VOL. 16, NO. 1, JANUARY 2021 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

FLUCTUATION MINIMIZATION IN THE INVERSE POINT KINETIC EQUATION WITH THE FIRST BERNOULLI NUMBER AND THE SAVITZKY-GOLAY FILTER

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ABSTRACT

In this work we present the first Bernoulli number for reactivity calculation with the Savitzky-Golay filter in order to reduce fluctuations that are present in the neutron density signal; this filter uses the second degree polynomial approximation of Gram $d=2$, with different widths of the sampling window between $N=25$ y $N=225$. The fluctuations are simulated numerically considering noise with a Gaussian distribution around a mean value for the neutron density, with different standard deviations. We compare the numerical experiments using the proposed method and the Savitzky-Golay filter with the different filters reported in the literature, such as the first order delayed low-pass filter and the exponential filter.

Keywords: neutron density, euler-maclaurin, savitzky-golay filter, reactivity, numerical simulation.

INTRODUCTION

Safety in a nuclear reactor depends on monitoring the reactivity, a parameter that describes the temporal evolution of the neutron density present during the nuclear reaction and the interaction with the fissile material. The study of this behavior is done using the point kinetic equations, and the solution to these equations being an integral-differential equation, which has all the neutron density points at each point in time. This equation is called the inverse point kinetic equation, since it describes the reactivity as a function of the neutron density (Duderstadt and Hamilton, 1976).

Several papers have solved the inverse point kinetic equation in order to calculate reactivity, which is the most important parameter inside a nuclear reactor, making use of diverse methods that discretize the integral term that contains the neutron density (Shimazu *et al.*, 1987; Hoogenboom, 1989; Ansari, 1991; Suescún *et al.,* 2008; Hessam and Vosoughi, 2013). A recent paper shows high precision by using a matrix formulation in order to calculate the reactivity (Suescún *et al.*, 2018), without considering the fluctuations in the neutron density present inside the nuclear reactor. This work makes use of the Euler-Maclaurin formula (Kuen Kwok, 2010) with the first Bernoulli number approximation, in order to discretize the neutron density; at the same time, that employ the Savitzky-Golay filter (Madisetti V., 2010) in order to reduce the fluctuations. This filter is designed to reduce the noise in two phases: applying least squares to a set of given samples and processing a fix polynomial by making use of a linear function (Cadan *et al.,* 2014).

THEORETICAL ASPECTS

The point kinetic equations are a set of seven nonlinear differential coupled equations (Stacey, 2018):

$$\frac{dP(t)}{dt}=\left[\frac{\rho(t)-\beta}{\Lambda}\right]P(t)+\sum_{i=1}^{6}\lambda_i C_i(t) \tag{1}$$

$$\frac{dC_i(t)}{dt}+\lambda_i C_i(t)=\frac{\beta_i}{\Lambda}P(t) \quad ; \quad i=1,2,...,6 \tag{2}$$

$$P(t=0)=P_0 \tag{3}$$

$$C_i(t=0)=\frac{\beta_i}{\Lambda\lambda_i}P_0 \tag{4}$$

$P(t)$ being the neutron density, C_i the precursor concentration, β_i the i-th fraction of delayed neutrons, $\rho(t)$ the reactivity, Λ the neutron generation time, β the total effective fraction of delayed neutrons and λ_i the decay constant of the i-th delayed neutron precursor group. When using the initial conditions, given by equations (3-4), we must set the reactivity at the initial time to be null. This to obtain a reactor at a critical state. Solving the equations (1-4), we can obtain the reactivity as a function of the neutron density:

$$\rho(t)=\beta+\frac{\Lambda}{P(t)}\frac{dP(t)}{dt}-\frac{P(0)}{P(t)}\sum_{i=1}^{6}\beta_i e^{-\lambda_i t}-\frac{1}{P(t)}\sum_{i=1}^{6}\int_0^t \lambda_i\beta_i e^{-\lambda_i(t-t')}P(t')dt' \tag{5}$$

