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OPTIMIZATION OF LOW SPEED WIND TURBINE BLADE PROFILE ON THE BASIS OF LIFT COEFFICIENT

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ABSTRACT

Design and optimization of airfoil is the essential part of aerodynamic analysis. Airfoil optimization is also very important for manufacturing wind turbine blades and to leverage the power output. Although plenty of research has already been conducted on high speed airfoil optimization, the research in very low speed airfoil optimization is limited. Moreover, wind energy sources are one of most popular energy sources now and low-speed airfoil designing is becoming a fascination for the researcher in this field. In this paper, an airfoil optimization analysis has been performed using genetic algorithm (GA) and PARSEC parameterization technique with vortex panel method. Four airfoils, namely NACA-2412, NACA-4412, NACA-4418, and NACA-4421 have been considered as these are very popular in practice. The result shows that, the optimized airfoils generated from the original airfoils have better lift performance with reduced area which will help in reducing raw material in manufacturing process. Thus, these optimized airfoils will be suitable for manufacturing lift-driven low-speed wind turbine blades and for other related low speed applications also.

Index term: wind turbine, optimization, genetic algorithm and lift coefficient.

1. INTRODUCTION

Wind energy is one of the most popular renewable energy sources in the world. Maximizing energy from a particular wind speed is challenging because of the low cut-off speed of wind turbine blades. Shape optimization of the airfoils is often considered to overcome this barrier and is found to be important to design wind turbine blades. Airfoil shape optimization is used in the aerospace and mechanical engineering industry for decades. Recent advancement of optimization techniques and computational power enhance the accuracy of airfoil optimizations.

There are many airfoil shape optimization techniques available in practice. Gradient based approaches are popular for precise optimizations. Evolutionary algorithms are also becoming popular now-a-days and popular algorithms are Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) algorithm. An evolutionary algorithm-based approach using PARSEC parameterization technique is conducted by Vecchia *et al.* [1]. They used PARSEC to parameterize the airfoil and use GA to find the Nash Equilibria (NE). Shahrokhi and Jahangirian [2] conducted an optimization study using GA but they used unstructured grid Navier-Stokes flow solver with a two-equation k-ε turbulence model. Another study conducted by Saleem and Kim [3] using Reynolds Averaged Navier-Stokes (RANS) equations along with k-ω SST turbulence model along with GA. A similar work has been conducted by Ivanov *et al.* [4] for vertical axis wind turbine (VAWT) using GA and Class-Shape Transformation (CST) parameterization techniques. Ebrahimi and Jahangirian [5] proposed a new parameterization technique with GA and compared the outcomes of the new method with popular method PARSEC. Multi-objective optimizations are also very popular for complex and sophisticated shape optimization problems. He *et al.* [6] proposed a new multipoint infill criterion with preference-driven kringing-based multi-objective optimization method to optimize shape of airfoils. Lim and Kim [7] conducted a multi-objective aerodynamic shape optimization using Multi-objective Genetic Algorithm (MOGA). Their approach of adaptive hybrid algorithm performs better for multi-objective optimizations. Wickramasinghe *et al.* [8] performed particle swarm optimization (PSO) to a multi-objective aerodynamic shape optimization problem and found a better result. A multi-objective genetic algorithm (MOGA) with gradient based method had studied by Ariyarit *et al.* [9]. They had found that, this combined approach is very useful for real-world problems.

Some data driven approaches are also getting attentions from the researchers. Li *et al.* [10] showed a data driven approach and demonstrated that this approach provides better low-speed off-design performance without sacrificing on-design performances. Xiao and Wu [11] conducted a research on the data–driven CFD works showed that current data sets have to be improved and tailor-made to perform better with machine learning models. Renganathan *et al.* [12] also studied machine learning for simulating flow past an airfoil in the transonic regime. They proposed a machine learning method to construct a reduced-order model via deep neural network which can produce results with higher accuracy in a lower computational cost. Artificial neural network (ANN) is also a great tool in data-driven problem solving. Bouhlel *et al.* [13] used Reynolds-averaged Navier-Stokes (RANS)-based CFD model with ANN to perform airfoil shape optimization problem in a short computational time.

