



# CALCULATING OPTIMAL GEAR RATIOS FOR THREE STAGE BEVEL HELICAL REDUCER FOR MINIMAL REDUCER CROSS-SECTION

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## ABSTRACT

This work deals with the determination of optimal gear ratios of three-step bevel helical reducers. To find the optimal ratios, an optimization problem was created and solved. In this problem, the minimal reducer cross section was selected for the target. Also, seven main design parameters counting the total ratio, the allowable contact stresses and the face width coefficients of all steps and the output torque were chosen for the examination of the effects of them to the optimal gear ratios. In addition, to estimate the weight of these parameters on the optimum ratios, a simulation experiment was carried out by programming. From the experimental results, the influence of the design factors on the optimal gear ratios was found, and an equation to find the optimal gear ratios was presented.

**Keywords:** reducer, gear ratio, optimum gear ratio, three-step bevel helical reducer.

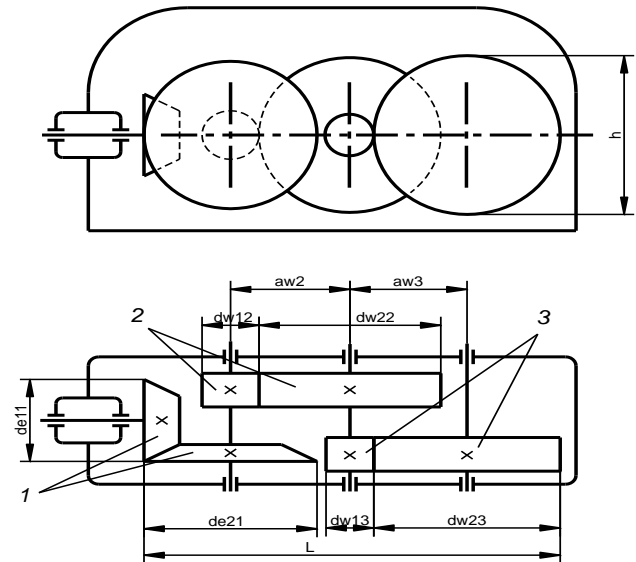
## 1. INTRODUCTION

In optimization design of a reducer, the calculation of the optimum gear ratios is very important. This is because the mass, the dimension, and the reducer cost are subject strongly to the gear ratios of each step. Therefore, the determination of the optimal gear ratios has attracted the attention of many scientists.

Until now, the optimal gear ratios have been calculated for mechanical systems with different reducer types counting the helical reducers [1-9], the bevel reducers [5, 6, 10-13] or the worm reducers [6, 14-17]. The gear ratios were computed for two-step reducers [2, 3, 5, 10, 11, 18], three-step reducers [1, 7, 9, 12, 19, 20] and four-step reducers [21-24]. Besides, the optimal gear ratios were determined for getting several targets such as minimal length of reducers [1, 14, 20-22, 25], minimal weight of gears [20, 26], the reducer height [27], minimal reducer cost [9], or minimal cross section of reducers [2-4, 19, 20, 23]. The optimal gear ratios have also found for mechanical systems with a reducer and a V-belt drive [28-31] or a chain drive [27, 32-35].

This paper deals with the influence of main design factors on the optimal gear ratios of a three step bevel helical reducer. Also, regression models to predict the optimal gear ratios to get the minimal cross reducer section were suggested.

## 2. METHODOLOGY



**Figure-1.** Schema of three-step bevel helical reducer.

For a three-step bevel helical reducer, the reducer cross section can be determined by:

$$A = L \cdot h \quad (1)$$

Wherein  $L$  and  $h$  are the length and the height of the reducer which are calculated by (see Figure-1):

$$L = d_{e21} / 2 + a_{w2} + a_{w3} + d_{w23} / 2 \quad (2)$$

$$h = \max(d_{e21}, d_{w22}, d_{w23}) \quad (3)$$



In Equations (2) and (3),  $a_{w2}$ ,  $a_{w3}$ ,  $d_{w22}$  and  $d_{w23}$  are the center distances and the pitch diameters of steps 2 and 3, respectively.  $d_{w22}$  and  $d_{w23}$  can be found as [25]:

$$d_{w22} = 2 \cdot a_{w2} \cdot u_2 / (u_2 + 1) \quad (4)$$

$$d_{w23} = 2 \cdot a_{w3} \cdot u_3 / (u_3 + 1) \quad (5)$$

In which,  $u_2$  and  $u_3$  are the gear ratios steps 2 and 3.

