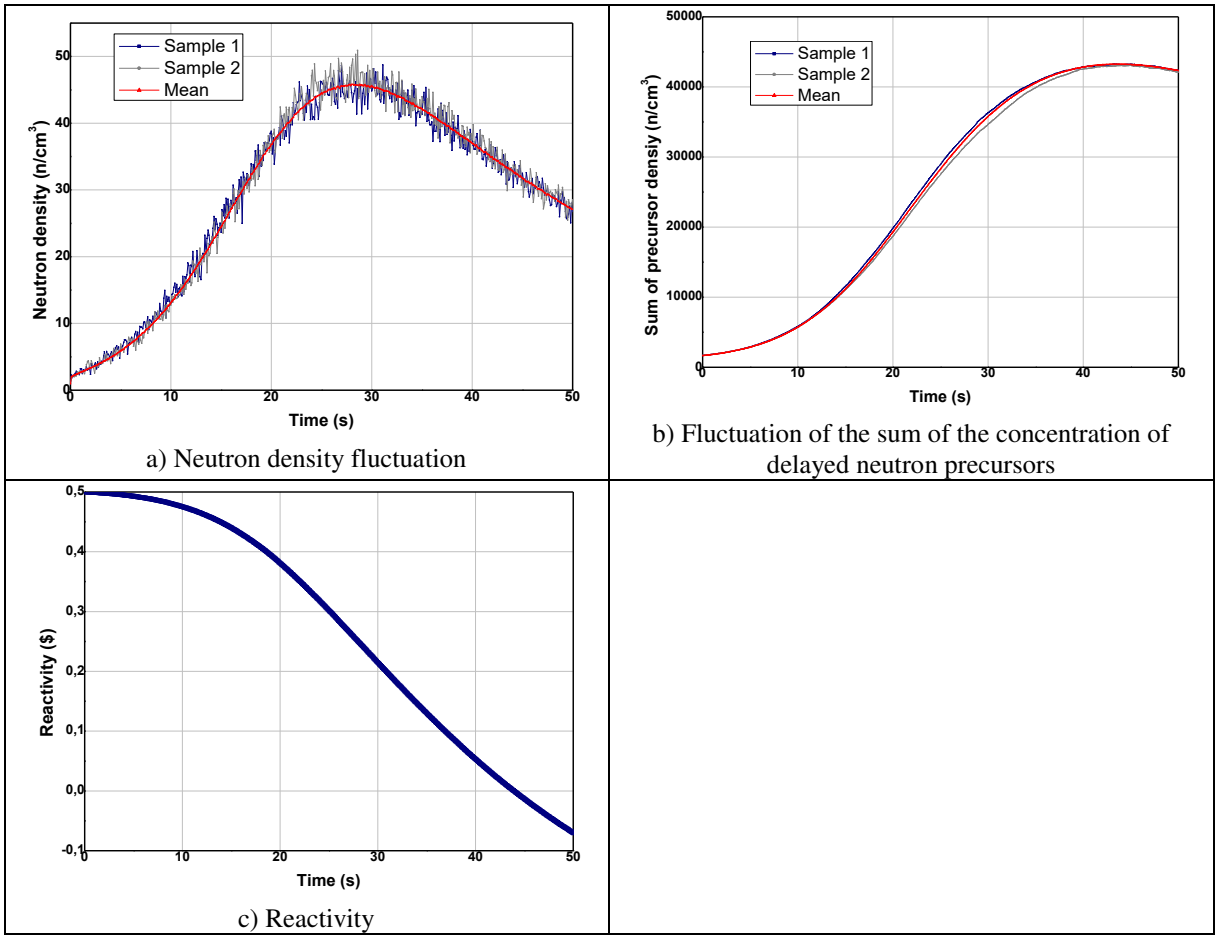
Table-2. Mean values and standard deviation of neutron density, the sum of the precursor groups and reactivity for different ρ_0.

