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CALCULATION OF REACTIVITY WITH RECURSIVE DEPENDENCE ON FEEDBACK TEMPERATURE

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ABSTRACT

The objective of this work is to calculate the reactivity with feedback temperature effects by means of the inverse equation of deterministic point kinetics using the first Bernoulli number in the Euler-Maclaurin series approximation. The Doppler effect with feedback temperature is considered, which is recursively calculated from an initial value. To validate the proposed method, several numerical experiments are carried out with different time steps, simulation times and forms for the density of neutron population.

Keywords: first bernoulli number, neutron density, reactivity, feedback temperature.

INTRODUCTION

There are several procedures for the safe control of a nuclear power plant; perhaps the most important parameter that needs to be known is reactivity, since it indicates the change in neutron population (Stacey, 2018). Several works have been reported in the literature that calculate the reactivity: discretisation of the dependence on neutron density (Shimazu *et al*., 1987; Hoogenboom and van der Sluijs, 1988; Binney and Bakir, 1989; Ansari, 1991; Kitano *et al*., 2000; Malmir and Vosoughi, 2013; Suescún *et al*., 2020), however, no work has considered the effects of feedback temperature. There are other works that calculate neutron density with feedback effects, among others we have (Aboanber and Nahla, 2002; Aboanber and Hamada, 2003; Nahla, 2009, 2011; Hamada, 2013; Ganapol, 2013). However, these works do not use the inverse point kinetic equation to calculate reactivity.

In this work, it is proposed to calculate the total reactivity, external reactivity and Doppler reactivity, by assuming the dependence on neutron density is known, using the inverse point kinetic equation considering six groups of delayed neutron precursors.

THEORETICAL CONSIDERATIONS INVERSE POINT KINETICS EQUATIONS WITH TEMPERATURE FEEDBACK

The nonlinear deterministic model of the point kinetics equations with feedback due to temperature effects is as follows (Stacey, 2018),

$$\frac{dP(t)}{dt} = \left[\frac{\rho(t)-\beta}{\Lambda}\right]P(t) + \sum_{i=1}^{6}\lambda_i C_i(t) \tag{1}$$

$$\frac{dC_i(t)}{dt} + \lambda_i C_i(t) = \frac{\beta_i}{\Lambda}P(t) \quad ; \quad i = 1,2,...,6 \tag{2}$$

$$\rho(t) = \rho_x(t) - \alpha[T(t) - T_0] \tag{3}$$

$$\frac{dT(t)}{dt} = HP(t) \tag{4}$$

Where P is the neutron density, C_i is the concentration of the *i-th* group of delayed neutron precursors, ρ is the reactivity, ρ_x is the external reactivity, Λ is the generation time of instantaneous neutrons, β_i is the effective fraction of the *i-th* group of delayed neutrons , β is the total effective fraction of delayed neutrons, λ_i is the decay constant of the *i-th* group of delayed neutron precursors, T is the fuel temperature, α is the Doppler temperature coefficient of reactivity, H is the reciprocal of the reactor heat capacity.

The initial conditions for equations (1-4) are described as follows:

$$P(t=0) = P_0 \tag{5}$$

$$C_i(t=0) = \frac{\beta_i}{\Lambda\lambda_i}P_0 \tag{6}$$

$$T(t=0) = T_0 \tag{7}$$

$$\rho(t=0) = \rho_0 \tag{8}$$

Equations (1) and (2) describe the time evolution of neutron density and precursor concentration, forming a stiff set of nonlinear equations. Equations (3) and (4) describe an adiabatic model of reactivity with feedback effects due to temperature.