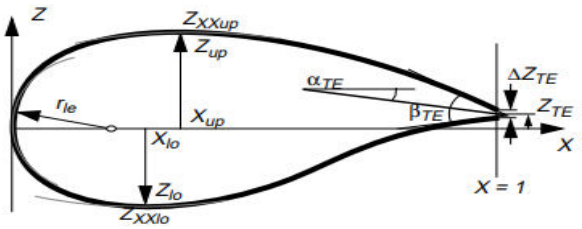

Figure-1. Airfoil Geometry defined by 11 design parameters in PARSEC method.

To perform a computational fluid dynamics (CFD) analysis, types of grid plays an important role. For a particular CFD problem, fine grids are called high-fidelity models and the same problem with coarse grid is called low-fidelity models and the type can be determined through a grid convergence study [14]. Han *et al.* [14] used a Multi-level Hierarchical Kriging (MHK) algorithm to perform an airfoil optimization task. They claimed to propose a new approach of variable-fidelity surrogate model which notably improves the optimization efficiency as well as outperformed single-fidelity or two-level-fidelity models. Madsen *et al.* [15] used 3-D RANS approach to perform high-fidelity multipoint shape optimization of a wind turbine blade and reported that the cross-sectional shape and the blade planform can be optimized simultaneously. Robustness means the ability of airfoil shape optimization process to converge to the actual shape starting from a wide range of different shapes and flow parameters. He *et al.* [16] conducted a research of robust aerodynamic shape optimization from an initial shape of a circle to an airfoil. They used a Reynolds-averaged Navier-Stokes (RANS) based CFD solver along with a B-spline FFD parameterization technique and conclude that, adaptive FFD approach leverages the robustness of optimization can be useful in explanatory design optimization.

Accurate parameterization is essential to achieve a good optimization result. Zhang *et al.* [17] compared the parameterization techniques and summarizes the results and showed that, parametric section or PARSEC is the most intuitive method. Sripawadkul *et al.* [18] also worked on a similar comparison study and also concluded that PARSEC method has the highest intuitiveness. The Class/Shape Transformation (CST) method is also a very popular method with a good accuracy. Lane *et al.* [19] used CST method for inverse airfoil design approach. Derksen and Rogalsky [20] used Bezier polynomial curve with PARSEC method and proposed a new approach of airfoil parameterization. They showed that, this technique improves the robustness and convergence speed or airfoil optimization problems. Oyama *et al.* [21] also constructed similar result and also agreed that, Bezier-PARSEC method is more suitable for genetic algorithm (GA). Kharal and Saleem [22] illustrated the way of determining airfoil geometry from a given C_p distribution using Bezier-PARSEC parameterization technique and feed-forward back-propagation neural network (NN).

A variety of methods are being used to compute the flow over an airfoil at a small angle of attack. Panel method is one of them and very convenient to use for optimization problems. Schmitz and Chattot [23] illustrated a novel approach of combining commercial CFD software with vortex panel method and also validated the results. Conlan-Smith *et al.* [24] also showed another way of optimizing airfoil shape using panel method.

A plenty of research works have been carried out to design, optimize and analyze the airfoils and new computational methods and algorithms are developed with the upgradation of computational platforms. Though a significant amount of study has been done on airfoil optimization, a very few studies only concentrate particularly on low speed airfoils and its aerodynamic behavior. In contrast, low speed airfoils are not only used in the aerospace industry but also used to design wind turbine blades now a days. To amplify the potential of harvesting wind energy in the low speed region, design and optimization of airfoil is very important. An in-depth research is needed to investigate the potential of airfoil optimization techniques for designing low Reynolds number flows. In this research work we are motivated to focus specially on very low speed airfoils and low Reynolds number flow behavior. Existing popular methods and algorithms are used to investigate the potential of low speed airfoil in practice. This study shows that, low speed airfoils can be optimized to maximize the lift and eventually can be very useful to design wind turbine blades. Additionally, this research work shows a comparison of different widely used low-speed airfoils.