ρ_0	**Time (s)**	**RK2-3st**				
		$E\left[n_{(t)}\right]$	$\sigma\left[n_{(t)}\right]$	$E\left[c_{i(t)}\right]$	$\sigma\left[c_{i(t)}\right]$	$\rho_{(t)}$
0.5$	100	7.3339	0.6788	22479.9319	213.4026	0.0137
0.75$	50	17.1675	0.9525	55804.2987	277.1445	-0.0375
1.0$	25	37.2910	1.4513	109934.2385	329.8327	-0.1536
1.5$	0.2	20752.7575	9952.4185	293391.5561	47817.4698	1.3552
2.0$	0.13	23347.5730	56385.7988	653413.4171	118181.8667	1.5949

Figures 1-4 a) shows the dynamical behaviour of the neutron density as a function of the time for the initial step reactivity $\rho_0 = 0.5\$$, $\rho_0 = 0.75\$$, $\rho_0 = 1.0\$$ and $\rho_0 = 1.5\$$, respectively, Figures 1-4 b) shows the concentration of the delayed neutron precursor density as a function of time and Figures 1-4 c) shows the reactivity along time. On Figures 1-4 a) and b) we show to different sample densities (blue and gray lines) and the median density (red line).

We observe on Figures 1-4 that there is low variation on the neutron density and the delayed neutron precursor density along time with respect to the median values, indicating that the RK2-3st method has good precision.

For the initial step reactivity $\rho_0 = 0.5\$$, $\rho_0 = 0.75\$$, $\rho_0 = 1.0\$$ and $\rho_0 = 1.5\$$, the reactivity values, the median neutron density values, and the sum of the delayed neutron precursor concentration are presented on Tables 3-6, respectively.



a) Neutron density fluctuation

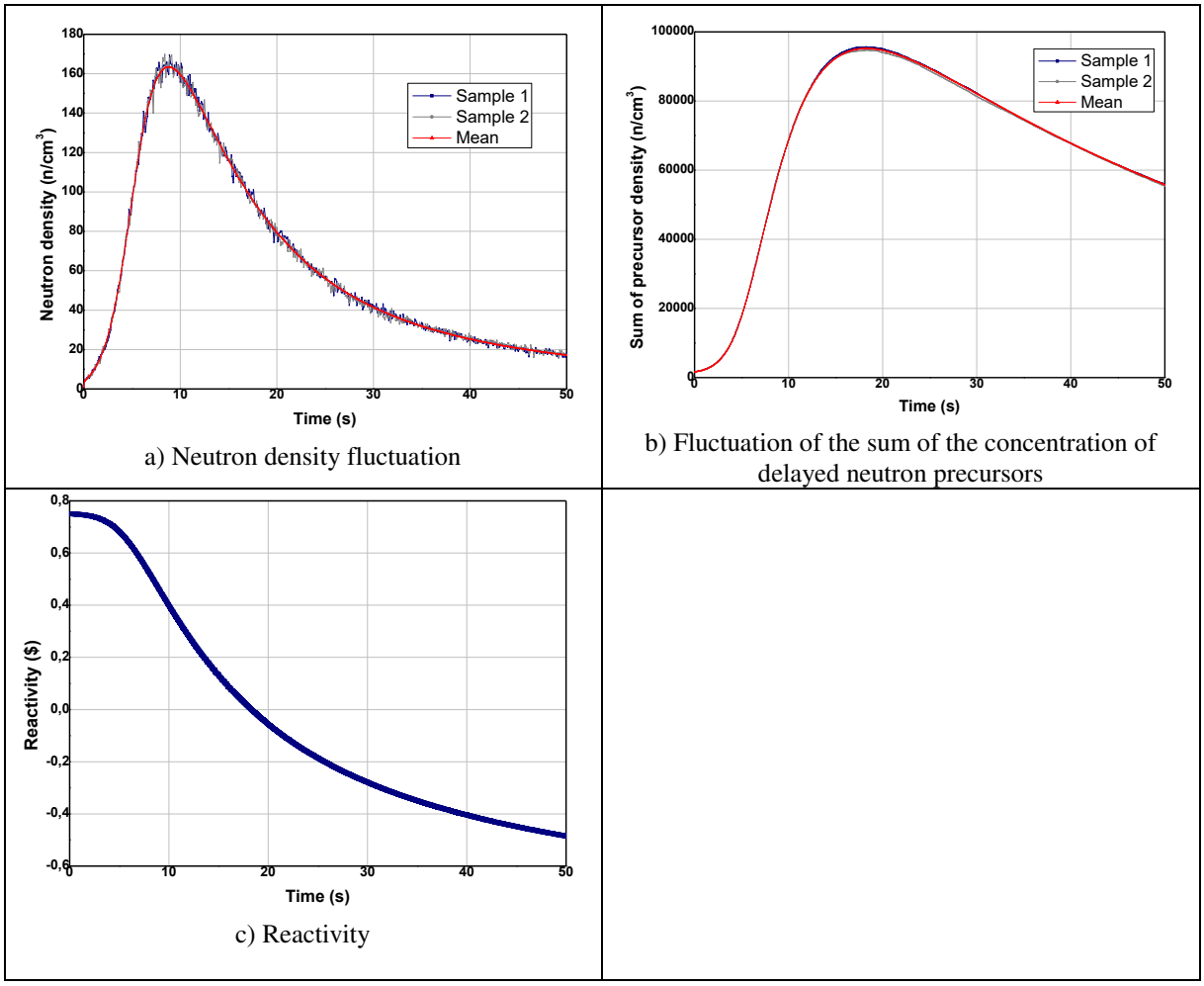
b) Fluctuation of the sum of the concentration of delayed neutron precursors

