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PULSED NEUTRONS APPLIED TO COVARIANCE MATRICES OF STOCHASTIC POINT KINETICS EQUATIONS

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ABSTRACT

This study presents the implementation of the theory of pulsed neutrons using different covariance matrices of stochastic point kinetics equations reported in literature, proposing the second order stage 2, stochastic Runge-Kutta method (RK2-2st) to numerically calculate mean values, such as the variance of neutron density and delayed neutron precursors. A number of numerical experiments were carried out with different values of reactivity. According to the results of the different numerical experiments, it was found that the pulsed neutron theory reduces the mean value fluctuations of neutron population density and delayed neutron precursors.

Keywords: stochastic point kinetics equations, pulsed neutrons, nuclear power reactor, numerical simulation.

1. INTRODUCTION

Nowadays nuclear power is an option for producing electricity almost without emitting greenhouse gases or air pollutants. Nuclear power plants are industrial facilities where electrical energy is produced from nuclear energy released in the fission process, and are located next to a water source because nuclear reactor systems can be cooled with water [1]. Nuclear power plants consist of one or more nuclear power reactors. In the core of the reactor is a set of pellets or a fuel network of Uranium 235 or Plutonium 239. There, fission occurs in the reactor core, which releases energy, which is used to heat water and generate high pressure steam, which causes the movement of a turbine connected to an electric power generator. This energy is then supplied to the electricity network [2]. Nuclear power plants must guarantee the controlled production of energy, and to achieve this, the density of neutrons obtained by solving the stochastic point kinetics equations [3] must be known, since these equations model the neutron population within the reactor.

Previous studies have proposed different covariance matrices for stochastic point kinetics equations [4, 5, 6]. This paper presents the study of the different covariance matrices reported in literature, applying pulsed neutron [7]. To solve the stochastic point kinetics equations, the explicit stochastic second order, stage 2 stochastic Runge-Kutta method is used.

2. MATERIALS AND METHODS

2.1 The Stochastic Point Kinetics Equations

Studies of the dynamic processes of a nuclear reactor are described by point kinetics equations which model the temporal evolution of the neutron population density and the concentration of delayed neutron precursors [2]. The processes involved in the reactor core are stochastic processes, and the stochastic point kinetics equations are given by:

$$d\hat{x}(t) = \left[A\hat{x}(t) + \hat{F}(t)\right]dt + B^{1/2}d\widehat{W}(t)dt \tag{1}$$

Where $\hat{x}(t)$, $\hat{F}(t)$ and $\widehat{W}(t)$ are random variable matrices, and these matrices are defined as

$$\hat{x}(t) = \begin{bmatrix} n(t) \\ c_1(t) \\ c_2(t) \\ \vdots \\ c_m(t) \end{bmatrix} \tag{2}$$

$$A = \begin{bmatrix} \frac{\rho(t)-\beta}{l} & \lambda_1 & \lambda_2 & \dots & \lambda_m \\ \frac{\beta_1}{l} & -\lambda_1 & 0 & \dots & 0 \\ \frac{\beta_2}{l} & 0 & \lambda_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\beta_m}{l} & 0 & 0 & \dots & -\lambda_m \end{bmatrix} \tag{3}$$

$$\hat{F}(t) = \begin{bmatrix} q(t) \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{4}$$

$$\widehat{W}(t) = \begin{bmatrix} W_1(t) \\ W_2(t) \\ W_3(t) \\ \vdots \\ W_m(t) \end{bmatrix} \tag{5}$$

Where $\rho(t)$ is the reactivity, β_i is the delayed neutron fraction of the i-thgroup of precursors, $\beta = \sum_{i=1}^{m}\beta_i$ is the total fraction of delayed neutrons, λ_i is the decay constant of the i-th group of precursors, l is the mean lifetime of neutron in reactor, and q is the neutron source. A previous work [5] proposed the following covariance matrix for stochastic point kinetics, based on the random variables presenting a normal distribution.