2. PARAMETERIZATION

Parameterization of airfoil is one of the most important tasks for initiating an optimization work. Proper parameterization gives the mobility to perform different optimization algorithm by generating various shapes. From the above-mentioned literature review and discussion on previous research works shows that, PARSEC is the most intuitive technique of airfoil parameterization. In this study, we have used PARSEC as a tool for parameterize given airfoils. Parameterization of airfoil is one of the most important tasks for initiating an optimization work. Proper parameterization gives the mobility to perform different optimization algorithm by generating various shapes. From the above-mentioned literature review and discussion on previous research works shows that, PARSEC is the most intuitive technique of airfoil parameterization. In this study, we have used PARSEC as a tool for parameterize given airfoils.

3. PARSEC

PARSEC is a parameterization technique that uses 11 design parameters [25, 26] to express an airfoil which gives higher flexibility to modify and generate arbitrary shapes. Figure-1 shows all the eleven design parameters labeled in an airfoil. The details of the figure summarize in the Table-1. The mathematical expression [1] defining the upper and lower curve of an airfoil is shown in equation (1):

$$Z_{Upper} = \sum_{i=i}^{6} a_{upper}^{i} \cdot x^{i-0.5} \quad (1)$$

$$Z_{lower} = \sum_{i=1}^{6} a_{lower}^{i} \cdot x^{i-0.5} \quad (2)$$

Z_{upper} and Z_{lower} are the coordinates of the upper and lower surface of the airfoil. x is the horizontal coordinate normalized in a range of 0 to 1. a_{upper} And a_{lower} are the coefficients and can be determined from the given 11 design parameters from the mathematical relation [1] as follows:

$$C_{upper} \times a_{upper} = b_{upper} \quad (3)$$

$$C_{lower} \times a_{lower} = b_{lower} \quad (4)$$

The definition of C_{upper}, C_{lower}, b_{upper}, and b_{lower} are as follows:

$$C_{upper} = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ p_2^{\frac{1}{2}} & p_2^{\frac{3}{2}} & p_2^{\frac{5}{2}} & p_2^{\frac{7}{2}} & p_2^{\frac{9}{2}} & p_2^{\frac{11}{2}} \\ \frac{1}{2} & \frac{3}{2} & \frac{5}{2} & \frac{7}{2} & \frac{9}{2} & \frac{11}{2} \\ \frac{1}{2}p_2^{-\frac{1}{2}} & \frac{3}{2}p_2^{\frac{1}{2}} & \frac{5}{2}p_2^{\frac{3}{2}} & \frac{7}{2}p_2^{\frac{5}{2}} & \frac{9}{2}p_2^{\frac{7}{2}} & \frac{11}{2}p_2^{\frac{9}{2}} \\ -\frac{1}{4}p_2^{-\frac{3}{2}} & \frac{3}{4}p_2^{-\frac{1}{2}} & \frac{15}{4}p_2^{\frac{1}{2}} & \frac{15}{4}p_2^{\frac{1}{2}} & \frac{63}{4}p_2^{\frac{5}{2}} & \frac{99}{4}p_2^{\frac{7}{2}} \\ 1 & 0 & 0 & 0 & 0 & 0 \end{vmatrix} \quad (5)$$

$$b_{upper} = \begin{vmatrix} p_8 + p_9/2 \\ p_3 \\ \tan(p_{10} - p_{11}/2) \\ 0 \\ p_4 \\ \sqrt{2p_1} \end{vmatrix} \quad (6)$$

$$b_{lower} = \begin{vmatrix} p_8 - p_9/2 \\ p_6 \\ \tan(p_{10} + p_{11}/2) \\ 0 \\ p_4 \\ \sqrt{2p_1} \end{vmatrix} \quad (7)$$

C_{lower} is defined in a similar manner like Eq. (5) using p_5 instead of p_2.

Table-1. Parameters and related definition.