From the above analysis, the problem to find the optimum gear ratios is given as:

$$\text{minimize } A = L \cdot h \quad (6)$$

With the constraints as

$$1 \leq u_2 \leq 9 \quad (7)$$

$$1 \leq u_3 \leq 9$$

From equations (1), (2), (4), (5) and (6), to solve the above problem, it is essential to define  $d_{e21}$ ,  $a_{w2}$  and  $a_{w3}$ .

### 2.1 Calculating $d_{e21}$

For the bevel gear set,  $d_{e21}$  can be found as [25]:

$$d_{e21} = 2 \cdot u_1 \cdot R_e / \sqrt{1 + u_1^2} \quad (8)$$

Where,  $R_e$  is the distance of external cone;  $R_e$  is calculated by [25]:

$$R_e = k_R \cdot \sqrt{u_1^2 + 1} \cdot \sqrt[3]{T_{11} \cdot k_{H\beta 1} / [(1 - k_{be}) \cdot k_{be} \cdot u_1 \cdot [\sigma_H]^2]} \quad (9)$$

In which,  $k_R = 50$  (MPa<sup>1/3</sup>) is material coefficient;  $u_1$  is the gear ratio of step 1;  $k_{be} = 0.25 \dots 0.3$  is the face width coefficient;  $[\sigma_{H1}]$  is the allowable contact stress (MPa);  $K_{H\beta 1}$  is the concentration factor of load; From the data in [25],  $K_{H\beta 1}$  is determined by:

$$K_{H\beta 1} = 0.25 \cdot k^2 + 0.2 \cdot k + 1.02 \quad (10)$$

Wherein,  $k = k_{be} \cdot u_1 / (2 - k_{be})$ . For the reducer, the following equation can be written:

$$T_{11} = T_{out} / (u_g \cdot \eta_{bg} \cdot \eta_{hg} \cdot \eta_b^3) \quad (11)$$

In which,  $T_{out}$  is the output torque (Nmm);  $u_g$  is the total ratio of the reducer;  $\eta_{hg} = 0.95 \dots 0.97$  is the bevel gear efficiency [25];  $\eta_{bg} = 0.96 \dots 0.98$  is the helical gear efficiency [25];  $\eta_b = 0.99 \dots 0.995$  is the bearing efficiency [25]. Choosing  $\eta_{bg} = 0.96$ ,  $\eta_{hg} = 0.97$ ,  $\eta_b = 0.992$  and replacing them into (11) provides:

$$T_{11} = 1.101 \cdot T_{out} / u_g \quad (12)$$

Replacing  $k_R = 50$  and (13) into (10) gets

$$R_e = 51.6296 \cdot \sqrt{u_1^2 + 1} \cdot \sqrt[3]{T_{out} \cdot k_{H\beta 1} / [(1 - k_{be}) \cdot k_{be} \cdot u_1 \cdot u_g \cdot [\sigma_H]^2]} \quad (13)$$

### 2.2 Calculating $a_{w2}$

For helical gear set,  $a_{w2}$  can be determined by [25]:

$$a_{w2} = k_m \cdot (u_2 + 1) \cdot \sqrt[3]{\frac{T_{12} \cdot k_{H\beta}}{[\sigma_H]^2 \cdot u_2 \cdot \psi_{ba2}}} \quad (14)$$

In which,  $K_{H\beta}$  is the concentration factor of load; it can be chosen by  $k_{H\beta} = 1.1$  as  $k_{H\beta} = 1.02 \div 1.28$  [25];  $[\sigma_H]$  is the allowable contact stress (MPa);  $k_m = 43$  is the material coefficient [25];  $\psi_{ba2}$  is the wheel face width coefficient of step 2.