c) Reactivity

Figure-1. Variation of neutron density with time for several initial step reactivities ρ_0.

Table-3. Mean values and standard deviation of neutron density, the sum of the precursor groups and reactivity for initial step reactivity $\rho_0 = 0.5\$$.

t(s)	$E\left[n_{(t)}\right]$	$\sigma\left[n_{(t)}\right]$	$E\left[c_{i(t)}\right]$	$\sigma\left[c_{i(t)}\right]$	$\rho_{(t)}$
0	1	-	1.6765E+03	-	0.5000E+00
10	1.3113E+01	1.1030E+00	5.7358E+03	1.5877E+02	4.7530E-01
20	3.6751E+01	1.7809E+00	1.9304E+04	4.7826E+02	3.8116E-01
30	4.5392E+01	1.7402E+00	3.5789E+04	5.0194E+02	2.1505E-01
40	3.6856E+01	1.5532E+00	4.2832E+04	3.5662E+02	5.3259E-02
50	2.7072E+01	1.2265E+00	4.2339E+04	3.0269E+02	-7.005E-02



a) Neutron density fluctuation

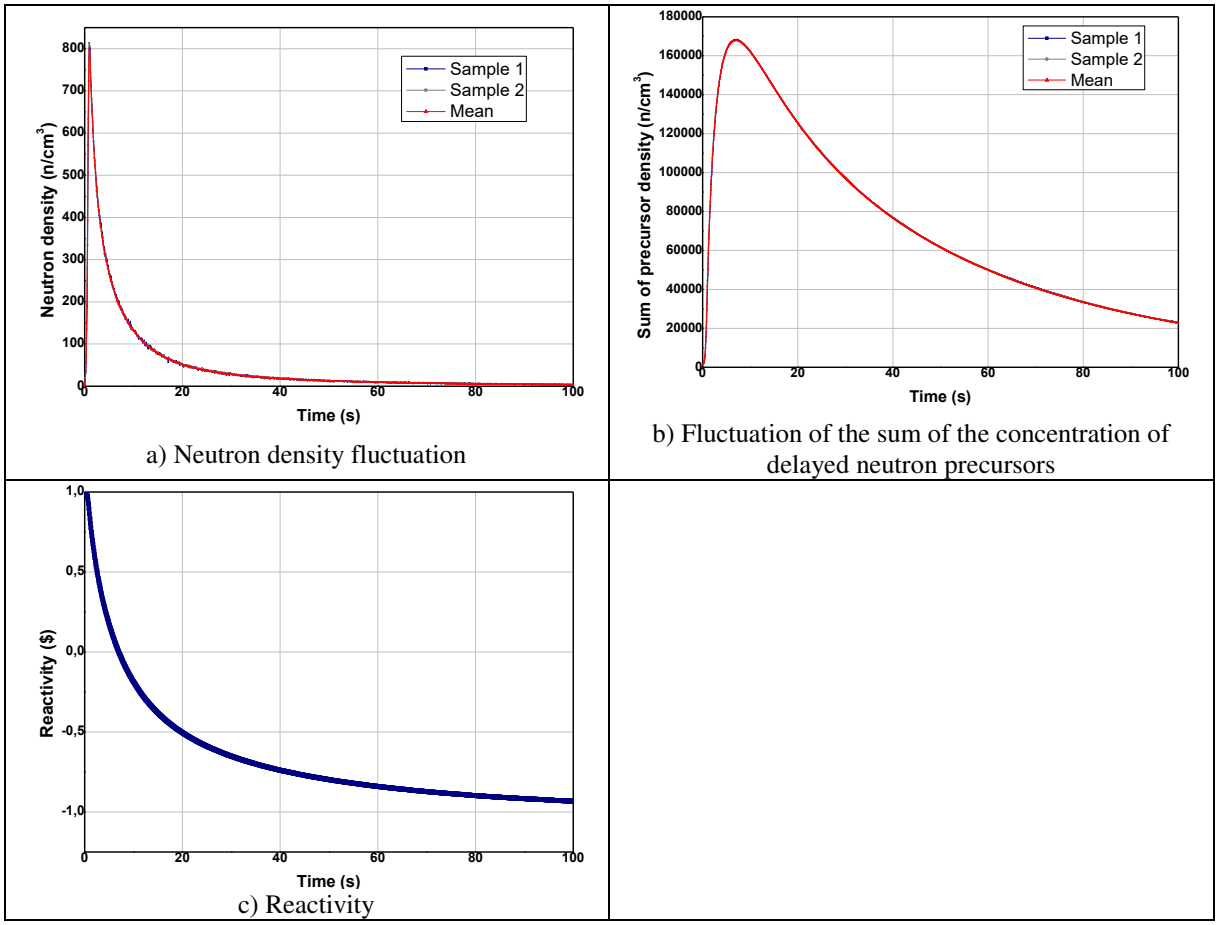
b) Fluctuation of the sum of the concentration of delayed neutron precursors