Equation (5) is known as the inverse point kinetic equation. The integral term is dependent of the neutron density. It is necessary to discretize this term in order to decrease the computational cost of the calculation. Equation (5) can be written as,

$$\rho(t)=\rho_{in}(t)-\frac{1}{P(t)}\sum_{i=1}^{6}\int_{0}^{t}\lambda_i\beta_i e^{-\lambda_i(t-t')}P(t')dt' \qquad (6)$$

Where,

$$\rho_{in}(t)=\beta+\frac{\Lambda}{P(t)}\frac{dP(t)}{dt}-\frac{P(0)}{P(t)}\sum_{i=1}^{6}\beta_i e^{-\lambda_i t} \qquad (7)$$

Equation (5) represents an expression for reactivity. It is used in the different methods which have been proposed, since it is the basis for the construction of digital reactivity meters, however there are difficulties in the implementation of this expression in real time, and it is therefore necessary to discretize the integral term known as neutron population density history.

PROPOSED METHOD

The Euler-Maclaurin formula consist in an equivalence between continuous and discrete time (Kuen Kwok, 2010) which contains the Bernoulli numbers B_x:

$$\int_0^n F(y)dy=\sum_{r=1}^{n-1}F[r]+\frac{1}{2}\left[F[0]+F[n]\right]-\sum_{x=1}^{\infty}\frac{B_x}{(2x)!}\left[F^{(2x-1)}[n]-F^{(2x-1)}[0]\right] \qquad (8)$$

In this work, we consider the approximation considering the first Bernoulli number on equation (8), this is, $x=1$, and we get:

$$\int_0^n F(y)dy=\sum_{r=1}^{n-1}F[r]+\frac{1}{2}\left[F[0]+F[n]\right]-\frac{B_1}{2!}\left[F^{(1)}[n]-F^{(1)}[0]\right] \qquad (9)$$

being $B_1=\frac{1}{6}$.

The function F considered on equations (8-9), represents the approximation between the continuous and discrete times F(t) and F[n] respectively. Comparing the integrand F(t) on equations (8-9) with the integrand on equation (6), we get:

$$F(t')=h_i(t-t')P(t') \qquad (10)$$

The discrete version of equation (10) is,

$$F[r]=h_i[n-r]P[r] \qquad (11)$$

Being h_i, the response of the system to a unitary impulse function (Haykin S, 1999) defined by:

$$h_i(t-t')=\lambda_i\beta_i e^{-\lambda_i(t-t')} \qquad (12)$$

It is necessary to derivate once the equations (10-11), and evaluate this result at *r=0 and r=n*; replacing this on equation (9) we get:

$$\int_0^n h_i(t-t')P(t')dt'=\Delta t\left[\sum_{r=0}^{n}h_i[n-r]P[r]-\frac{1}{2}[h_i[n]P[0]+h_i[0]P[n]]\right] -\frac{\Delta t^2}{12}\left[h_i^{(1)}[0]P[n]+h_i[0]P^{(1)}[n]-h_i^{(1)}[n]P[0]+h_i[n]P^{(1)}[0]\right] \qquad (13)$$

Being Δt the time step.

Replacing equation (13) into equation (6), the reactivity with the first Bernoulli number approximation is obtained:

$$\rho[n]=\rho_{ind}[n]-\frac{\Delta t}{P[n]}\sum_{i=1}^{6}\left[\sum_{r=0}^{n}h_i[n-r]P[r]-\frac{1}{2}[h_i[n]P[0]+h_i[0]P[n]]\right] +\frac{\Delta t^2}{12P[n]}\sum_{i=1}^{6}\left[h_i^{(1)}[0]P[n]+h_i[0]P^{(1)}[n]-h_i^{(1)}[n]P[0]-h_i[n]P^{(1)}[0]\right] \qquad (14)$$

Where,

$$\rho_{ind}[n]=\beta+\frac{\Lambda}{P[n]}P^{(1)}[n]-\frac{P[0]}{P[n]}\sum_{i=1}^{6}\beta_i e^{-\lambda_i nT} \qquad (15)$$

In order to analyze the fluctuations, present inside the reactor, noise with a Gaussian distribution around a median value of the neutron density is used (Kitano *et al.*, 2000), represented by the expression:

$$\overline{P_l}=\frac{1}{N}\sum_{j=1}^{N}P_j \qquad (16)$$

Recently, the Euler-Maclaurin method with a first order delayed low-pass filter (Suescún *et al.,* 2020) was used to reduce the fluctuations on the neutron density signal; such filter is represented, according to (Shimazu *et al.,* 1987), by the formula:

$$P_i=P_{i-1}+\frac{\Delta t}{\Delta t+\tau}(\overline{P_l}-P_{i-1}) \qquad (17)$$

Where τ is the filtering constant.