For the three step bevel helical reducer, the output torque can be calculated by:

$$T_{out} = T_{12} \cdot \eta_{hg}^2 \cdot \eta_b^3 \cdot u_2 \cdot u_3 \quad (15)$$

Replacing  $\eta_{hg} = 0.97$  and  $\eta_b = 0.992$  as in subsection 2.1 have

$$T_{12} = 1.0887 \cdot T_{out} / (u_2 \cdot u_3) \quad (16)$$

Substituting (16) and  $k_{H\beta} = 1.1$  into equation (14) with considering  $u_2 \cdot u_3 = u_g / u_1$  have:

$$a_{w2} = 45.6635 \cdot (u_2 + 1) \cdot \sqrt[3]{\frac{T_{out} \cdot u_1}{[\sigma_H]^2 \cdot u_g \cdot u_2 \cdot \psi_{ba2}}} \quad (17)$$

### 2.3 Calculating $a_{w3}$

For the third step,  $a_{w3}$  is calculated by [25]:



$$a_{w3} = K_m \cdot (u_3 + 1) \cdot \sqrt[3]{\frac{T_{13} \cdot k_{H\beta}}{[\sigma_H]^2 \cdot u_3 \cdot \psi_{ba3}}} \quad (18)$$

$$a_{w3} = 45.0814 \cdot (u_3 + 1) \cdot \sqrt[3]{\frac{T_{out}}{[\sigma_H]^2 \cdot u_3^2 \cdot \psi_{ba3}}} \quad (21)$$

Also

$$T_{out} = T_{13} \cdot \eta_{hg} \cdot \eta_b^2 \cdot u_3 \quad (19)$$

Selecting  $\eta_{hg} = 0.97$  and  $\eta_b = 0.992$  as in section 2.2 have

$$T_{12} = 1.0476 \cdot T_{out} / u_3 \quad (20)$$

Substituting (20),  $k_m = 43$  and  $k_{H\beta} = 1.1$  (as in section 2.2) into (18) gets:

**2.4 Experimental work**

In order to discover the impact of the design factors on the optimum ratios, a simulation experiment was planned and accomplished. For this work, the factorial design was taken and a 2 level factorial design with 1/2 fraction was carefully chosen. In addition, 8 main design factors were selected for the analysis (Table-1). Consequently, the experiment was organised with  $2^{8-1} = 128$  test runs. Also, based on equations (6) and (7), a computational program was built for solving the problem.

The plan of the experiment and the optimum ratios  $u_1$  and  $u_2$  are shown in Table-2.

**Table-1.** Input parameters.

Factor	Code	Unit	Low	High
Total reducer ratio	$u_g$	-	40	110
Face width coefficient of the bevel gear set	$K_{be}$	-	0.25	0.3
Wheel face width coefficient of step 2	$X_{ba2}$	-	0.3	0.35
Wheel face width coefficient of step 3	$X_{ba3}$	-	0.35	0.4
Allowable contact stress of step 1	$AS_1$	MPa	360	420
Allowable contact stress of step 2	$AS_2$	MPa	360	420
Allowable contact stress of step 3	$AS_3$	MPa	360	420
Output torque	$T_{out}$	Nmm	$10^5$	$10^7$

**Table-2.** Plan of experiment and output response.

Std Order	Run Order	Center Pt	Blocks	$u_g$	$K_{be}$	$X_{ba2}$	$X_{ba3}$	$AS_1$ (MPa)	$AS_2$ (MPa)	$AS_3$ (MPa)	$T_{out}$ (Nm)	$u_2$	$u_3$
26	1	1	1	90	0.25	0.3	0.4	420	360	360	10000	1.50	5.64
46	2	1	1	90	0.25	0.35	0.4	360	420	360	100	6.03	2.31
110	3	1	1	90	0.25	0.35	0.4	360	420	420	10000	6.60	2.82
8	4	1	1	90	0.3	0.35	0.35	360	360	360	10000	5.70	2.25
113	5	1	1	40	0.25	0.3	0.35	420	420	420	10000	3.99	2.25
98	6	1	1	90	0.25	0.3	0.35	360	420	420	10000	6.15	2.79
...													
122	127	1	1	90	0.25	0.3	0.4	420	420	420	10000	1.50	6.12
119	128	1	1	40	0.3	0.35	0.35	420	420	420	10000	3.96	1.98