Using equations (3) and (4) one can obtain an expression for Doppler reactivity as a function of neutron density, as follows:

$$\frac{d\rho_D(t)}{dt} = -\alpha HP(t) \tag{9}$$

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It is also possible to obtain a relation for the total reactivity $\rho(t)$ in equation (1) as dependent on the external reactivity $\rho_X(t)$, and the Doppler reactivity $\rho_D(t)$ represented by equation (9), in the following way

$$\rho(t) = \rho_X(t) + \rho_D(t) \tag{10}$$

Where the Doppler reactivity is:

$$\rho_D(t) = \alpha[T_0 - T(t)] \tag{11}$$

The following procedure is implemented to obtain the inverse equations of point kinetics: first equation (2) is solved by integrating factors or by the Laplace transform, applying the initial conditions given by equations (5-6), then an expression for the concentration of precursors is obtained. Second, the reactivity in equation (1) is cleared and then, when replacing the expression obtained for the concentration of precursors, an equation that describes the reactivity in terms of neutron density is obtained (Duderstadt and Hamilton, 1976),

$$\rho(t) = \beta + \frac{\Lambda}{P(t)}\frac{dP(t)}{dt} - \frac{P(0)}{P(t)}\sum_{i=1}^{6}\beta_i e^{-\lambda_i t} - \frac{1}{P(t)}\sum_{i=1}^{6}\int_0^t \lambda_i \beta_i e^{-\lambda_i (t-t')} P(t')dt' \tag{12}$$

Equation (12) is known as the inverse point kinetic equations; this equation will be used in this work as the reference method to validate the accuracy of the proposed method for calculating reactivity. In this work, the reactivity is calculated by discretising the dependency of the neutron density, which is represented by the integral term in equation (12), making use of the Euler-Maclaurin formula.

PROPOSED METHOD

In order to discretise the integral term in equation (12) we use the Euler Maclaurin's approximation in the following way (Kwok, 2010),

$$\int_0^n F(x)dx = \sum_{s=1}^{n-1} F[s] + \frac{1}{2}\left[F[0] + F[n]\right] - \frac{B_1}{2!}\left[F^{(1)}[n] - F^{(1)}[0]\right] \tag{13}$$

where $B_1 = 1/6$ is the first Bernoulli number.

By replacing equation (13) into equation (12) the reactivity is obtained in a discrete way, with the approximation of the first Bernoulli number (Suescún *et al.*, 2013)

$$\begin{aligned} r[n] &= \beta + \frac{\Lambda}{P[n]}P^{(1)}[n] - \frac{P[0]}{P[n]}\sum_{i=1}^{6}\beta_i e^{-\lambda_i n\Delta t} \\ &- \frac{\Delta t}{P[n]}\sum_{i=1}^{6}\left[\sum_{r=0}^{n} h_i[n-r]P[r] - \frac{1}{2}[h_i[n]P[0] + h_i[0]P[n]]\right] \\ &+ \frac{\Delta t^2}{12P[n]}\sum_{i=1}^{6}[h_i^{(1)}[0]P[n] + h_i[0]P^{(1)}[n] - h_i^{(1)}[n]P[0] - h_i[n]P^{(1)}[0]] \end{aligned} \tag{14}$$

Being Δt the time step and h_i the system´s response to a unitary impulse (Haykin and Veen, 2002), whose continuous version is given by,

$$h_i(t-t') = \lambda_i \beta_i e^{-\lambda_i (t-t')} \tag{15}$$

It is possible to find a recurrence formula for temperature, using the discrete versions of equation (3) and equation (4) we obtain:

$$T[n] = T\left[n-1\right] + HP[n]\Delta t \quad ; \quad n \geq 1 \tag{16}$$

$$\rho_X[n] = \rho[n] + \alpha\left[T[n] - T[0]\right] \quad ; \quad n \geq 0 \tag{17}$$

Summarizing, in this work the total reactivity is calculated using equation(14) with the criticality condition for a reactor $\rho[0] = 0$. Doppler reactivity is calculated by the discrete version of equation (11), using the recurrence relation given by equation (16) and the external reactivity is calculated using equation (17). All these calculations were performed considering the feedback temperature.

