



MODEL PREDICTIVE CONTROL OF THREE-PHASE VOLTAGE SOURCE INVERTERS WITH LC OUTPUT FILTER

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ABSTRACT

This paper deals with the problem of voltage and current control of voltage source inverters (VSIs) with LC output filter. The objective is to achieve voltage regulation and reference tracking while ensuring current stability and regulation despite the unknown disturbances. In this work a model predictive control approach is proposed for three-phase VSIs with LC output filter control. The proposed controller has a cascade structure and it is designed in the synchronous reference frame d-q. The outer loop is for voltage control and the inner loop is for current control. A model predictive controller is developed for robust voltage regulation and reference tracking despite the unknown disturbances. The current controller uses PI controllers for current regulation and feedforward terms to decouple the direct and quadrature components of voltage and current control loops. Unlike the conventional PI-based synchronous reference frame VSI control, the proposed approach provides better control performance with smaller and smoother control signal and enables the d-q components of voltage and current to be fully decoupled. A comparative simulation study with the conventional control system is provided to highlight the efficiency of the proposed approach.

Keywords: current control, model predictive control, voltage control, voltage source inverter (VSI).

INTRODUCTION

Voltage source inverters (VSIs) are among the main building blocks of power generation systems and they are commonly used for interfacing renewable energy sources and energy storage systems with the primary grid [1], [2]. VSIs have many applications including voltage and current regulation [2]–[6], active power filtering [7]–[9] and power factor control [10], [11]. VSIs require proper control to achieve the desired objectives in presence of the system nonlinearities and the unknown disturbances.

Several works addressed VSIs output voltage and current control in the last decade. The main idea is to design a cascade control system where the inner loop is for current control and the outer loop is for voltage control. In [2], a voltage harmonic mitigation strategy is proposed for VSIs using cascade control with voltage, current and harmonic compensation loops for accurate harmonic current sharing. However, the voltage/current loops use PR (Proportional-Resonant) controllers in the stationary $\alpha - \beta$ reference frame. This means that the manipulated variables are sinusoidal and only the targeted frequencies are controlled which may hinders the overall control performance with undesired signals in the output voltage/current. The conventional synchronous reference frame $d - q$ VSIs control system has been proposed in [3]. In inverter-based distributed generators control, synchronous (direct-quadrature d-q) reference frame is advantageous as the manipulated variables are not sinusoidal which facilitates the control. The conventional control system also uses standard PI regulators which simplifies the tuning process. Yet, several voltage and current feedforward and cross-terms are used in the conventional approach for voltage and current regulation and the control system requires measures of the output

voltage and current and the inductance current which implies several measurement equipment's. Some works investigated different approaches as internal model control in [5] and model predictive control in [6]. However, only current control was addressed.

Model predictive control (MPC) is an optimization-based control technique widely used in several industrial applications including electrical drives and power inverters control [6], [12]–[13], motors control [14]–[15] and renewable energy [12], [16]–[17]. Among the commonly used MPC schemes, the continuous model-predictive control is suitable for continuous-model systems such as VSIs for its numerous advantages. First, the continuous-time design is less sensitive to the choice of sampling-time. Second, robust output regulation and reference tracking by predicting the system's behavior on the desired horizon and by optimizing the control signal. In light of these advantages, this paper proposes a continuous model predictive control cascade control system for voltage and current control of VSIs with LC output filter in the d-q reference frame. There are four technical contributions in this work:

- Flexible dynamic modeling of VSIs in the d-q synchronous reference frame.
- Multiobjective control through optimal voltage regulation and reference tracking while ensuring current stability by maintaining control of the current reference.
- Fully decoupled control of voltage and current direct and quadratic components.



- Faster regulation and reference tracking with a smaller and smoother control signal compared to conventional control.

The rest of the paper is organized as follows. In Section 2 the VSI model is presented. Section 3 is devoted to the design of the proposed control system. In Section 4, a comparative simulation study with the conventional control system is provided to highlight the proposed controller performance. Finally, the study is summarized and concluded in Section 5.

VSI MATHEMATICAL MODEL

Three-phase VSI with LC output filter can be modeled, based on the abc/dq transformation, in two single-phase systems [18] as depicted in Figure-1.