**3. RESULTS AND DISCUSSIONS**

Figure-2 shows the impact of the main design factors on the optimal ratios of the second step  $u_2$  (Figure-2a) and the third step  $u_3$  (Figure-2b). From the

Fig.,  $u_2$  increases significantly with the increase of  $u_g$ ,  $\psi_{ba2}$  and  $AS_2$ . On the other hand,  $u_3$  upsurges considerably with the growth of  $u_g$ ,  $\psi_{ba3}$  and  $AS_3$ . In



addition, with the rise of  $\psi_{ba2}$  and  $AS_2$ ,  $u_3$  declines remarkably, whereas, with the increase of  $\psi_{ba3}$  and  $AS_3$ ,  $u_2$  reduces significantly. Besides, both  $u_2$  and  $u_3$  decrease with the increase of the  $AS_1$ . Additionally, they are nearly not affected by  $k_{be}$  and  $T_{out}$ .

Figure-3 presents the Normal Plot of the standardized effects for  $u_2$  (Figure-3.a) and  $u_3$  (Figure-3b). It is found from the figure that  $u_g$  (factor A),  $\psi_{ba2}$  (factor C),  $\psi_{ba3}$  (factor D),  $AS_2$  (factor F) and  $AS_3$  (factor

G) are the most weighty factors for both  $u_2$  and  $u_3$ . Furthermore,  $u_g$  (factor A),  $\psi_{ba2}$  (factor C),  $AS_2$  (factor F) and the interactions CG, FG, DF and CD have a positive impact for  $u_2$ . Besides,  $\psi_{ba3}$  (factor D),  $AS_3$  (factor G) and the interactions DG and CF have a negative impact for  $u_2$ . On the other hand, for  $u_3$ , factors A, D, G and the interactions AD, AG and DG a positive standardized effect and factors C, F and the interactions AC, AF, CG and FG have a negative influence.

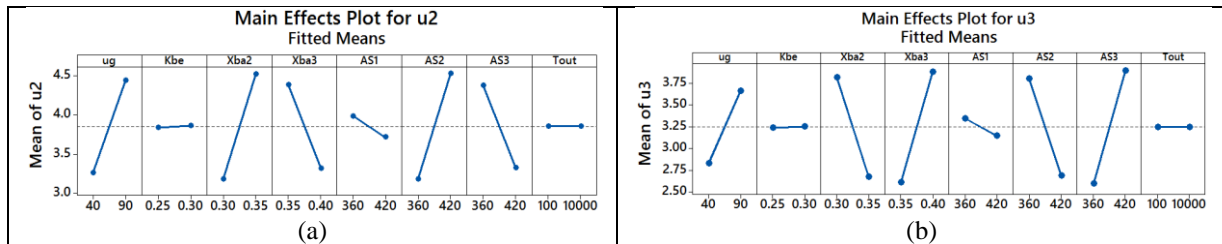


Figure-2. Graph of main influences for  $u_2$  and  $u_3$ .

