STABILITY ANALYSIS AND HEAT TRANSFER OF RAYLEIGH - BÉNARD CONVECTION OF BINGHAM FLUID THROUGH A VERTICAL CHANNEL IN A POROUS MEDIA

J. Murali kumar[1], Y. V. K. Ravi Kumar[2] and M. N. Rajasheker[3]
[1]Vivekananda Institute of Technology and Science, Karimnagar, India
[2]Practice School Division, Birla Institute of Technology and Science, Hyderabad, India
[3]Department of Mathematics, JNTUH College of Engineering, Jagtial, India
E-Mail: yvkravi@gmail.com

ABSTRACT

The studies on gravity modulation are made for non-internal heating systems. However, in many practically important situations the material offers its own source of heat and this leads to a setting up of different convective flow in a fluid layer through internal heating. Here we considered Stability Analysis and Heat Transfer of Rayleigh convection in a Bingham Fluid in a porous media with heat generation is explored by making a linear stability investigation. The steadiness of a flat layer of liquid heat-generation from inside is analyzed by accepting time-period of the power within the sight of the source. The impact of gravity regulation on the beginning of Rayleigh-Bénard convection a standard perturbation technique is used to show up at an articulation to process the basic Rayleigh number for little and dimensionless inner heat source. The Venezian approach is embraced to acquire the eigen estimation of the issue. The buoyancy effects will occur when the Rayleigh number exceeds a certain critical values are found.

Keywords: stability analysis, heat transfer of Rayleigh.

INTRODUCTION

A classical Rayleigh problem on the onset of convective instabilities in a horizontal thin layer of fluid heated from below is of fundamental importance and becomes a prototype to a more complex configuration in experiments and industrial processes. It has its origin in the experimental observations of Eringen [1] and [2]. The convective flows in a liquid layer are driven by buoyancy forces due to temperature gradients. Rayleigh's paper is the pioneering work for almost all modern theories of convection. Rayleigh [5] showed that Bénard convection, which is caused by Bingham fluids, the fluids with microstructure, are introduced and developed by Eringen [6]. Physically, these fluids represent fluids consisting randomly oriented particles suspended in a medium, where the deformation of the fluid particles is ignored. This constitutes a substantial generalization of the Navier-Stokes model and opens a new field of potential applications including a large number of complex fluids. A detailed survey of the theory of micropolar fluid and its applications are considered in the books of Lukasazewicz [7] and Power [8], which has become an important field of research especially in many industrially important fluids like paints, polymeric suspensions, colloidal fluids, and also in physiological fluids such as normal human blood and fluids in alveoli.

The main aim of the present study is to investigate the effects of gravity modulation and internal heat generation on the onset of Rayleigh-Bénard convection in a Bingham fluid. This analysis based on the linear stability theory and the resulting eigenvalue problem [10] is solved using the Venezian [20] approach by considering free-free, isothermal and no spin boundaries.

MATHEMATICAL FORMULATION

Consider an infinite horizontal layer of plastic Bingham fluid of depth d, where the fluid is heated from below with the internal heat generation exists with the fluid system. Let ΔT be the temperature difference between the lower and upper surfaces with the lower boundary at a higher temperature than the upper boundary. These boundaries maintained at constant temperature. The system is considered as a Cartesian co-ordinate system.
The governing equations are

$$\nabla.\vec{q} = 0 \tag{1}$$

$$\rho_0\left[\frac{\partial\vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q}\right] = -\nabla p + \rho\vec{g}(t)\hat{k} + (2\zeta+\eta)\nabla^2\vec{q} + \zeta(\nabla\times\vec{\omega}) \tag{2}$$

$$\vec{g}(t) = -g_0[(1+\varepsilon\cos(\gamma t)] \tag{3}$$

Conservation of Angular Momentum:

$$\rho_0 I\left[\frac{\partial\vec{\omega}}{\partial t} + (\vec{q}.\nabla)\vec{\omega}\right] = (\lambda'+\eta')\nabla(\nabla.\vec{\omega}) + \eta'\nabla^2\vec{\omega} + \zeta(\nabla\times\vec{q} - 2\vec{\omega}) \tag{4}$$