RESULTS

In this section, the results of the different numerical experiments of the proposed method are presented. The constants in this work are the typical constants of ^{235}U, whose values for the decay constant are $\lambda_i =$ (0.0127, 0.0317, 0.115, 0.311, 1.4, 3.87) s^{-1}, the *i-th* fraction of backward neutrons $\beta_i =$ (0.000266, 0.001491, 0.001316, 0.002849, 0.000896, 0.000182) and the neutron generation time is $\Lambda = 2\times10^{-5}\ s$. The parameters $H = 0.05\,K\,cm^3 s^{-1}$ and $\alpha = 5\times10^{-5}\,K^{-1}$ were used in the temperature and reactivity equations with feedback effects. The initial temperature is fixed at $T_0 = 1000$.

Table-1 shows the results obtained for the maximum difference in the calculation of reactivity, considering a density of neutron population of the form $P(t) = \exp(\omega t)$ for different values of ω. It is observed for $\omega = 0.00243$ and $\omega = 0.006881$ that the external reactivity has an order of magnitude of 10^{-6} *pcm* with a time step of $\Delta t = 1\,s$. As the value of ω increases, there is a slight change in the significant figures for the external reactivity $\rho_X(t)$. For large values of ω, it was necessary to decrease the final simulation time because after a certain time the external reactivity suffers an abrupt rise, as illustrated in Figure-1, which shows the reactivity curve for for a neutron density of the form $P(t) = \exp(\omega t)$ with $\omega = 0.12353$, for a final simulation time $t_f = 200\,s$, initial temperature $T_0 = 1000$ and a time step $\Delta t = 1\,s$. Considering a higher value $\omega = 1.00847$. Figure-2 shows

that the external reactivity $\rho_x(t)$ also tends to increase after a certain time; the final simulation time is $t_f = 27\ s$.

Table-2 shows the maximum differences in reactivity obtained for a density of the neutron population of the form $P(t) = a + bt^3$ with $a = 1$ and the value of b is varied, with a time step $\Delta t = 0.1\ s$. The calculation is made up to a final simulation time $t_f = 100000\ s$, for reactivity without feedback $\rho(t)$, and for external reactivity $\rho_x(t)$, in order to evaluate the behaviour of external reactivity for longer times. It is possible to observe that the maximum difference in Doppler reactivity $\rho_D(t)$ is in the order of $10^{-6}\ pcm$ for $b = (0.0127)^5/9$, the reactivity values without feedback and the external reactivity have the same order of magnitude of $10^{-4}\ pcm$. In Figure-3 $\rho(t)$ and $\rho_x(t)$ are compared with for a neutron density of the form $P(t) = a + bt^3$ with $a = 1$, $b = (0.0127)^5/9$, initial temperature $T_0 = 1000$, time step of $\Delta t = 0.1\ s$ and an ample final time of $t_f = 100000\ s$, a step size of $\Delta t = 0.1\ s$ and an initial temperature $T_0 = 1000\ K$ from which it is evident that after a certain time the externalreactivity $\rho_x(t)$ tends to increase exponentially as the neutron density does.

Table-1. Maximum difference in reactivity for a neutron population density of the form $P(t) = \exp(\omega t)$ with different values of ω, initial temperature $T_0 = 1000$ and time step $\Delta t = 1\ s$.

$P(t) = \exp(\omega t)$, $\Delta t = 1\ s$, $T_0 = 1000$			**Max. Diff [*pcm*]**		
ω	$t_f\,[s]$	$t_{fT}\,[s]$	$\rho(t)$	$\rho_x(t)$	$\rho_D(t)$
0.00243	1000	1000	4.6670×10^{0}	4.6670×10^{0}	1.0670×10^{-6}
0.006881	500	500	4.6835×10^{0}	4.6835×10^{0}	1.1011×10^{-6}
0.01046	800	800	4.6967×10^{0}	4.6968×10^{0}	1.0346×10^{-4}
0.02817	600	600	4.7628×10^{0}	4.9599×10^{0}	1.9710×10^{-1}
0.12353	300	150	5.1315×10^{0}	5.1314×10^{0}	2.3990×10^{-1}
1.00847	150	20	9.7634×10^{0}	9.9896×10^{0}	2.2620×10^{-1}
11.6442	60	2	2.0639×10^{2}	2.0965×10^{2}	3.2506×10^{0}