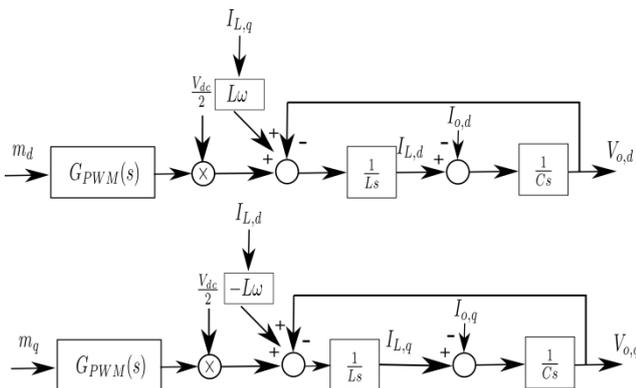


Figure-1. VSI with LC output filter model in the d-q synchronous reference frame.

The model contains the following elements:

- The voltage V_{dc} of the DC voltage source.
- The PWM transfer function $G_{PWM}(s)$ given as follows [19]

$$G_{PWM}(s) = \frac{1}{1+1.5T_s s} \tag{1}$$

Where T_s is the sampling time.

- The filter inductance L and capacity C.
- $V_{inv,d}$ and $V_{inv,q}$ are the direct and quadratic components of the control inputs of the VSI. $I_{L,d}$ and $I_{L,q}$ are respectively the inductance current I_L direct and quadratic components. $I_{o,d}$ and $I_{o,q}$ are respectively the VSI output current I_o direct and quadratic components. $V_{o,d}$ and $V_{o,q}$ are the direct and quadratic components of the output voltage V_o . ω is the rotation pulsation of the reference frame.

THE PROPOSED VSI CONTROL SYSTEM

In this work we propose a model predictive control system of three-phase VSIs. A schematic of the proposed controller is depicted in Figure-2.

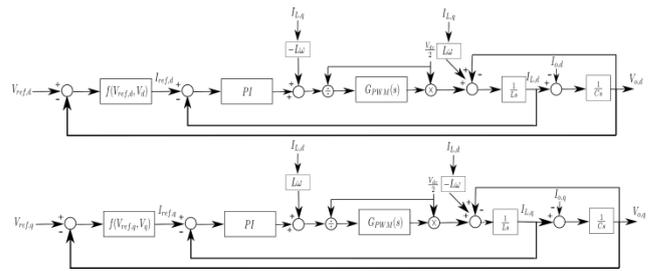


Figure-2. The proposed control system of three-phase VSI with LC output filter.

$V_{ref,d}$ and $V_{ref,q}$ are the references for the output voltage V_o direct and quadrature components respectively. Since output voltage magnitude verifies $V_{o,magn} = \sqrt{V_{o,d}^2 + V_{o,q}^2}$, the voltage references will be set as follows $V_{ref,d} = V_{ref}$ and $V_{ref,q} = 0$ where V_{ref} is the desired voltage magnitude. V_{ref} is supposed constant as in practice voltage magnitude in power networks is standardized and fixed.

Current feedforward terms are introduced to decouple the control of voltage and current direct and quadrature components. PI controllers are used for current regulation. To achieve the multiobjective control of voltage and current through optimal voltage regulation and reference tracking while maintaining current stability, an MPC controller will be designed. To do so, a model of the system with input $I_{ref,d}$ and $I_{ref,q}$ and output $V_{o,d}$ and $V_{o,q}$ will be established. Denote the load current equations $V_{o,d} = Z_{load} I_{o,d}$ and $V_{o,q} = Z_{load} I_{o,q}$ where $Z_{load} = \sqrt{R_{load}^2 + (L_{load}\omega)^2}$ is the load impedance. Using the control system model in Fig. 2 one can write

$$\frac{V_{o,d}}{I_{ref,d}} = \frac{V_{o,q}}{I_{ref,q}} = \frac{N(s)}{D(s)} \tag{2}$$

Where

$$N(s) = \frac{K_p}{1.5T_s LC} s + \frac{K_i}{1.5T_s LC} \tag{3}$$

And

$$D(s) = s^4 + \left(\frac{1}{1.5T_s} + \frac{1}{Z_{load}C}\right) s^3 + \left(\frac{K_p}{1.5T_s L} + \frac{1}{LC} + \frac{1}{1.5T_s C Z_{load}}\right) s^2 + \frac{1+K_i C + \frac{K_p}{Z_{load}}}{1.5T_s LC} s + \frac{K_i}{1.5T_s LC Z_{load}} \tag{4}$$

Consider a commendable and observable state-space representation of system (2-4) defined as follows:

$$\begin{aligned} \dot{x}_s &= A_s x_s + B_s u \\ y &= C_s x_s \end{aligned} \tag{5}$$