Conservation of Energy:

$$\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = \frac{\beta}{\rho_0 C_\upsilon}\nabla\times\vec{\omega}.\nabla T + \chi\nabla^2 T + Q(T-T_0) \tag{5}$$

Equation of State:

$$\rho = \rho_0[1-\alpha(T-T_0)] \tag{6}$$

Here, ρ_0 is density of the fluid at temperature $T = T_0$, p is the pressure, ρ is the density, $\vec{g}$ is acceleration due

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to gravity, g_0 is the mean gravity, ε is the small amplitude of gravity modulation, γ is the frequency, ζ is coupling viscosity coefficient or vortex viscosity, $\vec{\omega}$ is the angular velocity, I is moment of inertia, λ' and η' are bulk and shear spin viscosity coefficients, T is the temperature, χ is the thermal conductivity, β is heat conduction coefficient, α is coefficient of thermal expansion, Q is the internal heat source and t is time.

Basic State:

The basic state of the fluid is quiescent and is described by
$\vec{q} = \vec{q_b}(0,0,0),\ \vec{\omega} = \vec{\omega_b}(0,0,0),\ p = p_b(z),\ \rho = \rho_b(z),\ T = T_b(z)$.

Now, we obtain the following quiescent state solutions as

$$\frac{dp_b}{dz} = -\rho_b g_0[1 + \varepsilon\cos(\gamma t)]\hat{k} \tag{7}$$

$$\chi\frac{d^2T_b}{dz^2} = -Q(T - T_0) \tag{8}$$

$$\rho_b = \rho_0[1 - \alpha(T_b - T_0)] \tag{9}$$

Solution of equation (9) subject to the conditions

$$T_b = T_0 + \Delta T \text{ at } z = 0 \text{ and } T_b = T_0 \text{ and at } z = d \tag{10}$$

are obtained as $T_b(z) = T_0 + \Delta T\frac{\sin\sqrt{Ri}(1-z/d)}{\sin\sqrt{Ri}}$, where $Ri = \frac{Qd^2}{\chi}$ (11)

Linear Stability Analysis:

The stability of the basic state is analyzed by introducing the following perturbation

$$\vec{q} = \vec{q_b} + \vec{q'}, \vec{\omega} = \vec{\omega_b} + \vec{\omega'}, p = p_b + p', T = T_b + T', \rho = \rho_b + \rho' \tag{12}$$

where the prime indicates that the quantities are infinitesimal perturbations.

Substituting equation (12) into the equations (1)-(6) and using the basic state solutions, we get linearized equations governing the infinitesimal perturbations in the form:

$$\nabla.\vec{q'} = 0 \tag{13}$$

$$\rho_0\frac{\partial\vec{q'}}{\partial t} = -\nabla p' - \rho' g_0[1 + \varepsilon\cos(\gamma t)]\,\hat{k} + (2\zeta + \eta)\nabla^2\vec{q'} + \zeta(\nabla\times\vec{\omega'}) \tag{14}$$

$$\rho_0 I\left[\frac{\partial\vec{\omega'}}{\partial t} + (\vec{q'}.\nabla)\vec{\omega'}\right] = (\lambda' + \eta')\nabla(\nabla.\vec{\omega'}) + \eta'\nabla^2\vec{\omega'} + \zeta(\nabla\times\vec{q'} - 2\vec{\omega'}) \tag{15}$$

$$\frac{\partial T'}{\partial t} = \frac{\Delta T}{d}\frac{\sqrt{Ri}\cos\sqrt{Ri}(1-z/d)}{\sin\sqrt{Ri}}\left[W' - \frac{\beta}{\rho_0 C_\vartheta}\nabla\times\vec{\omega'}\right] + \chi\nabla^2 T' + QT' \tag{16}$$

$$\rho' = -\alpha\rho_0 T' \tag{17}$$

The perturbations equations (13)-(17) are non-dimensionalised using the following definitions