PARSEC Parameter	Geometry Parameter	Definition
p_1	r_{le}	Leading edge radius
p_2	X_{up}	Upper crest location in horizontal coordinates
p_3	Z_{up}	Upper crest location in vertical coordinates
p_4	Z_{XXup}	Upper crest curvature
p_5	X_{lo}	Lower crest location in horizontal coordinates
p_6	Z_{lo}	Lower crest location in vertical coordinates
p_7	Z_{XXlo}	Lower crest curvature
p_8	Z_{TE}	Trailing edge offset in vertical sense
p_9	ΔZ_{TE}	Trailing edge thickness
p_{10}	α_{TE}	Trailing edge direction
p_{11}	β_{TE}	Trailing edge wedge angle

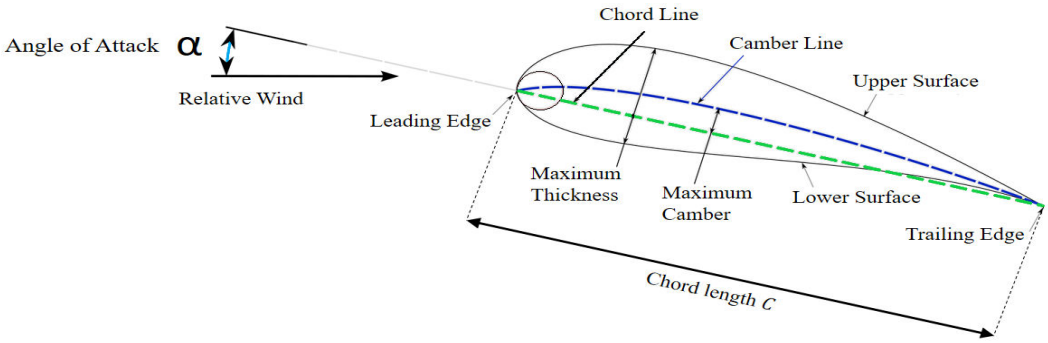

Figure-2. Schematic of an airfoil with relative wind.

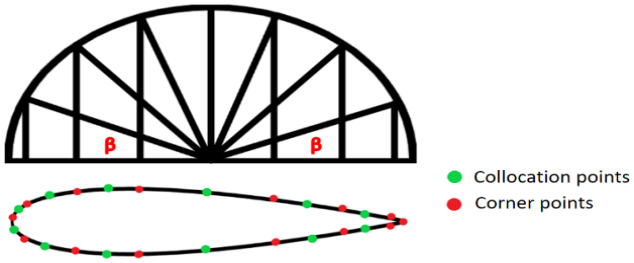

Figure-3. Schematic of discretization of airfoil in the panel method.

4. COMPUTATIONAL METHOD

Numerical computation is one of the most important part for analyzing and optimizing an aerodynamic problem. Plenty of methods are available to analyze a flow situation in practice. Panel method is one of the widely used method for solving linear, in viscid, irrotational flows [27]. The fundamental analytic solution to the Prandtl-Glauert equation is knows as source, doublet, and vorticity singularities. Panel method is based on the principle of superimposing surface distributions of these singularities over small quadrilateral sections known as panels, of the airfoil surface. Boundary conditions are imposed at discrete points of the panels and the points are also known as collocation points or control points.

Figure-2 shows the schematic of an airfoil with angel of attack α and chord length C. The governing equation of panel method is defined as follows:

$$\nabla \phi_P \cdot n = 0 \tag{8}$$

$$(u_\infty, v_\infty) \cdot n + \int \gamma \frac{\partial \phi_V}{\partial n} ds = 0 \tag{9}$$

here, ϕ_V is the potential of a unit strength vortex and defined as $\phi_V = -\frac{1}{2\pi}\theta$ is polar coordinate and u_∞, v_∞ are the horizontal and vertical components of free stream velocity U_∞.

The body of the airfoil is then divided in N panels. To get a better discretization of airfoil, a denser paneling is used near the leading edge and the trailing edge. Full cosine method is a well-known method for discretization of airfoil in N panels which is shown in Figure-3. If there is N panel for discretization then, $\beta = \pi / N$ and the corresponding x coordinates are [28]

$$x = \frac{c}{2}(1 - \cos\beta) \tag{10}$$

The lift coefficients are calculated as follows [28]:

$$C_L = -\sin\alpha \cdot F_x + \cos\alpha \cdot F_y \tag{11}$$

The lift and drag can easily be determined using the vortex panel method. In this work, vortex panel method is used for the numerical analysis of the flow field of the low Reynolds number airfoils.