Another filter used to reduce the fluctuations present inside a nuclear reactor, is the exponential filter (Mathews, 2000):

$$u(y)=Ce^{Ay} \qquad (18)$$

where A, C are constants obtained using the least squares criterion. This is achieved with the neutron density signal, obtained by means of the different sensors located externally on the reactor; the average of these values is adjusted to an exponential shape.

Due to convenience, it is better to work with linear equations. Equation (18) can be linearized by using the following substitutions,

$$U = \ln(u) \quad , \quad Y = y \quad , \quad B = \ln(C) \tag{19}$$

Constants *A* and *B* on equation (19) is calculated making use of the least square method using the normal Gauss equations, having:

$$\left(\sum_{k=1}^{N} x_k^2\right) A + \left(\sum_{k=1}^{N} x_k^2\right) B = \sum_{k=1}^{N} x_k y_k \tag{20}$$

$$\left(\sum_{k=1}^{N} x_k\right) A + FB = \sum_{k=1}^{N} y_k \tag{21}$$

In order to reduce the fluctuations that are originated in the neutron density, the Savitzky-Golay filter is used. We suppose there is a vector *x* that contains the information of the data of a signal with noise (Cadan *et al*., 2014) given by,

$$x = \left[x_{-M}, ..., x_{-1}, x_0, x_1, x_2, x_3, ..., x_M\right]^T \tag{22}$$

where *M* is the half width, which is related with the number of samples *N* in each block of information, given by $N = 2M + 1$, for a total number L of samples to be filtered.

It is necessary to consider a linear combination of base vectors, set in the following way:

$$s_i(n) = n^i \quad , \quad -M \le n \le M \tag{23}$$

where $i = 0,1,2,...,d$. Each value of d, indicates the degree of the polynomial being used to reduce the fluctuations present on vector *x*.

Now, we define a column matrix $S_{(2M+1)\times(d+1)}$ such that each component contains a base vector s_i:

$$S = \left[s_0, s_1, s_2, ..., s_d\right] \tag{24}$$

On equation (24) the noise is present, and it is necessary to decrease it; for such task, we build a polynomial $\hat{x}$ of degree *d*, having:

$$\hat{x} = \sum_{i=0}^{d} a_i s_i = \left[s_0, s_1, s_2, ..., s_d\right] \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_d \end{bmatrix} \tag{25}$$

Being a_i the polynomial coefficient vector.

It is possible to define an error as the difference between the estimated by equation (25) and the signal that contains the vector with the noise, given by equation (24), as:

$$e = x - \hat{x} = x - S_C \tag{26}$$

Minimizing the error on equation (26) by using the least squares method, we obtain:

$$S^T S_C = S^T x \tag{27}$$

Solving for *c* on equation (27), we obtain:

$$c = \left(S^T S\right)^{-1} S^T x = G^T x = \left[g_0, g_1, g_3, ..., g_d\right]^T x \tag{28}$$

It is possible to solve for matrix G on equation (28) to obtain,

$$G = SF^{-1} = S\left(S^T S\right)^{-1} \tag{29}$$

Using equation (17), we can estimate the vector $\hat{x}$, which has the filtered signal given by,

$$\hat{x} = S_C = SG^T x = S\left(S^T S\right)^{-1} S^T x = Bx \tag{30}$$

The estimated signal can be written as:

$$\hat{x} = Bx \Leftrightarrow \hat{x}_m = b_m^T x \quad , \quad -M \le m \le M \tag{31}$$

Equation (31) can be written as a sum of convolutions such:

$$\hat{x}(n) = \sum_{m=-M}^{M} b_0(m) x(n+m) = \sum_{m=-M}^{M} b_0(-m) x(n-m), \quad n = 0,1,...,L-1 \tag{32}$$