Where x_s is the state vector and A_s , B_s and C_s are the state-space matrix. $u \in \{I_{ref,d}, I_{ref,q}\}$ is the control input and $y \in \{V_{o,d}, V_{o,q}\}$ is the system's output. An



augmented state-space system of (5) can be designed as follows:

$$\begin{aligned} \dot{z} &= Az + Bu \\ y &= Cz \end{aligned} \quad (6)$$

Where $z = \begin{pmatrix} \dot{x}_s \\ y \end{pmatrix}$, $A = \begin{pmatrix} A_s & 0 \\ C_s & 0 \end{pmatrix}$, $B = \begin{pmatrix} B_s \\ 0 \end{pmatrix}$ and $C = (0 \ 1)$. For single input single output systems as system (2-4), the MPC control signal u is given as follows [20]

$$u = -K_x \tilde{x}_s + K_y \int (y_o - y) dt \quad (7)$$

Where K_x is a real vector of order four and K_y is a real scalar. y_o is the output reference to be reached. \tilde{x}_s is the estimation of the state vector x_s and it is given by the Luenberger observer as follows:

$$\dot{\tilde{x}}_s = A_s \tilde{x}_s + B_s u + K_o (y - C \tilde{x}_s) \quad (8)$$

Where K_o is the observer gain. The vector $K_{mpc} = (K_x \ K_y)$ is calculated using the following equation [20]

$$K_{mpc} = \sqrt{2p} \Omega \Psi \quad (9)$$

Where $\Omega = \int_0^{T_p} \phi(\tau) Q \phi(\tau)^T d\tau + R$ and $\Psi = \int_0^{T_p} \phi(\tau) Q e^{A\tau} d\tau$ with $p > 0$ the tuning gain, the weighting matrix $Q = C^T C$, $T_p > 0$ the prediction horizon and $R > 0$ control signal gain. ϕ is a real function calculated recursively on the prediction interval $[0 \ T_p]$ as follows:

$$\phi(kh)^T = e^{Ah} \phi((k-1)h)^T + e^{(k-1)Ah} \int_0^h (e^{A(h-\gamma)} B L^T(\tau)) d\gamma \quad (10)$$

Where h is the numerical step size and k is the iteration index. L is the Laquerre functions matrix defined

$$\text{by } L(\tau) = \sqrt{2pe^{A_p \tau}} \mathbf{1}_N \text{ with } A_p = \begin{pmatrix} -p & 0 & \dots & 0 \\ -2p & -p & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -2p & \dots & -2p & -p \end{pmatrix}$$

a N order triangular matrix and $\mathbf{1}_N = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$ a N order

vector. N is the number of orthonormal basis function used to approximate the derivative $\dot{u}(\tau)$, $0 \leq \tau \leq T_p$ of the control trajectory.

Remark 1. The convergence speed can be tuned using the gain p . The gain R is to be chosen small enough to optimize the control signal. The prediction horizon T_p is to be chosen sufficiently high to have better prediction of the system's behavior, however, choosing very high values of T_p does not directly imply high regulation and reference tracking performance, rather use the gains p and R for that matter as they directly affect the control performance as

stated earlier. An example of values of p , R and T_p will be provided in the simulation section.

Remark 2. There is no straightforward method for the choice of the number of Laguerre functions N . A sufficiently high value is to be chosen heuristically so that the approximation of \dot{u} on the prediction interval $[0 \ T_p]$ is sufficiently accurate. An example of values of N will be provided in the simulation study.

Remark 3. The gains K_x and K_y are calculated offline and thus, the numerical implementation of the control law is simple as the mathematical expression of the controller (7) is short and contains one integration and basic mathematical operations.

COMPARATIVE SIMULATION STUDY

The effectiveness of the proposed control strategy is verified using a comparative study with the conventional PI-based voltage control with feedback and feedforward terms [3]. The test system is 400V RMS electrical network containing a DC voltage source $V_{dc} = 700V$, a VSI, an LC filter with $L = 2 \text{ mH}$ and $C = 29.8 \mu F$ and a resistive charge with power $P = 1 \text{ kW}$. The control gains used in the simulations are as follows $p = 100$, $R = 1 \times 10^{-5}$, $T_p = 10$, $N = 4$, $K_p = 20$ and $K_i = 1 \times 10^{-5}$ where K_p and K_i are the proportional and integral gains of the current PI controllers. The control gains of the proposed control system and the conventional one were chosen to highlight the ability of the proposed control system to provide better performance with smaller control signal. The MPC MATLAB algorithm in [20] was used to conduct the simulation.

The results of the simulations are shown in Figure-3 and Figure-4.