$$(x^*, y^*, z^*) = \left(\frac{x}{d}, \frac{y}{d}, \frac{z}{d}\right),\ \vec{q^*} = \frac{\vec{q'}}{\frac{\chi}{d}},\ T^* = \frac{T'}{\Delta T},\ t^* = \frac{t}{\frac{d^2}{\chi}},\ \Omega^* = \frac{\nabla\times\vec{\omega^*}}{\frac{\chi}{d^3}},\ \vec{\omega^*} = \frac{\vec{\omega'}}{\frac{\chi}{d^2}} \tag{18}$$

Using equation (17) in equation (14) and operating curl twice on the resulting equation, operating curl once on equation (15) and non-dimensionalization the two resulting equations and equation (16), using equation (18), we get

$$\frac{1}{Pr}\frac{\partial}{\partial t}(\nabla^2 W) = R[1 + \varepsilon\cos(\gamma t)]\,\nabla_1^2 T + (1 + N_1)\nabla^4 W + N_1\nabla^2\Omega \tag{19}$$

$$\frac{N_2}{Pr}\frac{\partial\Omega}{\partial t} = N_3\nabla^2\Omega - N_1\nabla^2 W - 2N_1\Omega \tag{20}$$

$$\frac{\partial T}{\partial t} = g(z)[W - N_5\Omega] + \nabla^2 T + Ri\,T \tag{21}$$

where the asterisks have been dropped for simplicity

Various Constants Used in Solving the Equations:

$Pr = \dfrac{\zeta + \eta}{\chi\rho_0}$ (Prandtl Number) , $R = \dfrac{\rho_0\alpha g_0\Delta T d^3}{\chi(\zeta + \eta)}$ (Rayleigh Number)

$N_1 = \dfrac{\zeta}{\zeta + \eta}$ (Coupling Parameter) , $N_2 = \dfrac{I}{d^2}$ (Inertia Parameter)

$N_3 = \frac{\lambda' + \eta'}{(\zeta + \eta)d^2}$ (Couple Stress Parameter), $N_5 = \frac{\beta}{\rho_0 C_\vartheta d^2}$ (Heat Conduction Parameter)

Equations (19) to (21) are solved subject to the conditions, free-free, isothermal and no spin boundary conditions, given by

$$W = \frac{\partial^2 W}{\partial z^2} = \Omega = T = 0 \text{ at } z = 0 \text{ and } z = 1 \tag{22}$$

Eliminating T and Ω from equations (23) to (25), we get an equation for W in the form

$$\left[\left(\frac{N_2}{Pr}\frac{\partial}{\partial t} - N_3\nabla^2 + 2N_1\right)\left(\frac{\partial}{\partial t} - \nabla^2 - Ri\right)\left(\frac{1}{Pr}\frac{\partial}{\partial t} - (1 + N_1)\nabla^2\right)\nabla^4 + N_1^2\left(\frac{\partial}{\partial t} - \nabla^2 - Ri\right)\nabla^6\right]W = R\nabla^2\nabla_1^2\left(\frac{N_2}{Pr}\varepsilon f' + (-N_3\nabla^2 + 2N_1 - N_1N_5\nabla^2)(1 + \varepsilon f)\right)g(z)W \tag{23}$$

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where f = Real part of ($e^{-i\Omega t}$) and $f' = (-i\Omega)$ Real part of ($e^{-i\Omega t}$)

In the dimensionless form, the velocity boundary conditions for solving equation (23) are obtainable from equations (19)-(21) and (22) in the form

$$W = \frac{\partial^2 W}{\partial z^2} = \frac{\partial^4 W}{\partial z^4} = \frac{\partial^6 W}{\partial z^6} = \frac{\partial^8 W}{\partial z^8} = 0 \text{ at } z = 0,1 \tag{24}$$

The Perturbation Procedure:

We now seek eigen-function W and eigen-values R of the equation (23) in the form

$$(R, W) = (R_0, W_0) + \varepsilon(R_1, W_1) + \varepsilon^2(R_2, W_2) + \dots \tag{25}$$

Substituting the expression (25) into equation (23) and equating like powers of ε on both sides, we get

$$L_1 W_0 = 0 \tag{26}$$

$$L_1 W_1 = \frac{N_2}{Pr} R_0 \nabla^2 \nabla_1^2 f' g(z) W_0 + \nabla^2 \nabla_1^2 (-N_3 \nabla^2 + 2N_1 + N_1 N_5 \nabla^2) g(z)(fR_0 + R_1) W_0 \tag{27}$$

$$L_1 W_2 = \nabla^2 \nabla_1^2 \frac{N_2}{Pr\, g(z) f'(R_0 W_1 + R_1 W_0)} + \nabla^2 \nabla_1^2 (-N_3 \nabla^2 + 2N_1 + N_1 N_5 \nabla^2) g(z)[(R_1 + fR_0) W_1 + (R_2 + fR_1) W_0] \tag{28}$$

where

$$L_1 = \left\{ \left[\frac{N_2}{Pr} \frac{\partial}{\partial t} - N_3 \nabla^2 + 2N_1 \right] \left[\frac{\partial}{\partial t} - \nabla^2 - Ri \right] \left[\frac{1}{Pr} \frac{\partial}{\partial t} - (1 + N_1) \nabla^2 \right] \nabla^4 + N_1^2 \left[\frac{\partial}{\partial t} - \nabla^2 - Ri \right] \nabla^6 - R_0 \nabla^2 \nabla_1^2 g(z) \left[\frac{N_2}{Pr} \frac{\partial}{\partial t} - N_3 \nabla^2 + 2N_1 + N_1 N_5 \nabla^2 \right] \right\} \tag{29}$$

Solution to the First Order Problem:

Equation (27) for W_1 now takes the form

$$L_1 W_1 = \frac{N_2}{Pr} k^2 a^2 f' G(z) R_0 W_0 + k^2 a^2 G(z)[(N_3 - N_1 N_5) k^2 + 2N_1](fR_0 + R_1) W_0 \tag{30}$$

If the above equation is to have a solution, the right hand side must be orthogonal to the null space of the operator L_1. This implies that the time independent part of the right hand side of the equation (32) must be orthogonal to $\text{Sin}(\pi z)$. Since f varies sinusoidal with time, the only steady term on the rt. hand side of equation (32) is $k^2 a^2 G(z)[(N_3 - N_1 N_5) k^2 + 2N_1] R_1$, so that $R_1 = 0$. It follows that all the odd coefficients, $R_1 = R_3 = \dots = 0$ in eqn (25).
Using equation (28), we find that

$$L_1\left[Sin(\pi z)\, exp\left(i(lx + my - \gamma t)\right)\right] = L_1(\gamma) Sin(\pi z)\, exp\left(i(lx + my - \gamma t)\right)$$

$$= \Big(\Big[-\frac{N_2 \gamma^2}{Pr} k^2 \Big[(1 + N_1) k^4 + \frac{1}{Pr}\Big]\Big] + \Big(N_3 k^2 + \frac{2N_1}{Pr}\Big)\Big[(1 + N_1) k^8 - k^4 \frac{\gamma^2}{Pr}\Big] + N_1^2 k^8 - R_0 k^2 a^2 G(z)[(N_3 - N_1 N_5) k^2 + 2N_1] + Ri\Big[\frac{N_2 \gamma^2}{Pr^2} k^2 - N_3(1 + N_1) k^8 - 2N_1(1 + N_1) k^6 - N_1^2 k^6\Big]\Big) + i\,\Big(\gamma\Big[\frac{N_2}{Pr}\Big[\frac{\gamma^2}{Pr} k^4 - (1 + N_1) k^8 + R_0 G(z)\Big] - (N_3 k^2 + 2N_1)\Big[(1 + N_1) k^6 + \frac{1}{Pr} k^6\Big] - N_1^2 k^6 + Ri\Big[\frac{N_2}{Pr}(1 + N_1) k^6 + \frac{N_3}{Pr} k^6 + \frac{2N_1}{Pr} k^4\Big]\Big]\Big) \tag{31}$$