Some difficulties can occur if the convolution given by equation (32) is used. One such difficulties can be, that depending on the half width *M*, it is possible that a high computational cost is required. Another possibility is that sparse matrices can occur, which also increase the computational cost. It is recommended to calculate an equivalent form, but with less effort. This is achieved using the Gram polynomial, represented by:

ARPN Journal of Engineering and Applied Sciences

$$\hat{x}(n) = \sum_{i=0}^{d} b_i f_d^M(n) \tag{33}$$

Being $f_d^M(n)$ a Gram polynomial of order d such that:

$$f_d^M(n) = \sum_{i=0}^{d} \left(\frac{(-1)^{i+1}(i+d)^{(2i)}(M+n)^i}{(i!)^2(2M)^{(i)}} \right) \tag{34}$$

Replacing equation (34) on equation (33) and applying the least squares method, it is possible to write an equation which facilitates the reduction of the neutron density signal fluctuation, simply put:

$$\hat{x}(n) = \sum_{r=-M}^{M} \sum_{i=0}^{d} \left(\frac{(2i+1)(2M)^{(i)}}{(2M+i+1)^{(i+1)}} \right) f_d^M(r) f_d^M(n) x_r \; ; \; n, r = 0,1,2,...,L-1 \tag{35}$$

The Gram polynomials on equation (35), are calculated taking into account the recursive formula:

$$f_d^M(r) = \left(\frac{2(2d-1)}{i(2M-d+1)} i g_{i-1}^M(r) \right) - \left(\frac{(d-1)(2M+d)}{i(2M-d+1)} g_{i-2}^M(r) \right) \tag{36}$$

RESULTS AND DISCUSSIONS

In this section we present the different numerical experiments considering the first Bernoulli number in the Euler-Maclaurin method, having noise in the neutron density and filtering using Savitzky-Golay. The noise is generated using a Gaussian distribution, with 10 samples and with a generating seed of random numbers of 2^{31}-1. The standard deviation is varied between *σ=0.001* and *σ=0.01*. The time step in the reactivity calculation is $\Delta t = 0.01s$ and $\Delta t = 0.1s$. The constants used in this work are commonly present in the interaction of neutrons with the fuel element ^{235}U, which are the decay constants $\lambda_1 = 0.0127\, s^{-1}$; $\lambda_2 = 0.0317\, s^{-1}$; $\lambda_3 = 0.115\, s^{-1}$; $\lambda_4 = 0.311\, s^{-1}$; $\lambda_5 = 1.4\, s^{-1}$ and $\lambda_6 = 3.87\, s^{-1}$, the delayed neutron fraction $\beta_1 = 0.000266$; $\beta_2 = 0.001491$; $\beta_3 = 0.001316$; $\beta_4 = 0.002849$; $\beta_5 = 0.000896$ and $\beta_6 = 0.000182$, the instantaneous neutron generation time $\Lambda = 2\times10^{-5}\, s$ .For the Savitzky-Golay filter, we employ a Gram polynomial of order *d=2* and a sample size that is varied between *N=25* and *N=225*. The accuracy of the method and the Savitzky-Golay method, S-G, is compared with the combination of the Euler-Maclaurin method and the first order delayed low-pass filter with a filtering constant $\tau = 1.5$ and the combination of the Euler-Maclaurin method with an exponential filter with a sample numbers of 10.

Tables 1-2 show the results for the mean absolute error and for the maximum difference between reactivity in pcm (per cent mille), respectively. The standard deviation is fixed at $\sigma = 0.001$ and the time step $\Delta t = 0.01s$. On Table-1 it is possible to evidence that the proposed method, with a sample size of *N=25*, reduces the fluctuations for the values $0.00243 \le \omega \le 11.6442$ when comparing with the first order delayed low-pass filter, which has good results only for small reactivity values. If the sample size is increased in the proposed method to *N=225*, it is evident the reduction in the fluctuations for $0.00243 \le \omega \le 0.12353$ compared with the exponential filter, which shows good results, even for big values of the reactivity. On Table-2 we note, that when the S-G filter is used with a sample size of *N=25*, the maximum difference in the reactivity calculation, shows good results for the range $0.00243 \le \omega \le 1.00847$.