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$$B_{(2005)} = \begin{bmatrix} \zeta & a_1 & a_2 & \dots & a_m \\ a_1 & b_{1,1} & b_{1,2} & \dots & b_{1,m} \\ a_2 & b_{2,1} & b_{2,2} & \dots & b_{2,m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_m & b_{m,1} & b_{m,2} & \dots & b_{m,m} \end{bmatrix} \quad (6)$$

After [4], proposed that the random variables of point kinetics were not correlated, and present a Poisson distribution, therefore, the different elements of the diagonal of the covariance matrix [5] take zero values:

$$B_{(2014)} = \begin{bmatrix} \zeta & 0 & 0 & \dots & 0 \\ 0 & b_{1,1} & 0 & \dots & 0 \\ 0 & 0 & b_{2,2} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & b_{m,m} \end{bmatrix} \quad (7)$$

Where

$$\zeta = \left[\frac{-1 - \rho(t) + 2\beta + (1-\beta)^2\upsilon}{l}\right] n(t) + \sum_{i=1}^{m} \lambda_i c_i(t) + q(t)$$

$$a_i = \frac{\beta_i}{l}[(1-\beta)\upsilon - 1]n(t) - \lambda_i c_i(t)$$

$$b_{i,j} = \frac{\beta_i \beta_j}{l} n(t) + \delta_{i,j}\lambda_i c_j(t)$$

Where υ is the average of fission neutrons and $\delta_{i,j}$ represents the Kronecker delta.

A next work [8] subsequently presented the following covarance matrix

$$B_{(2016)} = \begin{bmatrix} \mu_0 & -\mu_1 & -\mu_2 & \dots & -\mu_m \\ -\mu_1 & \mu_1 & 0 & \dots & 0 \\ -\mu_2 & 0 & \mu_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\mu_m & 0 & 0 & \dots & \mu_m \end{bmatrix}$$
(8)

where

$$\mu_0 = \left[\frac{\rho(t) + \beta}{l}\right] n(t) - \sum_{i=1}^{m} \lambda_i c_i(t) + q(t)$$

$$\mu_i = \left[\frac{\beta_i}{l}\right] n(t) - \lambda_i c_i(t)$$

A recent study [7], found the following approximation of pulsed neutrons:

$$\frac{1}{l} = \frac{\rho(t) - \beta}{\Lambda} \quad (9)$$

where l is the mean lifetime of neutron in reactor and Λ is the time of neutron generation. In that work [7], it was modified the covariance matrix proposed by [5], in which they considered the approach based on the pulsed neutron method.

$$B_{(2005\,pn)} = \begin{bmatrix} \zeta^* & a_{1^*} & a_{2^*} & \dots & a_{m^*} \\ a_{1^*} & b_{1,1^*} & b_{1,2^*} & \dots & b_{1,m^*} \\ a_{2^*} & b_{2,1^*} & b_{2,2^*} & \dots & b_{2,m^*} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{m^*} & b_{m,1^*} & b_{m,2^*} & \dots & b_{m,m^*} \end{bmatrix} \quad (10)$$

Where

$$\zeta^* = \left(\frac{\rho(t) - \beta}{\Lambda}\right)[-1 - \rho(t) + 2\beta + (1-\beta)^2\upsilon]n(t) + \sum_{i=1}^{m} \lambda_i c_i(t) + q(t)$$

$$a_{i^*} = \left(\frac{\rho(t) - \beta}{\Lambda}\right)[(1-\beta)\beta_i\upsilon - 1]n(t) - \lambda_i c_i(t)$$

$$b_{i,j^*} = \left(\frac{\rho(t) - \beta}{\Lambda}\right)\beta_i\beta_j n(t) + \delta_{i,j}\lambda_i c_j(t)$$

Now, the approximation of the pulsed neutron method to modify the covariance matrix [4], defined in equation (7) is to be considered

$$B_{(2014\,pn)} = \begin{bmatrix} \zeta^* & 0 & 0 & \dots & 0 \\ 0 & b_{1,1^*} & 0 & \dots & 0 \\ 0 & 0 & b_{2,2^*} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & b_{m,m^*} \end{bmatrix} \quad (11)$$