c) Reactivity

Figure-2. Neutron density variation, sum of the delayed neutron precursor density and reactivity as a function of time for the initial step reactivity $\rho_0 = 0.75\$$.

Table-4. Mean values and standard deviation of neutron density, the sum of the precursor groups and reactivity for initial step reactivity $\rho_0 = 0.75\$$.

t(s)	$E\left[n_{(t)}\right]$	$\sigma\left[n_{(t)}\right]$	$E\left[c_{i(t)}\right]$	$\sigma\left[c_{i(t)}\right]$	$\rho_{(t)}$
0	1	-	1.6765E+03	-	0.7500E+00
10	1.5955E+02	3.6675E+00	6.8706E+04	5.2872E+02	4.0007E-01
20	7.9328E+01	2.3645E+00	9.4657E+04	3.3662E+02	-5.6172E-02
30	4.1469E+01	1.5587E+00	8.2138E+04	3.3418E+02	-2.7902E-01
40	2.5244E+01	1.2020E+00	6.7808E+04	3.0052E+02	-4.0457E-01
50	1.7168E+01	9.5252E-01	5.5804E+04	2.7714E+02	-4.8505E-01



a) Neutron density fluctuation

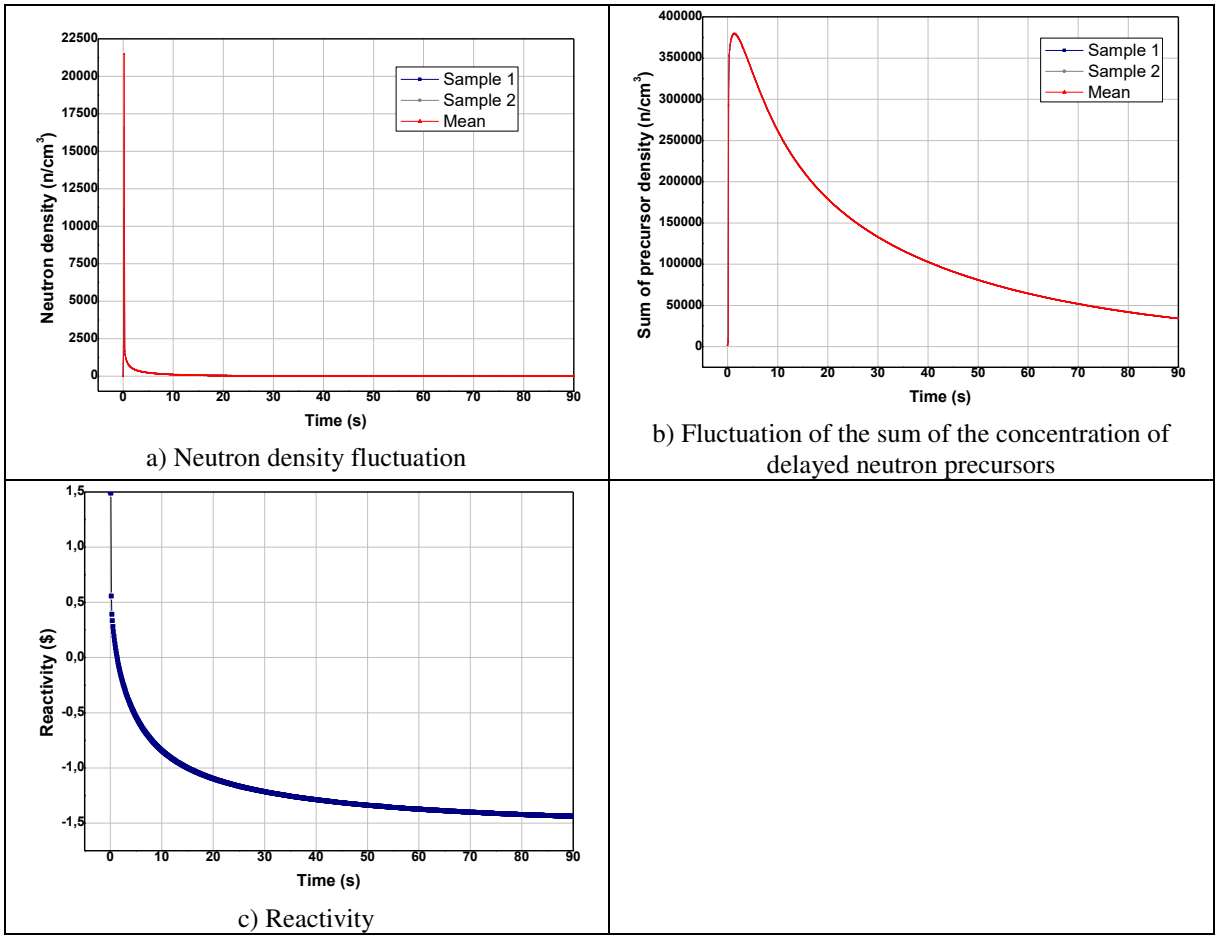
b) Fluctuation of the sum of the concentration of delayed neutron precursors

