# STUDY OF CAM SHAFT DESIGN METHOD FOR INDEXING TABLE 

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#### Abstract

The indexing tables have recently become an increasing necessity for improving the precision and accuracy of angle measurement. The indexing table employs a vee and 3 balls kinematic location system and the plate having 3 balls is moved by cam shaft. Generally, commercially available indexing tables (except worm and worm wheel type and optical grating type indexing table) contain two step operations for indexing (i.e. lift up and rotate), but this new type of indexing table employs one step operation, lifting up and rotating simultaneously by using a camshaft. The detailed profile of the cam shaft will be described in this paper and shows the mechanism of the indexing table. To hold the top disc on the body and lock the top disc on the vee, the indexing table employs a compression spring between the body and pindisc which is connected to the shaft by screws. To prevent movement of the spring during operating, a circular groove holds the one end of the spring, and the clearance between the spring and the bore of the pindisc is minimized to provide spring location. Also, a needle bearing is fitted between the spring and pin disc to prevent torsion and minimize friction.


Keywords: follower motion, camshaft profile, pressure angle, defection.

## 1. INTRODUCTION

The indexing tables have recently become an increasing necessity for improving the precision and accuracy of angle measurement, and circle dividing in various fields of science and technology. Most indexing tables use a worm wheel mechanism or a grating disc. The manufacturing process together with the requirements for precision is very difficult. Consequently, these indexing tables are expensive and usually used for manufacturing purpose of rotating [1]. The indexing table using a ball and pin was developed [2]. This performed very well in a laboratory environment, but it was not considered robust enough to reproduce as a commercial design. The problem with this indexing table is the balls which are located on the circumference of the body could move when the top plate was located, or any force was applied to the top plate. Consequently, a new mechanism should be considered for the indexing table. Thus 6 points contact kinematic design concept of ball and vee groove was used, because the ball and vee groove system can attain $0.1 \mu \mathrm{~m}$ repeatability [3] and has good rigidity [4].

## 2. DESIGN OF INDEXING TABLE

### 2.1 Concept of Kinematic Design

Kinematic design is widely recognised as one of the foremost design concepts in precision engineering. Most objects in space have three degrees of translatory and three degrees of rotational freedom. Two different philosophies for mechanical design exist - kinematic and elastic. While being quite different in approach they can be combined in a design. In the kinematic design philosophy, the aim is to locate all parts relative to each other, while allowing a degree of freedom as needed, by connecting points together without significant elastic deformation. Figure-1 shows three examples of kinematic couplings. A simple theorem that allows calculation of the number of points of contact was enunciated by Strong 1938. He defined.
"Kinematic design is correct when a body in contact with another has at least $6-\mathrm{n}$ points of contact where n is the number of degrees of freedom existing. If the system has more than $6-\mathrm{n}$ points of contact. It has mechanical redundancy" [6] In Figure-1(a), three rotations are possible for the ball, whereas in Figure-1(b) and (c) no freedom of movement is provided.


Figure-1. Kinematic coupling.

### 2.2 Feature of New Indexing

The automatic indexing table which was developed by Lin [3] employs pins and balls. The balls are in contact with the circumference of the body and the outer rings hold the balls fixed on the circumference of the body. The ball is moved by applying high speed rotating of camshaft. Consequently, the automatic indexing table cannot give good repeatability. This device applies only vertical calibration (axes of rotary table is vertical) due to, insufficient spring force. Thus, this project considers a new type of indexing table to improve previous automatic indexing table.