The same modification of pulsed neutrons is carried out for the covariance matrix proposed in [6] given in equation (8), results:

$$B_{(2016\,pn)} = \begin{bmatrix} \mu_{0^*} & -\mu_{0^*} & -\mu_{0^*} & \dots & -\mu_{0^*} \\ -\mu_{1^*} & \mu_{1^*} & 0 & \dots & 0 \\ -\mu_{2^*} & 0 & \mu_{2^*} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\mu_{m^*} & 0 & 0 & \dots & \mu_{m^*} \end{bmatrix} \quad (12)$$

where

$$\mu_{0^*} = \left(\frac{\rho(t) - \beta}{\Lambda}\right)(\rho(t) + \beta)n(t) - \sum_{i=1}^{m} \lambda_i c_i(t) + q(t)$$

$$\mu_{i^*} = \left(\frac{\rho(t) - \beta}{\Lambda}\right)\beta_i n(t) - \lambda_i c_i(t)$$

To give a numerical solution to the stochastic point kinetic equation in this work, we propose to implement the stochastic second order, stage 2 Runge-Kutta method (RK2-2st).

3. PROPOSED METHOD

The explicit second order stage 2 Runge-Kutta scheme [9], is given by

$$\bar{X}_{n+1} = \bar{X}_n + [\alpha_1 a + \alpha_2 a(t_n + \mu\Delta, \eta)]\Delta + [\beta_1 b + \beta_2 b(t_n + \mu\Delta, \bar{\eta})]\Delta\widehat{W}_n + R \quad (13)$$

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Where

$\eta = \bar{X}_n + \lambda a\varDelta + \gamma b\Delta\widehat{W}$

The functions $a(t_n + \mu\varDelta, \bar{X}_n)$ and $b(t_n + \mu\varDelta, \bar{X}_n)$ are obtained based on Taylor's truncated second order expansions defined as

$$f(t+\varDelta, X_t + \Delta X) \overset{(2)}{\simeq} f + f_{10}\varDelta + f_{01}\Delta x + f_{02}\frac{(\Delta X)^2}{2} + \left[f_{20} + b^2 f_{12} + b^3 b_{01} f_{03} + \frac{1}{4}b^4 f_{04}\right]\frac{\varDelta^2}{2} + \left(f_{11} + \frac{1}{2}b^2 f_{03}\right)\varDelta\Delta X \tag{14}$$

Where $g_{ij} = \frac{\partial^{i+j} g(t_n, \bar{x}_n)}{\partial t^i \partial x^j}$ and $\overset{(2)}{\simeq}$ refers to the third order approximation of the function. The combination of products of the form $\Delta^i\left(\Delta\hat{W}_n\right)^j$ with $i = j = 1,2,\ldots,n$ for the approximation two order corresponds to:

$$\Delta\left(\Delta\widehat{W}_n\right)^i\left(\Delta\widehat{W}_n\right)^j \overset{(2)}{\simeq} \begin{cases}\Delta^2 & si\ i \neq j\\ 0 & si\ i = j\end{cases}$$
$$\left(\Delta\widehat{W}_n\right)^i\left(\Delta\widehat{W}_n\right)^j\left(\Delta\widehat{W}_n\right)^k \overset{(2)}{\simeq} \begin{cases}3\Delta\left(\Delta\widehat{W}\right)^i & si\ i = j = k\\ \Delta\left(\Delta\widehat{W}\right)^i & si\ j = k \neq i\end{cases} \tag{15}$$
$$\Delta^i\left(\Delta\widehat{W}_n^1\right)^{j_1}\left(\Delta\widehat{W}_n^2\right)^{j_2}\ldots\left(\Delta\widehat{W}_n^m\right)^{j_m} \overset{(2)}{\simeq} 0\ si\ i + \frac{j_1 + j_2 + \cdots + j_m}{2} \geq \frac{5}{2}$$

From equation (14) and (15) $a(t_n + \mu\varDelta, \bar{X}_n)$ and $b(t_n + \mu\varDelta, \bar{X}_n)$, results:

$$a(t_n + \mu\varDelta, \eta)\varDelta \overset{(2)}{\simeq} a\varDelta + a_{10}\mu\varDelta^2 + aa_{01}\lambda\varDelta^2 + a_{01}b\gamma\varDelta\Delta\widehat{W}_n + \frac{1}{2}a_{02}b^2\gamma^2\varDelta^2 \tag{16}$$

$$b(t_n + \mu\varDelta, \eta)\varDelta \overset{(2)}{\simeq} b\Delta\widehat{W}_n + b_{01}b\gamma\varDelta\left(\Delta\widehat{W}_n\right)^2 + b_{10}\mu\varDelta\Delta\widehat{W}_n + ab_{01}\lambda\varDelta^2\Delta\widehat{W}_n + \frac{3}{2}b_{02}b^2\gamma^2\varDelta\Delta\widehat{W}_n + \left[b\left(b_{11} + \frac{1}{2}b_{03}b^2\right)\mu\gamma + abb_{02}\lambda\gamma\right]\varDelta^2 \tag{17}$$