c) Reactivity

Figure-3. Neutron density variation, sum of the delayed neutron precursor density and reactivity as a function of time for the initial step reactivity $\rho_0 = 1.0\$$.

Table-5. Mean values and standard deviation of neutron density, the sum of the precursor groups and reactivity for initial step reactivity $\rho_0 = 1.0\$$.

t(s)	$E\left[n_{(t)}\right]$	$\sigma\left[n_{(t)}\right]$	$E\left[c_{i(t)}\right]$	$\sigma\left[c_{i(t)}\right]$	$\rho_{(t)}$
0	1	-	1.6765E+03	-	1.0000E+00
10	1.3219E+02	2.9413E+00	1.6225E+05	3.8901E+02	2.657E-01
20	5.1700E+01	1.6937E+00	1.2539E+05	3.5270E+02	-5.428E-02
30	2.8164E+01	1.2382E+00	9.6960E+04	3.4131E+02	-2.278E-01
40	1.8058E+01	1.0273E+00	7.6669E+04	2.7849E+02	-3.497E-01
50	1.2759E+01	8.6643E-01	6.1539E+04	2.7173E+02	-4.312E-01
60	9.5020E+00	7.6797E-01	4.9893E+04	2.4833E+02	-4.946E-01
70	7.2266E+00	6.7143E-01	4.0700E+04	2.1911E+02	-5.473E-01
80	5.6110E+00	6.7237E-01	3.3374E+04	2.0079E+02	-5.891E-01
90	4.4246E+00	5.9833E-01	2.7479E+04	1.8647E+02	-6.248E-01
100	3.5626E+00	5.2597E-01	2.2716E+04	1.6714E+02	-6.553E-01



a) Neutron density fluctuation

b) Fluctuation of the sum of the concentration of delayed neutron precursors

c) Reactivity

Figure-4. Neutron density variation, sum of the delayed neutron precursor density and reactivity as a function of time for the initial step reactivity $\rho_0 = 1.5\$$.

Table-6. Mean values and standard deviation of neutron density, the sum of the precursor groups and reactivity for initial step reactivity $\rho_0 = 1.5\$$.

t(s)	$E\left[n_{(t)}\right]$	$\sigma\left[n_{(t)}\right]$	$E\left[c_{i(t)}\right]$	$\sigma\left[c_{i(t)}\right]$	$\rho_{(t)}$
0	1	-	1.6765E+03	-	1.5000E+00
10	1,0723E+02	2,6360E+00	2,6172E+05	4,7670E+02	-8,4186E-01
20	4,1373E+01	1,5697E+00	1,7907E+05	3,8904E+02	-1,0977E+00
30	2,3211E+01	1,1630E+00	1,3304E+05	3,6283E+02	-1,2155E+00
40	1,5179E+01	1,0552E+00	1,0267E+05	3,2266E+02	-1,2884E+00
50	1,0788E+01	8,9175E-01	8,0871E+04	2,9691E+02	-1,3380E+00
60	6,1845E+00	7,0942E-01	6,4476E+04	2,6137E+02	-1,3736E+00
70	6,1845E+00	7,0942E-01	5,1825E+04	2,4486E+02	-1,4016E+00
80	4,8080E+00	5,9848E-01	4,1909E+04	2,2002E+02	-1,4233E+00
90	3,6948E+00	5,6430E-01	3,4090E+04	1,9782E+02	-1,4388E+00