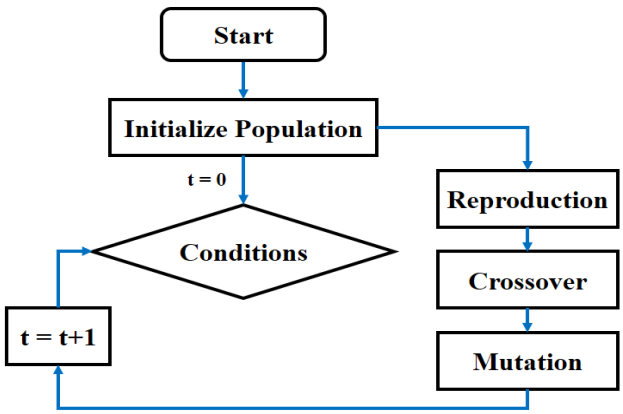

Figure-4. Flow chart of a simple genetic algorithm.

5. OPTIMIZATION ALGORITHM

There are many different algorithms available for single and multi-objective optimization. Though there are a lot of algorithms available, genetic algorithm (GA) is one of the widely used algorithm for aerodynamic optimization. John Holland, his student and his colleagues at University of Michigan developed Genetic Algorithm (GA) in 1960 [27]. Due to the enhancement of computational capability of advanced computers, GA has become a very popular algorithm to optimize complex problems.

Figure-4 illustrates a simple GA. For each and every GA problem, there must be one or more objectives. A fitness function is defined based on the objective. The GA starts with randomly generating a population which consists of the candidate solutions of the problem. Then the fitness is calculated for each chromosome of the population. Then the termination criterion is checked. If the result is not satisfactory, then reproduction take place and then crossover and mutation take place at a defined probability. The newly generated population is then used to replace the previous population for repeating the cycle until a satisfactory result is obtained.

In this study, Matlab 2016 is used for implementing genetic algorithm (GA) and vortex panel method. 3 ms^{-1} wind speed is considered as the average wind speed in most of the areas of Bangladesh is nearly ranges between 3 to 5 ms^{-1}. Angle of attack is considered 12 degree. Number of panel N=200 is considered for the numerical analysis. For genetic algorithm (GA), population size is considered 48, mutation rate is 20%, cross over rate is 75% and transcendence is 5%.

6. RESULTS AND DISCUSSIONS

This section illustrates the results of the optimization. Table-2 and Figure-5 shows the comparison of NACA 2412 airfoil and three optimized airfoils. The lift coefficient has increased notably in the optimized airfoil as well as the area of the airfoil is reduced. Though the area reduction is not so significant, it will be very helpful for rapid production and will the raw materials.

Table-2. Results of optimization of NACA 2412.

Airfoil	Original Lift Coefficient	Lift Coefficient of Optimized Airfoils	Area Reduction (%)
NACA 2412	1.6432	(a) 1.7498	5.22
		(b) 1.7427	2.13
		(c) 1.7494	3.57

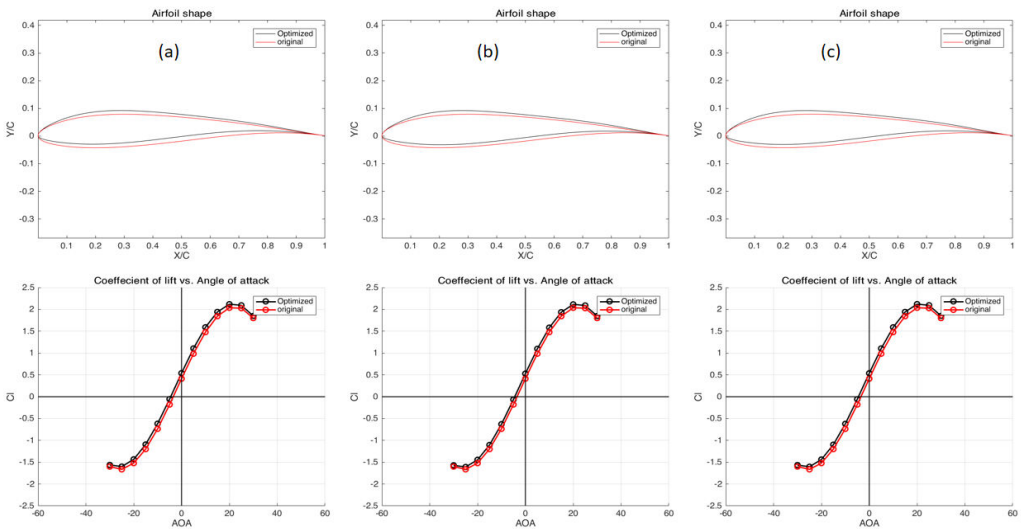

Figure-5. Comparison of NACA 2412 airfoil with optimized results.