Replacing equations (16) and (17) in equation (13), we obtain

$$\bar{X}_{n+1} = \bar{X}_n + (\alpha_1 + \alpha_2)a\varDelta + (\beta_1 + \beta_2)b\Delta\widehat{W}_n + \beta_2 b_{01}b\gamma\left(\Delta\widehat{W}_n\right)^2 + \left[\alpha_2\mu a_{10} + \alpha_2 a\lambda a_{01} + \beta_2\mu\gamma b\left(b_{11} + \frac{1}{2}b_{03}b^2\right) + \frac{1}{2}\alpha_2\gamma^2 a_{02}b^2 + \beta_2\lambda\gamma abb_{02}\right]\varDelta^2 + \left[\alpha_2\gamma a_{01}b + \beta_2\mu b_{10} + \beta_2\lambda ab_{01} + \frac{3}{2}\beta_2\gamma^2 b^2 b_{02} + \beta_2\lambda ab_{01}\right]\varDelta\Delta\widehat{W}_n + R \tag{18}$$

The scheme found in equation (18) must be equivalent to Taylor's second order simplified scheme [10]

$$\bar{X}_{n+1} = \bar{X}_n + a\varDelta + b\Delta\widehat{W}_n + \frac{1}{2}bb_{01}\left(\left(\Delta\widehat{W}_n\right)^2 - \varDelta\right) + \left[a_{10} + aa_{01} + \frac{1}{2}a_{02}b^2\right]\varDelta^2 + \frac{1}{2}\left[a_{01}b + b_{10} + ba_{01} + \frac{1}{2}b^2 b_{02}\right]\varDelta\Delta\widehat{W}_n \tag{19}$$

To obtain equivalence between the scheme found in equation (18) and that given by equation (19) the following equalities must be satisfied

$$\begin{matrix}\alpha_1 + \alpha_2 = 1 & \lambda\alpha_2 = \frac{1}{2} & \lambda\beta_2 = \frac{1}{2} & \mu\alpha_2 = \frac{1}{2} & \gamma^2\alpha_2 = \frac{1}{2}\\ \beta_1 + \beta_2 = 1 & \gamma\alpha_2 = \frac{1}{2} & \gamma\beta_2 = \frac{1}{2} & \mu\beta_2 = \frac{1}{2} & \end{matrix} \tag{20}$$

and $R = \frac{1}{2}bb_{01}\varDelta$. Note that the system has only one solution when $\gamma = \mu = \lambda = 1, \alpha_1 = \alpha_2 = \frac{1}{2}$ and $\beta_1 = \beta_2 = \frac{1}{2}$. Replacing the parameter values in equation (13), the stochastic Runge-Kutta second order stage 2 scheme (RK2-2st) is found:

$$\bar{X}_{n+1} = \bar{X}_n + \frac{1}{2}a\varDelta + \frac{1}{2}b\Delta\widehat{W}_n + \frac{1}{2}a\left(t_n + \mu\varDelta, \bar{X}_n + a\varDelta + b\Delta\widehat{W}\right) + \frac{1}{2}bb_{01}\left(\left(\Delta\widehat{W}_n\right)^2 - \varDelta\right) + \frac{1}{2}b\left(t_n + \mu\varDelta, \bar{X}_n + a\varDelta + b\Delta\widehat{W}\right)\varDelta\Delta\widehat{W}_n - \frac{1}{2}bb_{01}\varDelta. \tag{21}$$

The Runge-Kutta second order, stage 2 scheme given in equation (21) is equivalent to Taylor's second order scheme described by equation (19). Taylor's second order scheme requires the calculation of ten derivatives while the second order stage 2 Runge-Kutta scheme requires only two derivatives to be calculated, making the proposed method easy to implement.

4. RESULTS AND DISCUSSIONS

In this section, the precision of the second order, stage 2 Runge-Kutta method given by equation (21) will be measured by the approximations of the mean values and standard deviation of neutron density and the concentration of precursors. Covariance matrices reported in literature [4, 5, 8] are considered with the modification of the pulsed neutron method [7] defined by equations (10-12).