The basic principle of this new indexing table employs a vee and balls kinematic location system. The
chosen incremental indexing angle is 5 degree because 5 degrees incremental angle is considered to be the smallest increment used for the practical calibration of $360^{\circ}$ indexing tables. The detailed specification of this new table is as follows: An incremental angle indexing device which is small and light weight thus does not influence the machine on which it is being used. The size of the device compatible with the angle reflector optics of the laser interferometer. The table should be capable of operating in any attitude. (i.e. vertical, horizontal and up-side down). Accuracy of step angle of about $\pm 1$ seconds of arc but it is not essential using the calibration technique employed. Repeatability of indexing $\pm 0.2$ seconds of arc. Table can be easily manufactured and does not require high precision technique


Figure-2. Kinematic location of balls in radial vee.
An incremental indexing angle of 5 degrees was selected as being the smallest practical angle which could be manufactured using non-specialist equipment. Generally, commercially available indexing tables (except worm and worm wheel type and optical grating type indexing table) contain two step operations for indexing (i.e. lift up and rotate), but this new type of indexing table employs one step operation, lifting up and rotating simultaneously by using a camshaft and DC motor. The detailed profile of the cam shaft will be described in the next section. Figure-3 shows the mechanism of the indexing table.


Figure-3. Mechanism of indexing table.

To hold the top disc on the body and lock the top disc on the vee, the indexing table employs a compression spring between the body and pindisc which is connected to the shaft by screws. To prevent movement of the spring during operating, a circular groove holds the one end of the spring, and the clearance between the spring and the bore of the pindisc is minimized to provide spring location. Also, a needle bearing is fitted between the spring and pin disc to prevent torsion and minimize friction.

The vee and ball type of indexing table is designed to minimise the working space and to achieve this motor and camshaft are mounted in parallel. Spur gears are employed to translate motor motion into the camshaft and to increase rotation force (i.e. torque). The gear reduction ratio between the two spur gears is 1.36 to 1.

The indexing table employed a pindisc to transform the rotation motion into lift-up and rotation motion of the top disc. The pindisc had 36 equally spaced pins giving a 10 degrees incremental angle around the circular disc. The pindisc was fixed to the shaft by two screws spaced at $90^{\circ}$ degrees. Pins are designed to guide, lift-up and rotate. The pindisc follows the groove of the camshaft during camshaft rotation. The rotation of the camshaft is controlled by an opto-disc which is fixed to the camshaft end. Two slots in the opto-disc trigger an opto-switch when rotation takes place.


Figure-4. Rotation mechanism of indexing table.

## 3. CAMSHAFT PROFILE

Usually cams are designed for transforming one direction of motion into another. A cam mechanism consists of two elements, the cam and the follower. The follower is in direct contact with the cam. The follower system includes all of the elements to which motion is imparted by the cam. The pindisc is used as a follower in the indexing table system. The pindisc needs to produce lift up, rotation and lowering by the cam motion. The cam profile is designed accordingly. The cam specifications were as follows.

The cam most usually runs at a constant speed established by values of follower displacement, velocity and acceleration. The radial cam is usually employed for lift-up, rotation and lowering. Lift-up distance was
designed to be 1.5 mm . The cam size should be smaller than the size of the bearing case in order to install the indexing table easily, i.e. the size of the cam should be less than 18.5 mm . The follower acceleration at high speeds should be as low as possible to keep inertia forces and stresses small. The moving parts of the cam-follower mechanism should be made as light in weight and as rigid as possible. This helps to keep the inertia force, noise and wear at a minimum.

### 3.1 Concept of Kinematic Design

There are three basic types of cams classified in terms of the type of follower motion. Using the basic concept of cam type, the new cam which the indexing table employs, is modified to give dwell-rise (lift-up)-rotation-return (lower-down)-dwell during a half revolution of the camshaft. This is one in which parts of the cam action have a zero-displacement portion called dwell, followed by a rise contour, rotated by a groove, followed by a return contour, and finally followed by dwell.


Figure-5. Type of cams in terms of their follow motion.

### 3.2 Basic Curve of Cam Profile

The standard cam is designed according to the rule of a modified sine wave. The simple harmonic motion is one of the most popular primarily because of its simplicity in layout and understanding. It provides an acceptable performance at moderate speeds. The basis for the harmonic curve is the projection, (on a diameter) of the constant angular velocity movement of a point on the circumference of a circle. This circle is called the harmonic circle.