CONCLUSIONS

On this article, we have solved the point kinetic stochastic equations of a nuclear reactor with temperature feedback effects for the step change reactivity with the stochastic Runge-Kutta numerical method of order two-stage three. We have presented the results of several

different numerical experiments, on which we consider varied values of the initials step reactivity. We observed that the fluctuations are very close to the median neutron density values and the concentration of delayed neutron precursors, which indicates that the RK2-3st method is an efficient one. When comparing the values that are available on the literature for each numerical experiment, for both, the deterministic and stochastic method, we found that the RK2-3st method generates values with a good precision.

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REFERENCES

[1] Lamarsh J. 2002. Introduction to Nuclear Reactor Theory, Addison-Wesley, New York.

[2] Hamanda Y. M., Aboanber A. E. 2002. Generalization of the analytical inversion method for the solution of the point kinetics equations. Journal of Physics A: Mathematical and General. 35: 3245-3263.

[3] Stacey W. 2018. Nuclear Reactor Physics. Weinheim, Germany: Wiley-VCH Verlag.

[4] Saha R., Patra A. 2013. Numerical solution of fractional stochastic neutron point kinetic equation for nuclear reactor dynamics. Ann. Nucl. Energy. 54: 154-161.

[5] Aboanber A. E., Hamada Y. M. 2003. Power series solution (PWS) of nuclear reactor dynamics with newtonian temperature feedback. Ann. Nucl. Energy. 30, 1111.

[6] Ganapol B., Picca P., Previti A., Mostacci D. 2012. The solution of the point kinetics equations via converged accelerated Taylor series (CATS). In: Advances in Reactor Physics e Linking Research, Industry and Education (PHYSOR 2012), Knoxville, Tennessee, USA, April 15-20.

[7] Picca P., Furfaro R., Ganapol B. 2013. A highly accurate technique for the solution of the non-linear point kinetics equations, Ann. Nucl. Energy. 58(2013): 43-53.

[8] Leite S., De Vilhena M., Bodmann B. 2016. Solution of the point reactor kinetics equations with temperature feedback by the ITS2 method- Prog. Nucl. Energy. 91, 240-249.

[9] Suescún D., Rasero D., Lozano J. 2019. Neutron density calculation using the generalized Adams-Bash forth-Moulton Method, Universitas Scientiarum. 24(3): 543-563.

[10] Suescún-Díaz D., Espinosa-Paredes G. 2020. On the numerical solution of the point reactor kinetics equations. Nuclear Engineering and Technology. 52: 1340-1346.

[11] Nahla A. A., 2011. An efficient technique for the point reactor kinetics equations with Newtonian temperature feedback effects. Ann. Nucl. Energy. 38: 2810-2817.

[12] Singh S., Saha R. 2017. On the comparison of two split-step methods for the numerical simulation of stochastic point kinetics equations in presence of Newtonian temperature feedback effects, Ann. Nucl. Energy. 110: 865-873.

[13] Saha R., Singh S. 2019. Numerical solutions of stochastic nonlinear point reactor kinetics equations in presence of Newtonian temperature feedback effects. J. Theoretical Transport. 47-57.

[14] Tocino A., Ardanuy R. 2002. Runge-Kutta methods for numerical solution of stochastic differential equations. J. Computational and applied mathematics. 138: 219-242.

[15] Suescún D. D., Roa N. M. and Escobar F. H. 2019. Second order three stage stochastic Runge-Kutta method for nuclear reactors. ARPN J. Eng. App. Sci. 14: 3789-3796.

[16] Suescún D. D., Oviedo Y. M., Girón L. E. 2018. Solution of the stochastic point kinetics equations using the implicit Euler-Maruyama method. Ann. Nucl. Energy. 117: 45-52.

[17] Bird R. B., Stewart W. E., Lightfoot E. N. 2002. Transport phenomena. Second Edition, Jhon Wiley & Sons, United State of America.

[18] Kloeden P. E.; Platen E. 1992. Numerical solution of stochastic differential equations. Springer Verlag, New York.