In the following numerical experiments, the stochastic point kinetics equations defined in equation (1) are solved for three different situations with constant reactivity, 5000 Brownian trajectories are considered, and the pseudo-random numbers present a normal distribution for seed 200. The approximations obtained with the proposed RK2-2st method are compared with stochastic methods reported in literature such as PCA, Monte Carlo [5], Taylor 1.5 [11], AEM [6], ESM [8], DDDM [12] and implicit EM [7].

The first numerical experiment with reactivity $\rho = -1/3$ considers the following kinetic parameters, $\lambda_1 = 0.1s^{-1}$, $\beta_1 = \beta = 0.05$, $\Lambda = 2/3s$, $\nu = 2.5$ fission neutrons, $q = 200s^{-1}$ fission neutrons, for a time step $\Delta = 0.05s$, in a time interval $[0,2]s$, with the initial condition $\hat{x}_{(0)} = [400,300]^T$. Figure-1 shows the

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variation of neutron density and the concentration of the first group of precursors when implementing the covariance matrix $B_{(2005)}$ [5] and the modified matrix with pulsed neutron theory $B_{(2005\ pn)}$ [7].

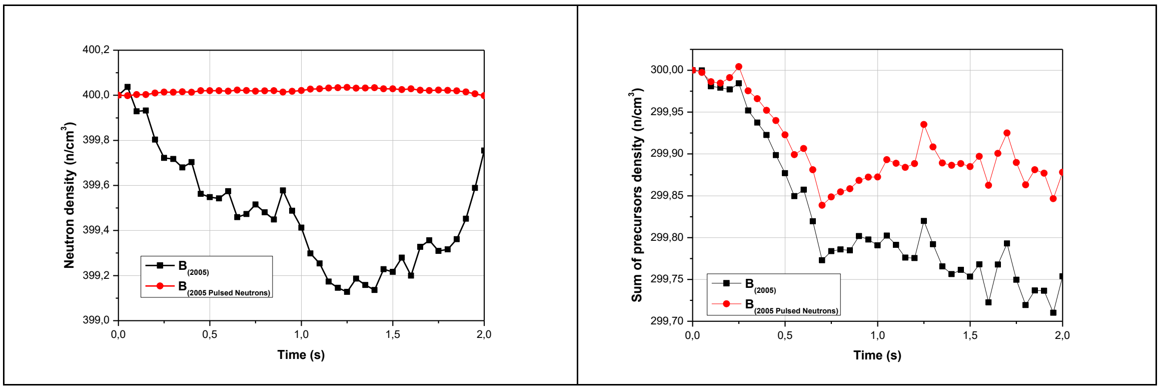

Figure-1. Variation of neutron density and concentration of precursors based on time for a nuclear reactor with reactivity $\rho = -1/3$.

Table-1 presents the average deviations and standard of neutron density and precursors for the first example. The results of the RK2-2st method with the covariance matrix $B_{(2005)}$ are compared with those obtained with the modified matrix$B_{(2005\ pn)}$. It can be seen that the proposed RK2-2st method is efficient, and that the results obtained with matrix $B_{(2005\ pn)}$ present less fluctuation than those obtained with matrix $B_{(2005)}$.

Table-1. Comparison of solutions for each method with a reactivity of $\rho = -1/3$ for one group of precursors.

Method	$E\left[n_{(2)}\right]$	$\sigma\left[n_{(2)}\right]$	$E\left[c_{1(2)}\right]$	$\sigma\left[c_{1(2)}\right]$
Montecarlo	400.0300	27.3110	300.0000	7.8073
PCA	395.3200	29.4110	300.6700	8.3564
EM	412.2300	34.3910	315.9600	8.2656
AEM	396.2800	31.2120	300.4200	7.9576
ESM	396.6200	0.91990	300.3000	0.0016
Double DDM	402.3500	28.6100	305.8400	7.9240
Implicit EM	399.9874	0.5439	299.8730	6.8405
RK2-2st $B_{(2005)}$	399.7550	24.5511	299.7538	8,6142
RK2-2st $B_{(2005\ \mathrm{pn})}$	399.9985	1.3402	299.8781	6.7549
RK O(h^4)	400.0000	-	300.0000	-