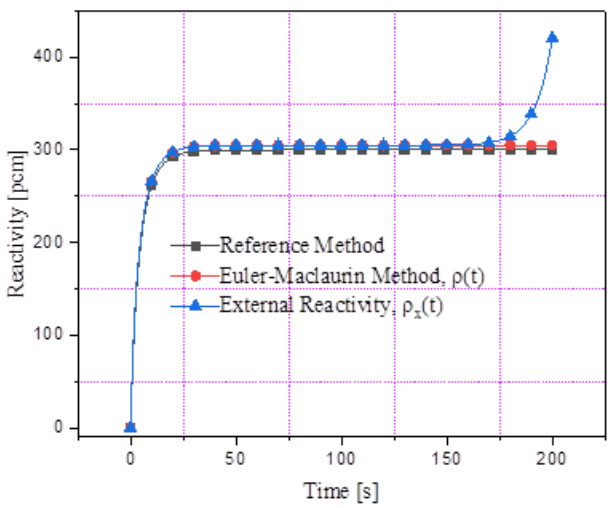

Figure-1. Comparison between $\rho_x(t)$ and $\rho(t)$.

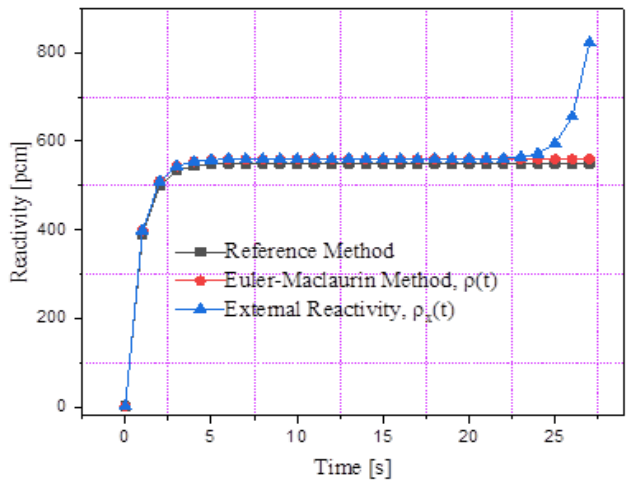

Figure-2. Comparison between $\rho_x(t)$ and $\rho(t)$.

Table-2. Maximum difference in reactivity for a neutron population density of the form $P(t) = a + bt^3$ with initial temperature $T_0 = 1000$ and time step $\Delta t = 0.1\,s$.

$P(t) = a + bt^3, \quad \Delta t = 0.1\,s, \quad T_0 = 1000$				**Max. Diff [*pcm*]**		
a	b	$t_f\,[s]$	$t_{fT}[s]$	$\rho(t)$	$\rho_x(t)$	$\rho_D(t)$
1	$(0.0127)^5/9$	100000	100000	6.1343×10^{-4}	6.1498×10^{-4}	1.5461×10^{-6}
1	$(0.0127)^4/40$	100000	100000	6.1389×10^{-4}	6.2443×10^{-4}	1.0883×10^{-5}
1	$(0.0127)^4/4$	100000	100000	6.1475×10^{-4}	7.1350×10^{-4}	9.9944×10^{-5}

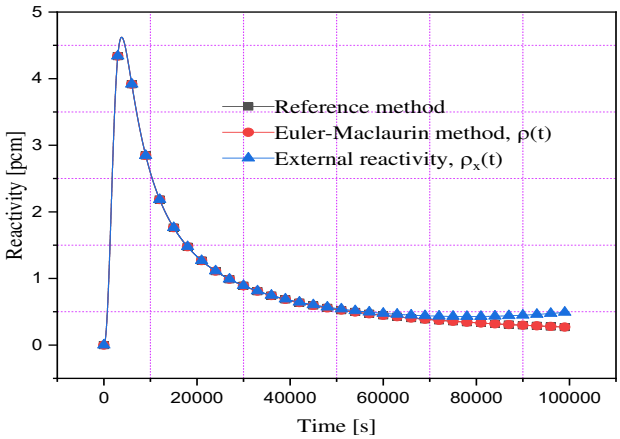

Figure-3. Comparison between $\rho_x(t)$ and $\rho(t)$.