Figure-6. Simple harmonic motion construction.
Referring to Figure-6, we see that the displacement diagram shows a cosine curve plotted from points projected from the harmonic circle of radius $h / 2$, giving

$$
\begin{equation*}
\mathrm{y}=(\mathrm{h} / 2)(1-\operatorname{Cos} \phi) \tag{1}
\end{equation*}
$$

Since the cam rotates $\beta$ radians while harmonic circle vector turns $\pi$ radians
$\frac{\phi}{\pi}=\frac{\theta}{\beta}$
where
$\phi \quad$ : angle of rotation of harmonic circle vector (radians)
$\beta$ : cam rotates while the harmonic circle vector turns through $\pi \operatorname{radian}\left(60^{\circ}\right)$
h : lift-up distance( 1.5 mm )
Solving yields
$\emptyset=\frac{\pi \theta}{\beta}$ radian
where $\phi$ : angle of rotation of harmonic circle vector
$y=\frac{h}{2}\left(1-\cos \frac{\pi \theta}{\beta}\right)$
Differentiating gives the velocity
$v=\frac{d y}{d t}=\frac{h \pi \omega}{2 \beta} \sin \frac{\pi \theta}{\beta} \mathrm{~mm} / \mathrm{sec}$
The velocity curve is a sine curve with point plotted from a rotating vector $\mathrm{h} \pi \omega / 2 \beta$ in length. Differentiating again for the acceleration
$a=\frac{d v}{d t}=\frac{h}{2}\left(\frac{\pi \omega}{\beta}\right)^{2} \sin \frac{\pi \theta}{\beta} \mathrm{~mm} / \sec ^{2}$

So, the acceleration is a cosine function with a rotating vector $(h / 2)(\pi \omega / \beta)^{2}$ in length.

### 3.3 Pressure Angle

The minimum cam size is basically affected by the pressure angle, the curvature of profile, and the size of the cam shaft. Minimum size is desirable because of space limitations and the size of the bearing case.

If the pitch curve of the displacement diagram is plotted on a radial cam, it is distorted towards the cam centre, i.e. the displacement - diagram pressure angle is not the same as the pressure angle measured on the actual radial cam. Figure-7 shows this clearly.


Figure-7. Radial cam translation flat-faced follower force distribution.

It is obvious that their direction change depends on the distance q. It follows that the pressure angle is zero all times, allowing the cam to be much smaller.

### 3.4 Undercutting Phenomenon

A further limiting the cam size of this follower is undercutting, because the pressure angle has only a minor effect on the flat-faced follower. From kinematics, the center of rotation between two bodies is a point common to these bodies, and has the same velocity related to the ground considered on either body.

Therefore, the velocity (v) of the translating follower or center is
$v=\frac{d y}{d t}=q \omega$
The center of curvature of the cam at this instant has assumed a distance $u$ from the center. And the velocity of this point in the direction of the tangent is
$\frac{d q}{d t}=u \omega$
The acceleration (a) of the follower,

$$
\begin{equation*}
a=\frac{d^{2} y}{d t^{2}}=\frac{d q}{d t} \omega \tag{9}
\end{equation*}
$$



Figure-8. Cam curvature limitation with translating flatfaced distribution.
and consequently
$u=\frac{1}{\omega^{2}} \times \frac{d^{2} y}{d t^{2}}$
Therefore, the location of the curvature is directly related to the follower acceleration. For flat-faced follower cam construction, there is no limitation of curvature during the positive acceleration period. This requires that the distance $u$ is positive below the center (Figure-9 a). The restriction occurs when the acceleration is negative, which gives u a minimum value (Figure-9 b)

To eliminate undercutting in the cam profile, the minimum radius of curvature is
$\rho_{c(\min )}=R_{b}+(y+u)>0$
i.e. $\rho_{c(\min )}=R_{b}+\left(y+\frac{1}{\omega^{2}} \cdot \frac{d^{2} y}{d t^{2}}\right)>0$

Since $R b$ and $y$ are always positive, the undercutting $\left(\rho_{c}(\min )<0\right)$ is due to excessive negative acceleration. If undercutting exists, it is necessary to employ either a larger cam or a smaller maximum negative acceleration.