The second numerical experiment considers a constant reactivity of $\rho = 0.003$for six groups of precursors.In this example, the following kinetic parameters are used, $\lambda_1 =$ [0.0127; 0.0317; 0.115; 0.311; 1.4; 3.87]s^{-1},$\beta_1 =$ [0.000266; 0.001491; 0.001316; 0.002849; 0.0000896; 0.000182], $\beta = 0.007$, $\Lambda = 0.00002\ s$, fission neutrons, $q = 0s^{-1}$external neutron source, the initial condition of the system is$\hat{x}_{(0)} = 100\left[1, \frac{\beta_1}{\lambda_1\Lambda}, \frac{\beta_2}{\lambda_2\Lambda}, \dots, \frac{\beta_6}{\lambda_6\Lambda}\right]^T$. For the simulations,$N = 40$steps in the time interval$[0,0.001]s$ were implemented.

Figure-2 shows the variation in neutron density and precursors when implementing the $B_{(2014)}$covariance matrix [4] and the modified matrix with the pulsed neutron theory$B_{(2014\ pn)}$. Table-2 compares the reults obtained with the RK2-2st method for the covariance matrix $B_{(2014)}$and the modified matrix with the pulsed neutron theory $B_{(2014\ pn)}$ with other methods reported in literature. It can be seen that the level of precision of the RK2-2st method is good. Wheapplying the pulsed neutron theory to the covariance matrix $B_{(2014)}$fluctuations are reduced and better approximations are obtained in the mean values, together with reductions in the variance of neutron population and precursors density.

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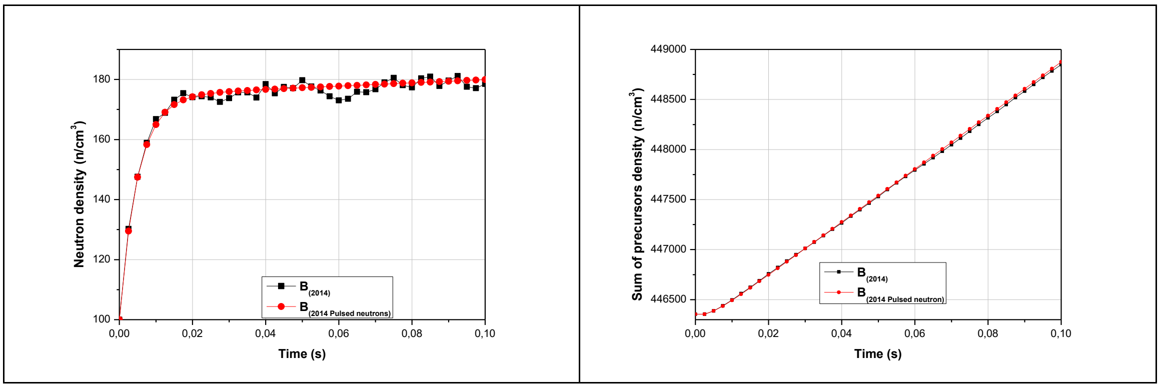

Figure-2. Variation of neutron density and concentration of precursors based on time for a nuclear reactor with reactivity $\rho = 0.003$.

Table-2. Comparison of solutions for each method with a reactivity of $\rho = 0.003$ for six groups of precursors.

Method	$E\left[n_{(0.1)}\right]$	$\sigma\left[n_{(0.1)}\right]$	$E\left[c_{i(0.1)}\right]$	$\sigma\left[c_{i(0.1)}\right]$
Montecarlo	183.0400	168.7900	447800	1495.7000
PCA	186.3100	164.1600	449100	1917.2000
EM	208.6000	255.9500	449800	1233.3800
Taylor 1.5	199.4080	168.5470	449700	1218.820
AEM	186.3000	164.1400	449000	1911.9100
ESM	179.9300	10.5550	448900	94.7500
Double DDM	187.0500	167.8300	448800	1475.6000
Implicit EM	179.9461	0.2178	448880	60.4267
RK2-2st $B_{(2014)}$	188.3292	998.9062	448944	7802.3181
RK2-2st $B_{(2014\ \text{pn})}$	179.9085	4.7218	448876	73.2165
RK O(h^4)	179.9528	-	448877	-

The third numerical experiment considers the same initial conditions and kinetic parameters of the second experiment, with a constant reactivity of $\rho = 0.007$with six groups of delayed neutron precursors. Figure-3 shows the variation of neutrons and delayed neutron precursors in the time interval $[0,0.001]s$ with $N = 40$steps. The results are obtained by implementing the covariance matrix [8] and the modified matrix with the pulsed neutron theory$B_{(2016\ pn)}$.