Table-1. Mean absolute error for a standard deviation of $\sigma = 0.001$ and a time step of $\Delta t = 0.01s$.

$P(t) = e^{\omega t}$		Mean Absolute Error				
		E-M	**E-M with Low-Pass Filter**	**E-M with Exponential Filter**	**E-M with S-G Filter *N=25***	**E-M with S-G Filter *N=225***
$\omega = 0.00243$	t_f*=1000 s*	0.54	0.05	0.23	0.16	0.05
$\omega = 0.006881$	t_f*=500 s*	0.52	0.18	0.22	0.15	0.05
$\omega = 0.01046$	t_f*=800 s*	0.50	0.16	0.21	0.15	0.04
$\omega = 0.02817$	t_f*=600 s*	0.44	0.39	0.19	0.13	0.04
$\omega = 0.12353$	t_f*=300 s*	0.32	1.43	0.14	0.09	0.03
$\omega = 1.00847$	t_f*=150 s*	0.12	2.99	0.05	0.04	1.01
$\omega = 11.6442$	t_f*=60 s*	0.02	3.33	0.01	0.66	288.19
$\omega = 52.80352$	t_f*=10 s*	0.06	5.77	0.01	107. 22	642.74

VOL. 16, NO. 1, JANUARY 2021 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

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Table-2. Maximum differences for a standard deviation of $\sigma = 0.001$ and a time step of $\Delta t = 0.01s$.

$P(t) = e^{\omega t}$		Maximum Differences [pcm]				
		E-M	**E-M with Low-Pass Filter**	**E-M with Exponential Filter**	**E-M with S-G Filter *N=25***	**E-M with S-G Filter *N=225***
$\omega = 0.00243$	t_f=*1000 s*	3.25	1.53	1.73	0.88	0.23
$\omega = 0.006881$	t_f=*500 s*	3.10	4.24	1.65	0.84	0.22
$\omega = 0.01046$	t_f=*800 s*	3.01	6.38	1.60	0.81	0.21
$\omega = 0.02817$	t_f=*600 s*	2.67	16.46	1.41	0.72	0.19
$\omega = 0.12353$	t_f=*300 s*	2.14	60.27	0.94	0.50	0.18
$\omega = 1.00847$	t_f=*150 s*	1.49	228.08	0.74	0.33	146.22
$\omega = 11.6442$	t_f=*60 s*	1.12	452.41	0.41	372.59	1440200.00
$\omega = 52.80352$	t_f=*10 s*	7.71	322.45	0.45	49344.00	350990.00

On Tables 3-4, the time step in the reactivity calculation takes the value of $\Delta t = 0.1s$ maintaining a standard deviation of $\sigma = 0.001$. It is evident that when there are small values for the reactivity, the recommended sample size is *N=225* since it has better reduction to the fluctuations, the mean absolute error is 0.03 pcm at $\omega = 0.01046$. With respect to the maximum difference in the reactivity, we observe that the sample size *N=25* works well, even for big values of ω. For a value of $\omega = 52.80352$ the maximum difference reaches 8.53×10^3, contrary to what happens if the window width is *N=225* where it does not converge to a value.

On Tables 5-6 we increase the standard deviation to $\sigma = 0.01$ and we use a time step of $\Delta t = 0.01s$. The S-G filter reduces the fluctuations well, with a window width of *N=25*, when the reactivity is $\rho \leq 700\ pcm$. However, when we increase the sample size to *N=225*, the method increases precision, but it is reduced for reactivities such that $\rho \geq 550\ pcm$, this mean, for $\omega \geq 1.00847$.

Table-3. Mean absolute error for a standard deviation of $\sigma = 0.001$ and a time step of $\Delta t = 0.1s$.