Table-3 and Figure-6 illustrates that, for NACA 4412 the improvement of lift coefficient is very small but the area of the optimized airfoil is reduced significantly which will eventually reduce the material cost.

Table-3. Results of optimization of NACA 4412.

Airfoil	Original Lift Coefficient	Lift Coefficient of Optimized Airfoils	Area Reduction (%)
NACA 4412	1.7887	(a) 1.8297	16.82
		(b) 1.8326	17.47
		(c) 1.8283	14.42

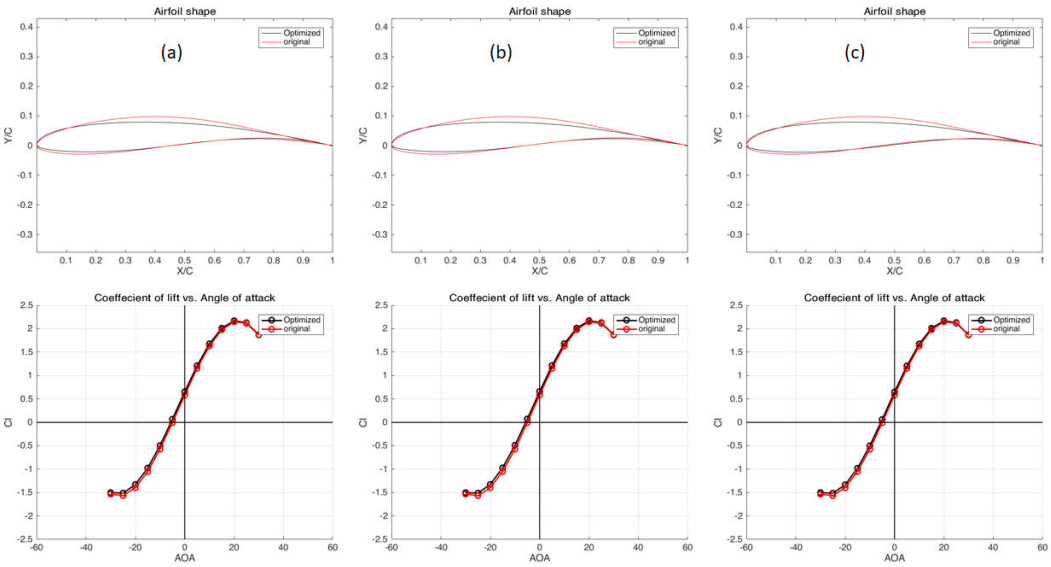

Figure-6. Comparison of NACA 4412 airfoil with optimized results.

Table-4 and Figure-7 indicate that for NACA 4418 the optimized results are not so appealing. The change of lift coefficient is slightly improved but the area reduction is very minor.

Table-4. Results of optimization of NACA 4418.

Airfoil	Original Lift Coefficient	Lift Coefficient of Optimized Airfoils	Area Reduction (%)
NACA 4418	1.9500	(a) 2.0643	2.77
		(b) 2.0808	2.43
		(c) 2.0702	3.71