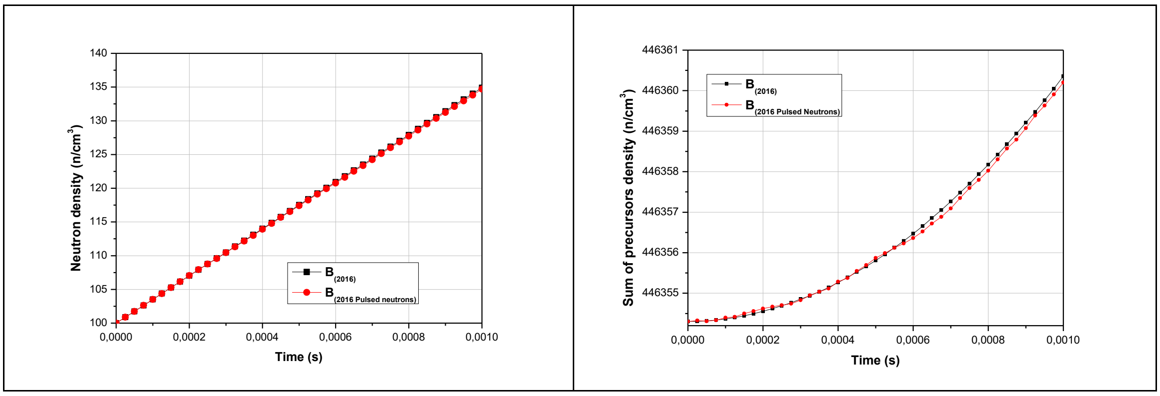

Figure-3. Variation of neutron density and concentration of precursors based on time for a nuclear reactor with reactivity $\rho = 0.007$.

Table-3 shows the comparison of the results obtained with the RK2-2st method for the covariance matrix $B_{(2016)}$and the modified matrix with the pulsed

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neutron theory with other methods reported in literature; it is observed that the proposed RK2 -2st method provides adequate approximations in the mean values of neutron density and precursor concentration. The results obtained with the covariance matrix $B_{(2016)}$[8] and the modified matrix with the pulsed neutron theory $B_{(2016\ pn)}$show very small fluctuations, thus generating good approximations.

Table-3. Comparison of solutions for each method with a reactivity of $\rho = 0.007$ for six groups of precursors.

Method	$E\left[n_{(0.001)}\right]$	$\sigma\left[n_{(0.001)}\right]$	$E\left[c_{i(0.001)}\right]$	$\sigma\left[c_{i(0.001)}\right]$
Montecarlo	135.6500	93.3760	446400	16.2260
PCA	134.5500	91.2420	446400	19.4440
EM	139.5680	92.0420	446300	6.0710
Taylor 1.5	139.5690	92.0470	446300	18.3370
AEM	134.5400	91.2340	446400	19.2350
ESM	134.9600	6.8527	446400	2.5290
Double DDM	135.8600	93.2100	446300	17.8450
Implicit EM	134.9218	5.9661	446360	6.0686
RK2-2st $B_{(2016)}$	134.9090	6.7014	446360	2.6480
RK2-2st $B_{(2016\ \mathrm{pn})}$	134.8164	5.9102	446360	13.4444
RK O(h^4)	135.0009	-	446360	-

4. CONCLUSIONS

In this study the stochastic second order stage 2 Runge-Kutta numerical derivation method (RK2-2st) is used to solve stochastic point kinetics equations, and for this purpose we proposed three different experiments with constant reactivity with one and six groups of delayed neutron precursors with different kinetic parameters and initial conditions. Pulsed neutron was used to modify the covariance matrices of the stochastic point kinetics equations reported in literature. It was found that by implementing the pulsed neutron theory with covariance matrices, the fluctuation of the mean values of neutron density, and concentration of delayed neutron precursors were reduced.

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