$$\text{Now, } G(z) = \int_0^1 g(z)\, \text{Sin}^2(\pi z)\, dz$$

The particular solution of equation (32) is

$$W_1 = \frac{R_0 k^2 a^2 G(z)}{|L_1(\gamma)|^2}\Big[-\frac{N_2 \gamma}{Pr}(Y_1 \text{Sin}[\gamma t] + Y_2 \text{Cos}[\gamma t]) + A_1(Y_1 \text{Cos}[\gamma t] - Y_2 \text{Sin}[\gamma t])\Big] \tag{32}$$

Where

$$A_1 = [(N_3 - N_1 N_5) k^2 + 2N_1]$$

The equation of W_2 is

$$L_1 W_2 = R_2 k^2 a^2 A_1 G(z) W_0 + R_0 k^2 a^2 \frac{N_2}{Pr} f' G(z) W_1 + R_0 k^2 a^2 A_1 f G(z) W_1 \tag{33}$$

Now, for the existence of a solution of equation (35), it is necessary that the steady part of its right hand side is orthogonal to Sin (πz). Implies that

$$\int_0^1 \Big[R_2 k^2 a^2 A_1 G(z) W_0 + R_0 k^2 a^2 \frac{N_2}{Pr} f' G(z) W_1 + R_0 k^2 a^2 A_1 f G(z) W_1\Big] \text{Sin}[\pi z] dz = 0$$

Taking time average, we get

$$R_2 = -\frac{N_2 R_0}{A_1 Pr} G(z) \int_0^1 f' W_1 \text{Sin}[\pi z] dz - R_0 G(z) \int_0^1 f W_1 \text{Sin}[\pi z] dz \tag{34}$$

Finally

$$R_2 = -\frac{R_0^2 k^2 a^2 G(z)}{2|L_1(\Omega)|^2}\Big[\Big(\frac{N_2}{Pr}\Big)^2 \frac{\Omega^2 Y_1}{A_1} + A_1 Y_1\Big] \tag{35}$$

RESULTS AND DISCUSSIONS

- The parameters N_1, N_2, N_3, N_5 arise due to the Bingham fluid, the parameters Pr and Ri arise due to the fluid. To study the effects of these parameters on gravity modulation, the following range of parameters are considered in this paper

$0 \le N_1 \le 1$, $0 \le N_2 \le r$, $0 \le N_3 \le m$, $0 \le N_5 \le n$
where the quantities r, m and n are finite positive real numbers[21].

- The values of Pr for fluid with suspended particles are taken greater than the fluid without suspended

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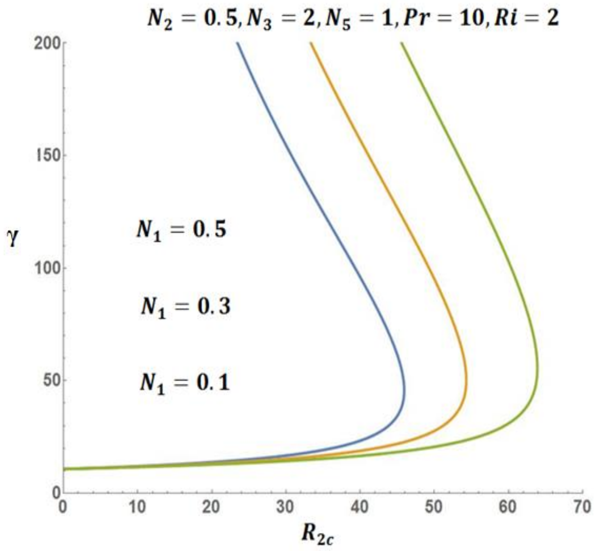
ARPN Journal of Engineering and Applied Sciences

particles because viscosity increases due to the presence of suspended particles.