$P(t) = e^{\omega t}$		Mean Absolute Error				
		E-M	**E-M with Low-Pass Filter**	**E-M with Exponential Filter**	**E-M with S-G Filter *N=25***	**E-M with S-G Filter *N=225***
$\omega = 0.00243$	t_f=*1000 s*	0.53	0.09	0.22	0.15	0.04
$\omega = 0.006881$	t_f=*500 s*	0.51	0.25	0.20	0.14	0.04
$\omega = 0.01046$	t_f=*800 s*	0.49	0.28	0.20	0.13	0.03
$\omega = 0.02817$	t_f=*600 s*	0.44	0.69	0.17	0.11	0.05
$\omega = 0.12353$	t_f=*300 s*	0.30	2.60	0.12	0.08	5.37
$\omega = 1.00847$	t_f=*150 s*	0.11	10.12	0.05	1.30	198.81
$\omega = 11.6442$	t_f=*60 s*	13.27	137.18	3.79	52.91	1046.9
$\omega = 52.80352$	t_f=*10 s*	2.19×10^{18}	8.60×10^{19}	29.10	222.67	Infinite

Table-4. Maximum differences for a standard deviation of $\sigma = 0.001$ and a time step of $\Delta t = 0.1s$.

$P(t) = e^{\omega t}$		Maximum Differences [pcm]				
		E-M	**E-M with Low-Pass Filter**	**E-M with Exponential Filter**	**E-M with S-G Filter *N=25***	**E-M with S-G Filter *N=225***
$\omega = 0.00243$	t_f=1000 s	2.56	1.79	1.24	0.66	0.14
$\omega = 0.006881$	t_f=500 s	2.44	4.69	1.19	0.63	0.13
$\omega = 0.01046$	t_f=800 s	2.36	7.04	1.15	0.60	0.12
$\omega = 0.02817$	t_f=600 s	2.08	18.07	1.04	0.51	1.33
$\omega = 0.12353$	t_f=300 s	1.45	66.00	0.78	0.33	228.51
$\omega = 1.00847$	t_f=150 s	1.09	250.02	0.47	187.22	81000.00
$\omega = 11.6442$	t_f=60 s	391.77	2144.75	29.19	10086.00	326000.00
$\omega = 52.80352$	t_f=10 s	4.83×10^{19}	1.09×10^{21}	259.69	8526.6	Infinite

Table-5. Mean absolute error for a standard deviation of $\sigma = 0.01$ and a time step of $\Delta t = 0.01s$.

$P(t) = e^{\omega t}$		Mean Absolute Error				
		E-M	**E-M with Low-Pass Filter**	**E-M with Exponential Filter**	**EM with S-G Filter *N=25***	**EM with S-G Filter *N=225***
$\omega = 0.00243$	t_f=1000 s	5.43	0.24	2.34	1.60	0.48
$\omega = 0.006881$	t_f=500 s	5.21	0.34	2.25	1.53	0.47
$\omega = 0.01046$	t_f=800 s	5.04	0.33	2.17	1.48	0.45
$\omega = 0.02817$	t_f=600 s	4.49	0.53	1.93	1.31	0.39
$\omega = 0.12353$	t_f=300 s	3.23	1.53	1.39	0.94	0.28
$\omega = 1.00847$	t_f=150 s	1.22	3.02	0.52	0.35	1.10
$\omega = 11.6442$	t_f=60 s	0.20	3.34	0.08	0.69	324.57
$\omega = 52.80352$	t_f=10 s	0.40	6.09	0.17	54.09	648.13

Table-6. Maximum differences for a standard deviation of $\sigma = 0.01$ and a time step of $\Delta t = 0.01s$.

$P(t) = e^{\omega t}$		Maximum Differences [pcm]				
		E-M	**E-M with Low-Pass Filter**	**E-M with Exponential Filter**	**E-M with S-G Filter *N=25***	**E-M with S-G Filter *N=225***
$\omega = 0.00243$	t_f=1000 s	34.02	1.95	16.96	8.87	2.35
$\omega = 0.006881$	t_f=500 s	32.50	4.55	16.19	8.47	2.23
$\omega = 0.01046$	t_f=800 s	31.49	6.66	15.68	8.20	2.15
$\omega = 0.02817$	t_f=600 s	27.95	16.69	13.88	7.26	1.88
$\omega = 0.12353$	t_f=300 s	22.13	60.54	9.26	4.98	1.25
$\omega = 1.00847$	t_f=150 s	14.59	228.31	7.38	2.80	146.46
$\omega = 11.6442$	t_f=60 s	11.44	452.42	4.08	365.16	1658300.00
$\omega = 52.80352$	t_f=10 s	94.38	320.22	5.11	31999.00	356360.00