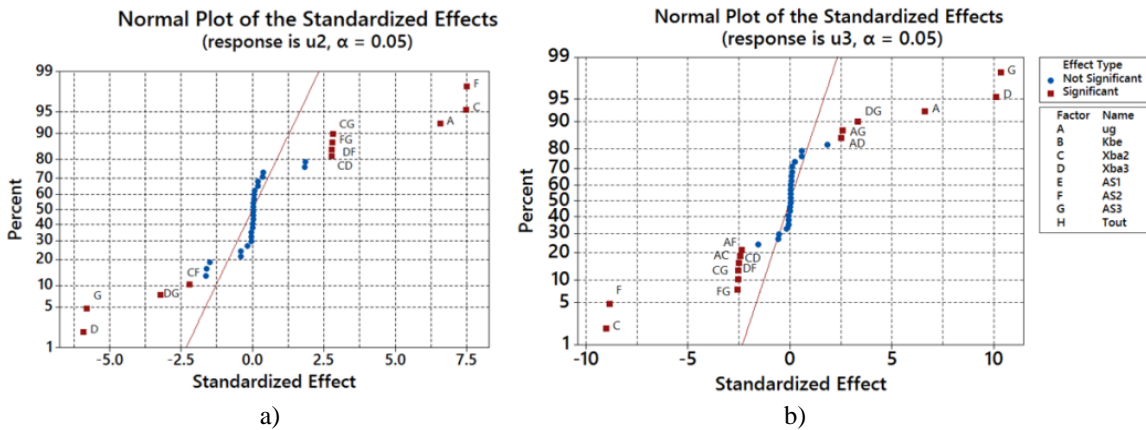


Figure-3. Normal plot for  $u_2$  and  $u_3$ .

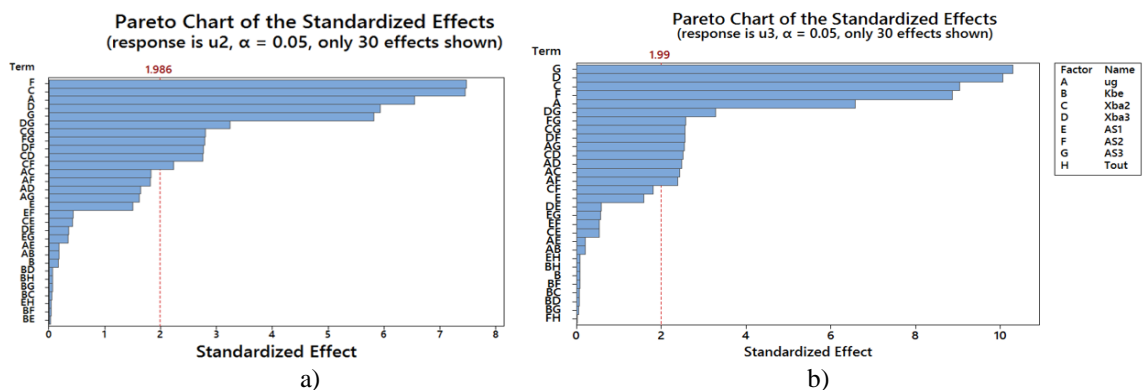


Figure-4. Pareto chart for  $u_2$  and  $u_3$ .



The Pareto chart for  $u_2$  (Figure-4a) and  $u_3$  (Figure-4b) is described in Figure-4. It is learned that the reference line crosses the bars which represent factors F, C, A, D, G and the interactions DG, CG, FG, DF and CD. As a result, these factors are significant with the optimum gear ratios of the second step  $u_2$ . Also, the bars which characterize factors G, D, C, F, A and the interactions DG, FG, CG, DF, AG, CD, AD, AC cut the reference line. Consequently, these parameters are weighty with  $u_3$ .

Figure-5 shows the expected effects and coefficients for  $u_2$  (Figure-5a) and  $u_3$  (Figure-5b). From this Fig., parameters which have P-values lesser than 0.05 are  $u_g$ ,  $\psi_{ba2}$  and  $\psi_{ba3}$ ,  $AS_2$  and  $AS_3$ , and the interactions  $\psi_{ba2} * \psi_{ba3}$ ,  $\psi_{ba2} * AS_2$ ,  $\psi_{ba2} * AS_3$ ,  $\psi_{ba3} * AS_2$ ,

$\psi_{ba3} * AS_3$  and  $AS_2 * AS_3$ . Subsequently, these parameters have a significant effect on the responses ( $u_2$  and  $u_3$ ). Also, the following models are proposed for finding  $u_2$  and  $u_3$ :

$$u_2 = 91.6 + 0.02382 \cdot u_g - 150.3 \cdot \psi_{ba2} - 129.2 \cdot \psi_{ba3} - 0.1248 \cdot AS_2 - 0.0905 \cdot AS_3 + 401 \cdot \psi_{ba2} \cdot \psi_{ba3} - 0.27 \cdot \psi_{ba2} \cdot AS_2 + 0.34 \cdot \psi_{ba2} \cdot AS_3 + 0.335 \cdot \psi_{ba3} \cdot AS_2 - 0.393 \cdot \psi_{ba3} \cdot AS_3 + 0.000282 \cdot AS_2 \cdot AS_3 \quad (22)$$