Total cam angle $\beta=60 \times \frac{\pi}{180}=1.047$ radian, Cam angular velocity $\omega=10 \times \frac{2 \pi}{60}=1.047 \mathrm{rad} / \mathrm{sec}$

The point of maximum negative acceleration for the simple harmonic motion from eq. (6) occurs at cam angle $\beta$, giving
$\left(\frac{\mathrm{d}^{2} \mathrm{y}}{\mathrm{dt}^{2}}\right)_{\text {max.neg }}=-\frac{\mathrm{h}}{2}\left(\frac{\pi \omega}{\beta}\right)^{2}=-\frac{1.5}{2}\left(\frac{\pi \times 1.047}{1.047}\right)^{2}=-7.40 \mathrm{~mm} / \mathrm{sec}^{2}$
The displacement at this angle $\beta$ is $y=1.5 \mathrm{~mm}$ and the radius of basic circle $\left(\mathrm{R}_{\mathrm{b}}\right)$ is 7.75 mm . Therefore, from eq. (12), the minimum radius of curvature of the cam profile

$$
\begin{aligned}
\rho_{c(\min )} & =R_{b}+\left(y+\frac{1}{\omega^{2}} \cdot \frac{d^{2} y}{d t^{2}}\right)_{\text {max.neg. }}>0 \\
& =7.75+\left(1.5-\frac{7.40}{1.047^{2}}\right)>0 \\
& =2.499 \mathrm{~mm} .>0
\end{aligned}
$$

Therefore, no undercutting is evident and the cam will be a proper design if the stresses are not excessive.

### 3.5 Cam Profile Determination

In this section methods of determining the cam shape are established. i.e. The cam profile and machine cutter location are determined exactly. The cam-follower mechanism has a constant pressure angle which measures zero degrees for the flat-faced follower. Also undercutting as referred to in the previous section, should not occur.


Figure-9. Radial cam-translating flat-faced follower.


Figure-10. Radial cam-translating flat-faced follower.
From the layout, the smallest size of flat-faced follower must maintain contact with the cam profile at all times. In other words, the cam should not go over the edge of the follower. Position 3 is the critical location where the contact point is farthest from the line of motion. It can be shown that the transition point is the critical location in establishing the necessary follower size. From the eq. (5), the maximum eccentricity of the point of contact from the cam center is
$q_{m}=\frac{v_{m}}{\omega}$
Where
$\mathrm{vm} \quad$ : maximum velocity of follower
$\omega \quad$ : cam angular velocity
i.e. The follower face may be made smaller by reducing the maximum follower velocity and increasing the cam speed.

### 3.6 Calculation Cam Profile

Point A is the centre of the cam. AB is the line of follower motion. Point C is the instantaneous contact point between the cam and follower.
$\mathrm{q} \quad:$ eccentricity of point of contact from cam centre
$\mathrm{r}_{\mathrm{c}} \quad$ : radial distance to cam profile
From the fig., point C is distance
$q=\frac{d y}{d t} \times \frac{1}{\omega}$
$r_{c}=\left[\left(R_{b}+y\right)^{2}+q^{2}\right]^{\frac{1}{2}}$
$\operatorname{Tan} \eta=\frac{q}{R_{b}+y}$
$\psi_{c}=\theta+\eta \Rightarrow$ on the rise period
$=\theta-\eta \Rightarrow$ on the rise period

## 4. MOTORLOAD ANALYSIS

DC motors provide excellent speed regulation, high torque and high efficiency therefore the one ideally suited for control application. DC motors can be designed to meet a wide range of power requirement so that DC motors allow precise control of either the speed and the torque. Torque requirement is calculated for DC motor selection.