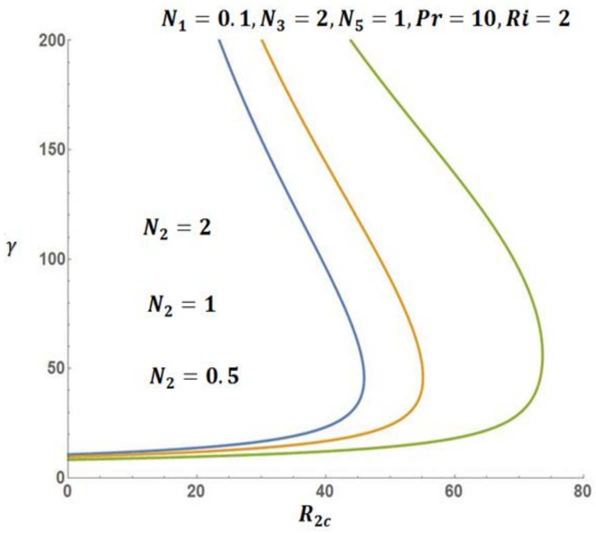
- The solutions obtained are based on the assumption that the amplitude of the gravity modulation is small, the result depends on the value of the modulating frequency γ.
- It is observed that when γ < 1, the gravity modulation affects the entire volume of the fluid resulting in the growth of the disturbance.
- On the other hand, the effect of modulation disappears for large frequencies. This is due to the fact the buoyancy force takes a mean value leading to the equilibrium state of the un-modulated case. In view of this, we choose only moderate value of γ in our present study.

The results obtained in this paper are depicted in the Plots (1) to (7).



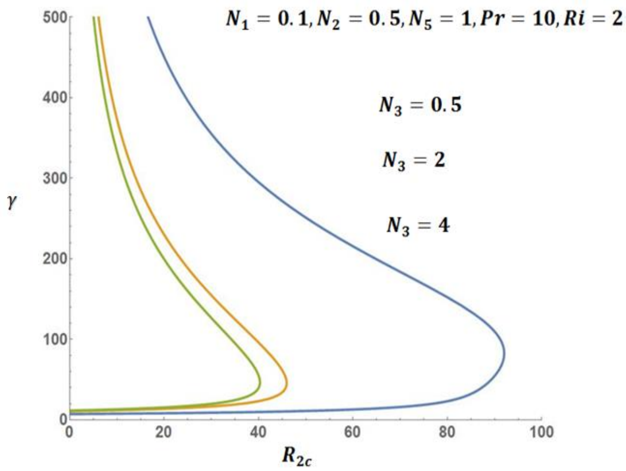
Plot-1. Increase in N_1 stabilizes the system w.r.to. R_{2c}.

Correction Rayleigh number R_{2c} versus frequency of modulation γ for different values of coupling parameterN_1. We observe that as N_1 increases, R_{2c} also increases. The increase in N_1 implies increase in the concentration of suspended particles.



Plot-2. Increase in N_2 stabilizes the system w.r.to. R_{2c}.

R_{2c} versus γ for different values of inertia parameter N_2. Increase in N_2 is representative of the increase in inertia of the fluid due to the suspended particles.



Plot-3. As N_5 increases R_{2c} increases.

For different values of the heat conduction parameter N_5. When N_5 increases, the heat induced into the fluid due to this, microelements also increases