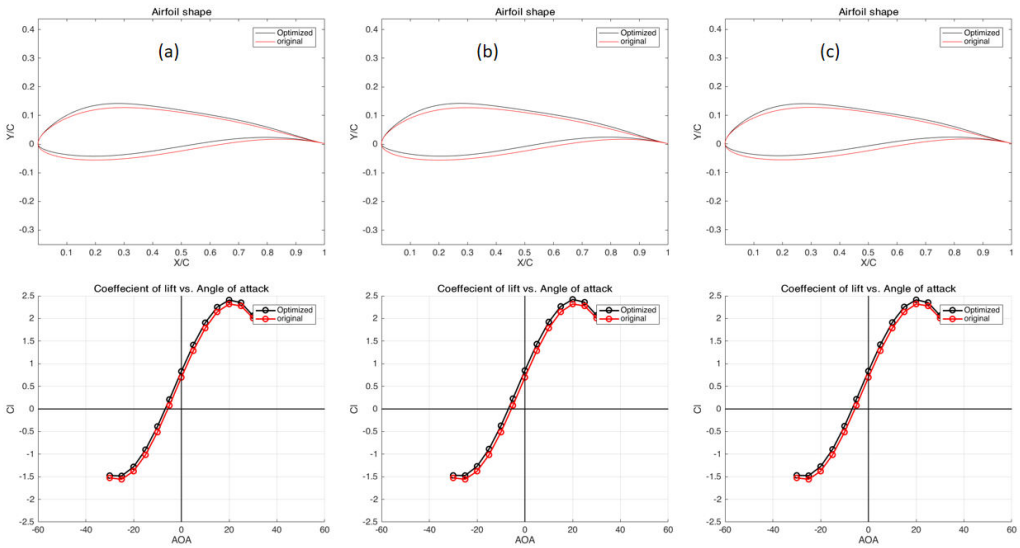

Figure-7. Comparison of NACA 4418 airfoil with optimized results.

Table-5 and Figure-8 illustrates the comparison of NACA 4421 and the optimized airfoil generated. The improvement of lift coefficient is significant. The area reduction is very minor but overall, the optimized airfoils are improved in design compared to the original one.

Table-5. Results of optimization of NACA 4421.

Airfoil	Original Lift Coefficient	Lift Coefficient of Optimized Airfoils	Area Reduction (%)
NACA 4421	2.0548	(a) 2.1730	3.48
		(b) 2.1870	1.21
		(c) 2.1787	2.73

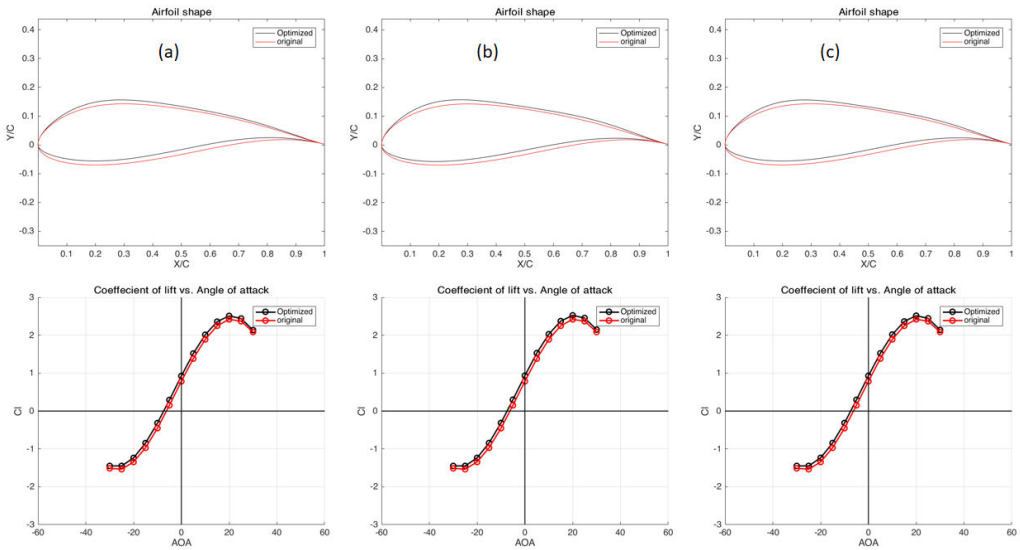

Figure-8. Comparison of NACA 4421 airfoil with optimized results.

7. CONCLUSIONS

Low-speed airfoil designing is one of the major interests of wind energy enthusiasts and manufacturers all over the world. Plenty of research are ongoing on this field but only a very few researches have shown the comparison of different low-speed airfoils. Four popular low-speed airfoils have been compared here. Genetic algorithm is used to generate optimized airfoils. PARSEC parameterization technique is used for parameterizing the airfoil surface and vortex panel method is used as the solver for flow analysis. The results show that, lift coefficient increases along with area reduction. Which is very useful for manufacturing wind turbine blades.

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