### 4.1 Force Analysis

From the cam characteristics eq. 6 (i.e. cam motion curve angle), the acceleration of the follower is calculated. The displacement diagram and acceleration diagram is shown plotted in Figure-11(a) and (b). The inertia load is obtained by the equation of $F_{i}=\frac{w}{g} a=m a$

Also, the external load is calculated by above equation. Where m is 3.6 Kg , external load includes weight of follower. The external load on the follower is 35.3 N in Figure-11(d). The spring force is calculated using the spring specification below. In section 3.2.2, the indexing table employs a compression spring between body and pindisc (i.e. follower) so that preload exists during the operation of the cam and follower. The spring specification is a follow.

Stiffness of spring: $2.38 \mathrm{~N} / \mathrm{mm}$
Free length: 48mm
Preload length: 12.4 mm
The preload of spring is 84.7 N . These forces can be combined and Figure-12(e) shown the superimposed values to give a combined force diagram.
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Figure-11. Force analysis diagram.


Figure-12. Diagram of cam and follower.

### 4.2 Torque Required for Pindise(follower) Lift-Up and Table Indexing

From Figure-12, the torque required for lift-up and indexing is calculated using the equation below.
$T_{L I F T-U P}=F_{c} r_{c}+\mu_{c} F_{c} R_{L}$
where
$\mathrm{Fc} \quad=$ external force + spring force + inertia force (123.6 N)
$r_{c} \quad$ : max. eccentricity of point contact from cam center
$\mathrm{R}_{\mathrm{L}} \quad:$ Radius of cam shaft $(9.25 \mathrm{~mm})$
$\mu \quad$ : Coefficient of friction (0.4)
Kinematic contacting surface between steel on steel

So, the result of torque required for lift-up is plotted, maximum torque is occurred at the $33^{\circ}$ of cam angle.


Figure-13. The result of torque required for lift-up.


Figure-14. DC motor characteristics.
Also torque requirement for table indexing is calculated with the equation below.
$\mathrm{T}_{\max }($ indexing $)=\mathrm{T}_{\text {acceleration }}+\mathrm{T}_{\text {friction }}$
where
$\mathrm{T}_{\text {acceleration }}$ : Torque necessary to accelerate (decelerate) all masses
$\mathrm{T}_{\text {friction: }}$ Torque necessary to overcome friction $\mathrm{T}_{\text {max }}$ (indexing) is calculated 74.8 Ncm ,

And the DC motor specification is plotted below. When the torque is compared to motor characteristic, DC motor torque is sufficient to operate the indexing table.

### 4.3 Deflection of Camshaft



Figure-15. Mechanism of camshaft and pindisc.
The balls which are fixed in the top plate, are located on the circular vee for locking. When the indexing table is operating, the top plate is rotated with a clearance between the tops of the vees and the bottom of balls. The clearance is 0.428 mm . If there is no clearance due to the deflection of the camshaft, the indexing table will jam between vee and ball. So, the deflection of the camshaft should be calculated.

P(spring force)


Figure-16. Deflection of camshaft.
From the Figure-16
In the region to the left of the force $P$
Bending moment
$M=E I \frac{d^{2} y}{d x^{2}}=\frac{P b}{L} x$ for $0<\mathrm{x}<\mathrm{a}$
Therefore, the deflection of the camshaft is
$E I y=\frac{P b}{L} \cdot \frac{x^{3}}{6}+C_{1} x+C_{2}$
(Assuming no influence from the bearing) In the region to the right of the force P
$M=\left(\frac{P b}{L}\right) x-P(x-a)$ for $0<\mathrm{x}<\mathrm{L}$
$E I \frac{d y}{d x}=\frac{P b}{L} \cdot \frac{x^{2}}{2}-\frac{P(x-a)^{2}}{2}+C_{3}$
$E I y=\frac{P b}{L} \cdot \frac{x^{3}}{6}-\frac{P(x-a)^{3}}{6}+C_{3} x+C_{4}$
From the condition, at $x=0, y=0$ and $x=L, y=0$
$\mathrm{X}=\mathrm{a}$ Slope given by (17) must be equal to that given by (19)
$\mathrm{X}=\mathrm{a}$ Slope given by (18) must be equal to that given by (20)