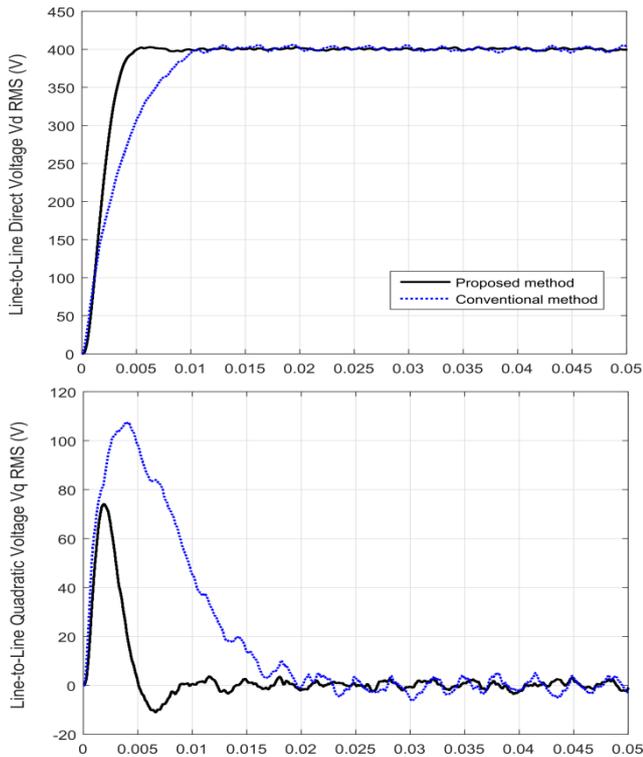


Figure-3. Output voltage direct and quadrature components RMS using the proposed and the conventional control methods.

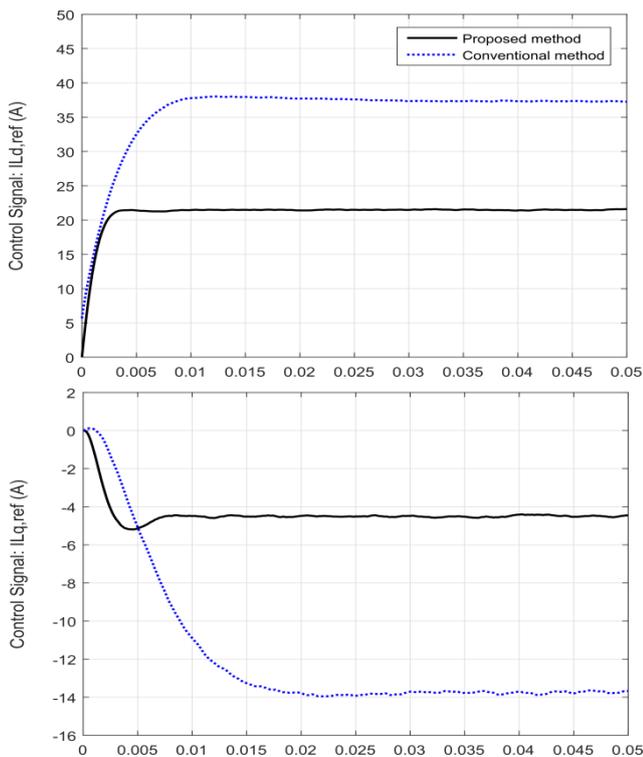


Figure-4. Control signals using the proposed and the conventional control methods.

As seen in Figure-3, the RMS of the output voltage direct component exhibits faster dynamics using the proposed approach and reaches the reference value

400V first compared to the conventional PI-based method. The quadrature component of the output voltage is canceled also faster using MPC technique. In addition, the proposed approach provides a smoother and less fluctuating output voltage compared to the conventional PI based voltage control. As for the control signals, Figure-4, shows that voltage control requires a control signal with lower values using the proposed control system. MPC voltage control signal is also smoother and less fluctuating compared to conventional control.

CONCLUSIONS

In this paper, a model predictive cascade-based control system of three-phase voltage source inverters (VSIs) with LC output filter is designed. For the outer loop, a model predictive controller is developed for robust voltage regulation and reference tracking despite the unknown disturbances while ensuring current stability by maintaining control of the current reference. The inner loop uses PI controllers for current regulation and feedforward terms to decouple the control of direct and quadrature voltage and current components. The design procedure of the proposed control system allows a flexible dynamic modeling of VSIs in the d-q synchronous reference frame and provides easy tuning via two gains that optimize independently the convergence speed and the control signal. A comparative simulation study with the conventional PI-based controller confirmed the theoretical results regarding fast voltage regulation and reference tracking. The simulations showed also that the proposed control strategy provides a smaller and less fluctuating control signal compared to conventional control.

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