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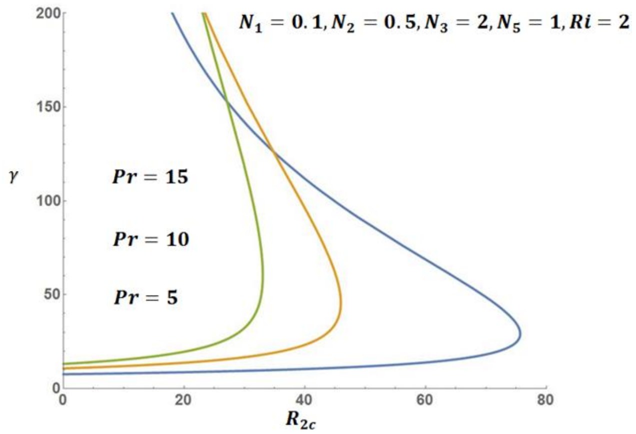
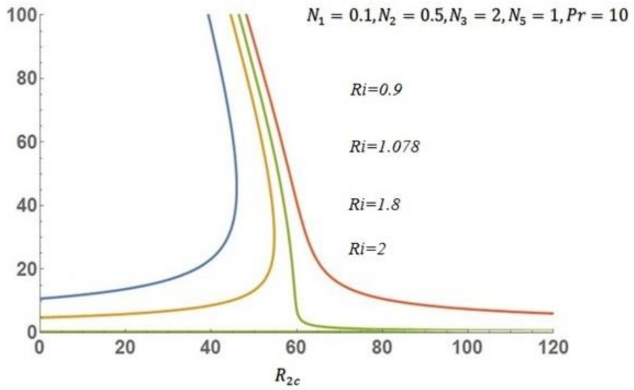


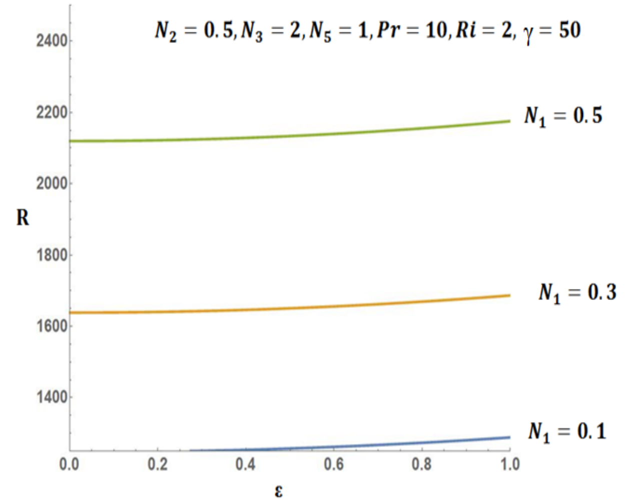
Plot-4. As N_5 increases R_{2c} increases.

It can be inferred from this that the effect of increasing the concentration of the suspended particle is to stabilize the system. The fluids with suspended particles are more susceptible to stabilization by modulation than clean fluids.



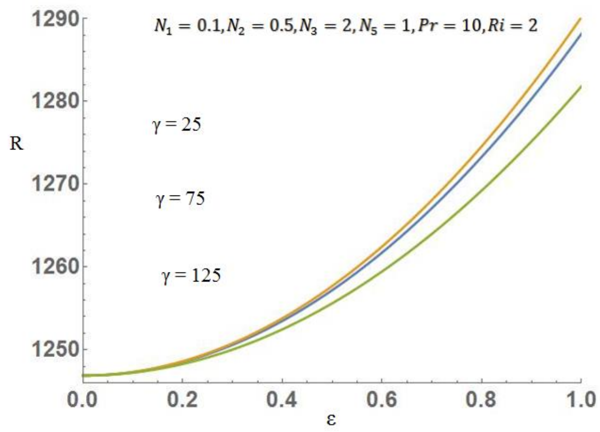
Plot-5. The plot of R_{2c}.

R_{2c} versus γ for different values of Prandtl number Pr. It is observed that as Pr increases R_{2c} calso increases. This means that the fluids with suspended particles are more susceptible to stabilization.



Plot-6. The plot of R_{2c}.

R_{2c}versus Ω for different values of internal Rayleigh number Ri. We observe that the increase in the internal Rayleigh number Ri increases the heat transport in the system.



Plot-7. The plot of R_{2c}.

Amplitude of modulation ε versus Rayleigh number R for different values of γ. It is observed that the amplitude of modulation ε increases, the Rayleigh number R also increases.