From eq. (18) $\mathrm{x}=0, \mathrm{y}=0 \quad \therefore \mathrm{C}_{2}=0$
From eq. (19) $x=L, y=0$
x
$0=\frac{P b}{6} L^{2}-\frac{b}{6}(L-a)^{3}+C_{3} L+C_{4}$
From eq. (17) and eq. (19), when x is equal to a, slope is equal to
$\frac{P b}{L} \cdot \frac{a^{2}}{2}+C_{1}=\frac{P b}{2 L} a^{2}-\frac{P}{2}(a-a)^{2}+C_{3} \therefore \mathrm{C}_{1}=\mathrm{C}_{3}$
From eq. (18) and eq. (20), when x is equal to a, deflection is equal to
$\frac{P b}{L} \cdot \frac{a^{3}}{6}+C_{1} a+C_{2}=\frac{P b}{6 L} a^{3}-\frac{P(a-a)^{3}}{6}+C_{3} a+C_{4}$
$\therefore \mathrm{C}_{4}=0$
From eq. (21)
$\frac{P b}{6} L^{2}-\frac{P}{6}(L-a)^{3}+C_{3} L=0$ Where $\mathrm{L}-\mathrm{a}=\mathrm{b}$
$\therefore C_{3}=-\frac{P b}{6 L}\left(L^{2}-b^{2}\right)=C_{1}$
Therefore
$E I y=\frac{P b}{6 L}\left[x^{3}-\left(L^{2}-b^{2}\right) x\right] \quad$ for $0<\mathrm{x}<\mathrm{a}$
$E I y=\frac{P b}{6 L}\left[x^{3}-\frac{L}{b}(x-a)^{3}-\left(L^{2}-b^{2}\right) x\right]$ For $\mathrm{a}<\mathrm{x}$
<b
Where, $\quad \mathrm{L}=88 \mathrm{~mm}, \quad \mathrm{a}=41 \mathrm{~mm}, \quad \mathrm{P}=85 \mathrm{~N}$, $\mathrm{E}=200 \mathrm{kNmm}^{-2}, \mathrm{I}=201.062$
$\therefore y=\frac{1}{E I} \cdot \frac{P b}{6 L}\left[x^{3}-\left(L^{2}-b^{2}\right) x\right]=-0.0297(\mathrm{~mm})$

It means that the deflection of the camshaft is $29.7 \mu \mathrm{~m}$. So, when the indexing table is operation, the clearance between vee and ball is theoretically 0.398 mm .

## 5. CONCLUSIONS

Many techniques and methods for calibrating angular measurement exist in engineering industry. The cost-effective systems are generally available using general purpose hardware. The kinematic design of the indexing table using the veeplate and the ball location gives good repeatability. Compact indexing table was achieved by driving the cam mechanism.

Usually cams are designed for transforming one direction of motion into another. A cam mechanism consists of two elements, the cam and the follower. The follower is in direct contact with the cam. The follower system includes all of the elements to which motion is imparted by the cam. The pindisc is used as a follower in the indexing table system. The pindisc needs to produce lift up, rotation and lowering by the cam motion. The cam profile is designed accordingly. To achieve manufacturing accuracy of the cam shaft, milling machine and indexer is used. The equipment required for economical manufacture is relatively simple and could be undertaken by most machine shop

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