Table-3 shows the numerical simulations for a neutron density of the form $P(t) = a + \sin(kt)$ for different values of $a = [50, 100, 150, 200]$, leaving the value of $k = \pi/10$ fixed and a time step $\Delta t = 0.1\,s$ for all the numerical experiments. It was not necessary to modify the final time to calculate the external reactivity $\rho_x(t)$ since there was no problem when calculating the reactivity for the established final time of $t_f = 160\,s$. It is observed that the maximum difference in Doppler reactivity $\rho_D(t)$ -for all numerical experiments- is in the order of 10^{-6} pcm. Figure-4 shows the difference in reactivities between the reference method, $\rho(t)$ and $\rho_x(t)$ for a neutron density $P(t) = a + \sin(k\,t)$ with $a = 100$, $k = \pi/10$, a time step of $\Delta t = 0.1\,s$ up to a final time of $t_f = 100\,s$. with an initial temperature $T_0 = 1000$.

Table-3. Maximum difference in reactivity for a neutron population density of the form $P(t) = a + \sin(k\,t)$ with initial temperature $T_0 = 1000$ and time step $\Delta t = 0.1\,s$.

$P(t) = a + \sin(kt), \quad \Delta t = 0.1\,s, \quad T_0 = 1000$				**Max. Diff [*pcm*]**		
a	k	$t_f\,[s]$	$t_{fT}[s]$	$\rho(t)$	$\rho_x(t)$	$\rho_D(t)$
50	$\pi/10$	160	160	6.1656×10^{-4}	6.1780×10^{-4}	1.2500×10^{-6}
100	$\pi/10$	160	160	6.1485×10^{-4}	6.1734×10^{-4}	2.5000×10^{-6}
150	$\pi/10$	160	160	6.1428×10^{-4}	6.1802×10^{-4}	3.7500×10^{-6}
200	$\pi/10$	160	160	6.1400×10^{-4}	6.1899×10^{-4}	5.0000×10^{-6}

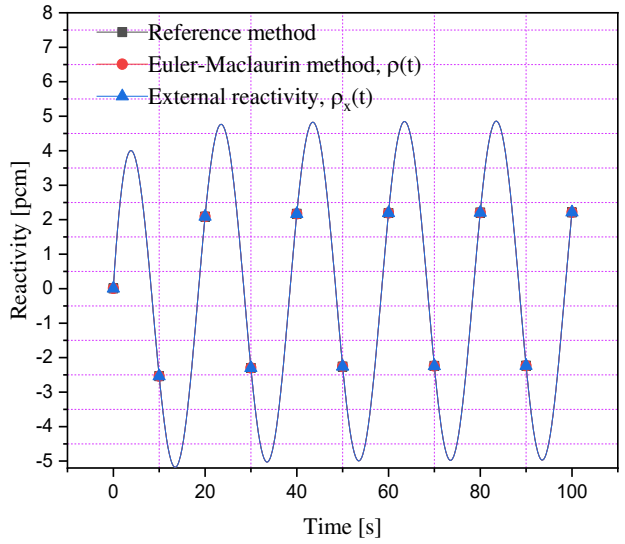

Figure-4. Comparison between $\rho_X(t)$ and $\rho(t)$.

CONCLUSIONS

In this work, the calculation of reactivity with feedback temperature dependence is shown. The reactivity is calculated recursively considering the Doppler effect. To this end, it was proposed to solve the inverse point kinetic equations with an approximation using the first Bernoulli number in the Euler-Maclaurin series. The different numerical results are valid depending on the form of the neutron density in a given simulation time interval. These simulation times can be increased for neutron density functions that produce small reactivities, the interval being shorter if the reactivity takes large values.

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