CONCLUSIONS

- From the Plots (1) to (7) it is observed that since R_{2c} remains always positive for all values of γ, gravity modulation leads to delay in onset of convection.
- The results of this study are helpful in the areas of crystal growth under microgravity conditions.
- The effect of coupling parameter N_1, inertia parameter N_2, Bingham heat conduction parameter N_5 and Prandtl number Pr is to reduce the amount of heat transfer whereas the opposite effect is observed in the case of couple stress parameter N_3 and internal Rayleigh number Ri.

- It is observed that gravity modulation or g-jitter leads to delay in convection and frequency of gravity modulation also plays an important role in controlling heat transfer in the system.
- The effect of internal heat generation has significant influence on the Rayleigh - Bénard convection and is clearly a destabilizing factor to make the system more unstable.

REFERENCES

[1] A. C. Eringen. 1966. Theory of Micro polar Fluids. Int. J. Eng. Sci.

[2] A. C. Eringen. 1966. A unified theory of thermomechanical materials. International Journal of Engineering Science. 4(2): 179-202.

[3] H. Benard. 1900. Les tourbillions cellulaires dans une nappe liquid. Revue generale des Sciences pures et appliqués. 11: 1261-1271.

[4] H. Benard. 1901. Les tourbillions cellulaires dans une nappe liquid transportant de la chaleur en regime permanent. Ann. Chem. Phys. 23: 62-144.

[5] L. Rayleigh. 1916. On convection currents in a horizontal layer of fluid, when the higher temperature is on the under side. Phil. Mag. 32(1916): 529-546.

[6] A. C. Eringen. 1964. Simple microfluids. International Journal of Engineering Science. 2(2): 205-217.

[7] G. Lukaszewicz. 1999. Micropolar fluid theory and applications. Boston: Birkhauser.

[8] H. Power. 1995. Bio-Fluid Mechanics: Advances in Fluid Mechanics. U.K.: W.I.T. Press.

[9] A. B. Datta and V. U. K. Sastry. 1965. Thermal instability of a horizontal layer of micropolar fluid heated from below. International Journal of Engineering Science. 14(7): 631-637.

[10] G. Ahmadi. 1976. Stability of micropolar fluid layer heated from below. International Journal of Engineering Science. 14, 81-85.

[11] R. V. R. Rao. 1980. Thermal instability in a micropolar fluid layer subject to a magnetic field. International Journal of Engineering Science. 18(5): 741-750.

[12] S. P. Bhattacharyya and S. K. Jena. 1983. On the stability of a hot layer of micropolar fluid. International Journal of Engineering Science. 21(9): 1019-1024.

[13] P. M. Gresho and R. L. Sani. 1970. The effects of gravity modulation on the stability of a heated fluid layer. J. Fluid Mech. 40(4): 783-806.

[14] Wheeler *et al.* 1991. Convection stability in the Rayleigh Benard and directional solidification problems: high frequency gravity modulation. Phys. Fluids. 3: 2847-2858.

[15] M. S. Malashetty and D. Basavaraja. 2002. Rayleigh-Benard convection subject to time dependent wall temperature/gravity in fluid-saturated anisotropic porous medium. Int. J. Heat Mass Transfer. 38, 551563.

[16] S. P. Bhattacharya and S. K. Jena. 1984. Thermal instability of a horizontal layer of micropolar fluid with heat source. Proc. Indian Acad. Sci. 93(1): 13-26.

[17] M. Takashima. 1989. The stability of natural convection in an inclined fluid layer with internal heat generation. Journal of the Physical Society of Japan. 58: 4431-4440.

[18] Yuji Tasaka and Yasushi Takeda. 2005. Effects of heat source distribution on natural Convection induced by internal heating. International Journal of Heat and Mass Transfer. 48: 1164-1174.

[19] B. S. Bhadauria *et al*. 2011. Natural convection in a rotating anisotropic porous layer with internal heat generation. Transp Porous Med. 90: 687-705.

[20] Venezian G. 1969. Effect of modulation on the onset of thermal convection. J. Fluid Mech. 35, 243-254.