$$u_3 = -76.7 - 0.0028 \cdot u_g + 172.7 \cdot \psi_{ba2} + 68 \cdot \psi_{ba3} + 0.1457 \cdot AS_2 + 0.0445 \cdot AS_3 - 0.2464 \cdot u_g \cdot \psi_{ba2} + 0.2509 \cdot u_g \cdot \psi_{ba3} - 0.000201 \cdot u_g \cdot AS_2 + 0.000215 \cdot u_g \cdot AS_3 - 254.6 \cdot \psi_{ba2} \cdot \psi_{ba3} - 0.2153 \cdot \psi_{ba2} \cdot AS_3 - 0.2153 \cdot \psi_{ba3} \cdot AS_2 + 0.2766 \cdot \psi_{ba3} \cdot AS_3 - 0.000181 \cdot AS_2 \cdot AS_3 \quad (23)$$

Coded Coefficients							Coded Coefficients						
Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF	Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		3.8515	0.0868	44.37	0.000		Constant		3.2454	0.0588	55.23	0.000	
ug	1.1911	0.5955	0.0868	6.86	0.000	1.00	ug	0.8302	0.4151	0.0588	7.06	0.000	1.00
Xba2	1.3552	0.6776	0.0868	7.80	0.000	1.00	Xba2	-1.1405	-0.5702	0.0588	-9.70	0.000	1.00
Xba3	-1.0786	-0.5393	0.0868	-6.21	0.000	1.00	Xba3	1.2698	0.6349	0.0588	10.81	0.000	1.00
AS2	1.3598	0.6799	0.0868	7.83	0.000	1.00	AS2	-1.1189	-0.5595	0.0588	-9.52	0.000	1.00
AS3	-1.0570	-0.5285	0.0868	-6.09	0.000	1.00	AS3	1.2989	0.6495	0.0588	11.05	0.000	1.00
Xba2*Xba3	0.5011	0.2505	0.0868	2.89	0.005	1.00	ug*Xba2	-0.3080	-0.1540	0.0588	-2.62	0.010	1.00
Xba2*AS2	-0.4055	-0.2027	0.0868	-2.34	0.021	1.00	ug*Xba3	0.3136	0.1568	0.0588	2.67	0.009	1.00
Xba2*AS3	0.5095	0.2548	0.0868	2.93	0.004	1.00	ug*AS2	-0.3014	-0.1507	0.0588	-2.56	0.012	1.00
Xba3*AS2	0.5020	0.2510	0.0868	2.89	0.005	1.00	ug*AS3	0.3220	0.1610	0.0588	2.74	0.007	1.00
Xba3*AS3	-0.5892	-0.2946	0.0868	-3.39	0.001	1.00	Xba2*Xba3	-0.3183	-0.1591	0.0588	-2.71	0.008	1.00
AS2*AS3	0.5067	0.2534	0.0868	2.92	0.004	1.00	Xba2*AS3	-0.3230	-0.1615	0.0588	-2.75	0.007	1.00
							Xba3*AS2	-0.3230	-0.1615	0.0588	-2.75	0.007	1.00
							Xba3*AS3	0.4148	0.2074	0.0588	3.53	0.001	1.00
							AS2*AS3	-0.3258	-0.1629	0.0588	-2.77	0.007	1.00

Model Summary				Model Summary			
S	R-sq	R-sq(adj)	R-sq(pred)	S	R-sq	R-sq(adj)	R-sq(pred)
0.982183	71.83%	69.16%	65.70%	0.664811	82.81%	80.68%	77.94%

Figure-5. Estimated effects and coefficients for  $u_2$  and  $u_3$ .

4. CONCLUSIONS

The present work deals with the determination of the optimum gear ratios of a three step bevel helical reducer to find the minimal reducer cross section was presented. In this work, the effect of the main design factors excluding the entire reducer ratio, the wheel face width coefficients and the allowable contact stresses of all steps, and the output torque were inspected. Also, regression models to determine the optimal gear ratios of each step of the reducer were suggested. The optimal ratios of the reducer can be calculated simply as they are explicit.

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