ARPN Journal of Engineering and Applied Sciences

Tables 7-8 show that the values obtained when increasing the time step to $\Delta t = 0.1s$, maintaining the same standard deviation. It is possible to observe that the fluctuations are reduced for a sample size of *N=225*, with the S-G filter. Good results are obtained for values of $\omega \leq 0.02817$ compared with the exponential filter, and even for a sample size of *N=25*.

Table-7. Mean absolute error for a standard deviation of $\sigma = 0.01$ and a time step of $\Delta t = 0.1s$.

$P(t) = e^{\omega t}$		Mean Absolute Error				
		E-M	**E-M with Low-Pass Filter**	**E-M with Exponential Filter**	**E-M with S-G Filter *N=25***	**E-M with S-G Filter *N=225***
$\omega = 0.00243$	t_f=1000 s	5.35	0.65	2.19	1.39	0.33
$\omega = 0.006881$	t_f=500 s	5.10	0.74	2.05	1.31	0.32
$\omega = 0.01046$	t_f=800 s	4.97	0.72	2.02	1.29	0.30
$\omega = 0.02817$	t_f=600 s	4.42	0.95	1.77	1.12	0.27
$\omega = 0.12353$	t_f=300 s	3.09	2.63	1.21	0.76	5.54
$\omega = 1.00847$	t_f=150 s	1.18	10.13	0.46	1.65	199.93
$\omega = 11.6442$	t_f=60 s	100.41	1141.1	8.12	139.11	833.75
$\omega = 52.80352$	t_f=10 s	2.19×10^{19}	8.62×10^{20}	39.79	224.74	Infinite

Table-8. Maximum differences for a standard deviation of $\sigma = 0.01$ and a time step of $\Delta t = 0.1s$.

$P(t) = e^{\omega t}$		Maximum Differences [pcm]				
		E-M	**E-M with Low-Pass Filter**	**E-M with Exponential Filter**	**E-M with S-G Filter *N=25***	**E-M with S-G Filter *N=225***
$\omega = 0.00243$	t_f=1000 s	25.13	3.66	12.59	6.55	1.35
$\omega = 0.006881$	t_f=500 s	24.17	6.34	12.08	6.19	1.27
$\omega = 0.01046$	t_f=800 s	23.64	8.59	11.74	5.96	1.22
$\omega = 0.02817$	t_f=600 s	21.32	18.91	10.54	5.13	1.42
$\omega = 0.12353$	t_f=300 s	14.21	66.16	7.92	3.32	228.79
$\omega = 1.00847$	t_f=150 s	10.57	249.85	4.65	182.66	81930.00
$\omega = 11.6442$	t_f=60 s	14568.00	20900.00	20.20	25678.00	158060.00
$\omega = 52.80352$	t_f=10 s	4.83×10^{20}	1.09×10^{22}	330.05	8525.6	Infinite

CONCLUSIONS

The numerical experiments done in this work, using the Euler-Maclaurin method with an approximation of the first Bernoulli number and Savitzky-Golay filter with a Gram polynomial of order *d=2*, were done so that we could reduce the fluctuations present in a nuclear reactor. Such fluctuations in the neutron density were simulated with a noise with a Gaussian distribution. We used values for the standard deviation between σ=0.001 and σ=0.01. The results were compared with the Euler-Maclaurin method combined with the first order delayed low-pass filter and with Euler-Maclaurin method combined with the exponential filter. We showed that when the sample size of *N=225* in the Savitzky-Golay filter, the precision of the method is reduced when the reactivity increases, however, when the sample size was *N*=25, the results are a better approximation.

ACKNOWLEDGEMENTS

The authors wish to thank the *Young Researcher* program of the Vice chancellor for research and social development of Universidad Surcolombiana.

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