



## NON-PARAMETRIC ALGORITHMS FOR DUAL CONTROL OF NONLINEAR DYNAMIC OBJECTS

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### ABSTRACT

The article devotes to the problem of controlling discrete nonlinear dynamical systems under non-parametric uncertainty. This term refers to the situation when the model structure of the object under study remains unknown to the parameters. Non-parametric models are considered, in which the difference equation degree of a dynamic process model is refined based on the rule of selection of significant variables. According to this rule, only those variables are included in the non-parametric model, whose optimal coefficient of kernel function is minimal. Non-parametric dual control algorithms are given. Control devices based on these algorithms perform not only the function of controlling the object itself, but also its study. The learning process of the dual control system with active accumulation of information is analyzed. The results of the computational experiment using the non-parametric adaptive dual control algorithm are given in detail. In the simulation, the characteristics of objects were described by equations with different degrees of nonlinearity whose form was unknown. Equation form in the process of active accumulation of information was automatically restored on the base of input and output process variables measurements. The above computational experiments confirmed the possibility of using non-parametric algorithms to control nonlinear systems.

**Keywords:** non-parametric algorithms, non-linear processes, bandwidth, kernel function, a priori information, dynamic system, dual control algorithm.

### INTRODUCTION

The solution of the control problem is inherently connected with the solution of the object under study identification problem. A large number of works of various Russian and foreign scientists are devoted to this topic. In particular, these issues are widely covered in [1-5]. The solution of the identification problem usually has two main stages. The first stage is structural identification, during which the model structure is selected with an accuracy of parameters. In this case, a dynamic object can be described, for example, in the form of differential or difference equations, transfer functions, sets of typical units, integral equations in convolutions, etc. The second step is to determine the parameters of the model from current experimental data.

The most studied today are the methods of parametric identification, in which even at the stage of problem statement the model structure of the process under study is assumed to be known and then it is necessary to determine model parameters. In conditions when it is impossible to choose the structure of the model, the use of the above-mentioned algorithms cannot be used. In this case, it is advisable to use non-parametric methods, the application of which requires knowledge only of the qualitative characteristics of the object under study.

The issues of using non-parametric theory for non-linear objects were discussed in the works of S. Chaika, where the problem was solved under conditions of both parametric and non-parametric uncertainty, since the parametric structure of the non-linear block was assumed to be known. Despite the relatively high efficiency of non-parametric methods in solving problems of identification and control of both linear dynamic objects and objects

belonging to the category of nonlinear, we can note a number of shortcomings inherent in these methods. In particular, it is possible to apply developed non-parametric algorithms only for some classes of dynamic objects. In some cases, it is necessary to implement partial parameterization of the model, which is based on a priori information, and to set the special input test signals, which is impossible in the normal operation of the object.

This article proposes a non-parametric dual control algorithm, which is based on the use of a non-parametric regression estimate taking into account information on the difference equation degree of a dynamic process model.

### THE PROBLEM STATEMENT

Figure-1 shows the flowchart of the process control.

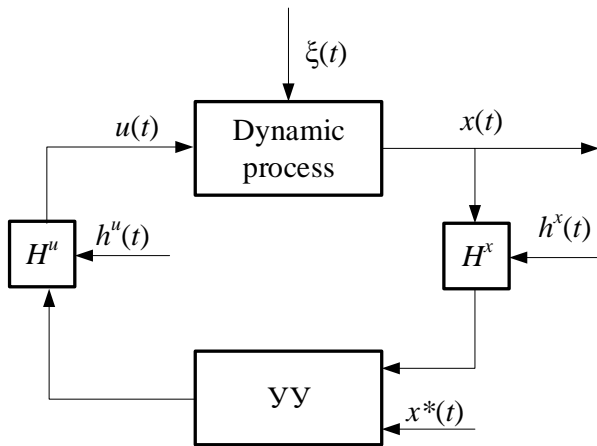


Figure-1. Dynamic object control block diagram.

In Figure-1, the following notation is used:  $u(t)$  is an input variable of the object,  $x(t)$  is an output variable of the object,  $t$  is continuous time, index  $t$  is discrete time,  $x_t^*$  is a set point, CU is control device,  $h_t^u$ ,  $h_t^x$  are random measurement noise, corresponding to process variables,  $\xi(t)$  is vector random interference. Control of variables is carried out through an interval of time  $\Delta t$ .

The paper considers classes of control objects that can be described by difference equations of the form:

$$x_t = F(x_{t-1}, \dots, x_{t-k}, u_t, \xi_t). \tag{1}$$

Here  $F$  is an unknown functional,  $k$  is the degree of a difference equation, which is limited  $k \leq k_{\max}$ . The input and output of a dynamic object are represented by measurements that form a sample of the form  $\{u_i, x_i\}, i = \overline{1, s}$ , where  $s$  is the sample size,  $u_i, x_i$  are the measurements of the input and output of the object at a time instant  $t_i$ .

**ALGORITHM**

Under these conditions, we take the following non-parametric estimation of the regression function from observational data  $\{x_i, u_i, i = \overline{1, s}\}$  as the non-parametric model of an object [6]:

$$x_s^t = \frac{\sum_{i=1}^s x_i \cdot \Phi\left(\frac{u_s - u_i}{c_s^u}\right) \prod_{j=1}^k \Phi\left(\frac{x_{s-j} - x_{i-j}}{c_s^{x[j]}}\right)}{\sum_{i=1}^s \Phi\left(\frac{u_s - u_i}{c_s^u}\right) \prod_{j=1}^k \Phi\left(\frac{x_{s-j} - x_{i-j}}{c_s^{x[j]}}\right)}, \tag{4}$$

where  $\Phi(\cdot)$  is a kernel function,  $c_s^u, c_s^{x[j]}$  are bandwidth parameters, satisfying the assumptions [7]:

$$\begin{aligned} c_s > 0; \quad \lim_{s \rightarrow \infty} c_s &= 0; \\ H(c_s^{-1}(t - t_i)) &< \infty; \\ c_s^{-1} \int_{\Omega(u)} H(c_s^{-1}(t - t_i)) dx &= 1; \\ \lim_{s \rightarrow \infty} s c_s &= \infty; \\ \lim_{s \rightarrow \infty} c_s^{-1} H(c_s^{-1}(t - t_i)) &= \delta(t - t_i), \end{aligned} \tag{5}$$

The optimal bandwidths are found by minimizing a quadratic error function by using the sliding exam method.

Non-parametric dual control algorithm has the following form [6, 8, 9]:

$$u_{s+1} = u_s^* + \Delta u_{s+1}, \tag{6}$$

where  $u_s^*$  is the component accumulating information about the object under study,  $\Delta u_{s+1} = \varepsilon(x_{s+1}^* - x_s)$  is the “studying” search step. The dual control scheme is shown in Figure-2.

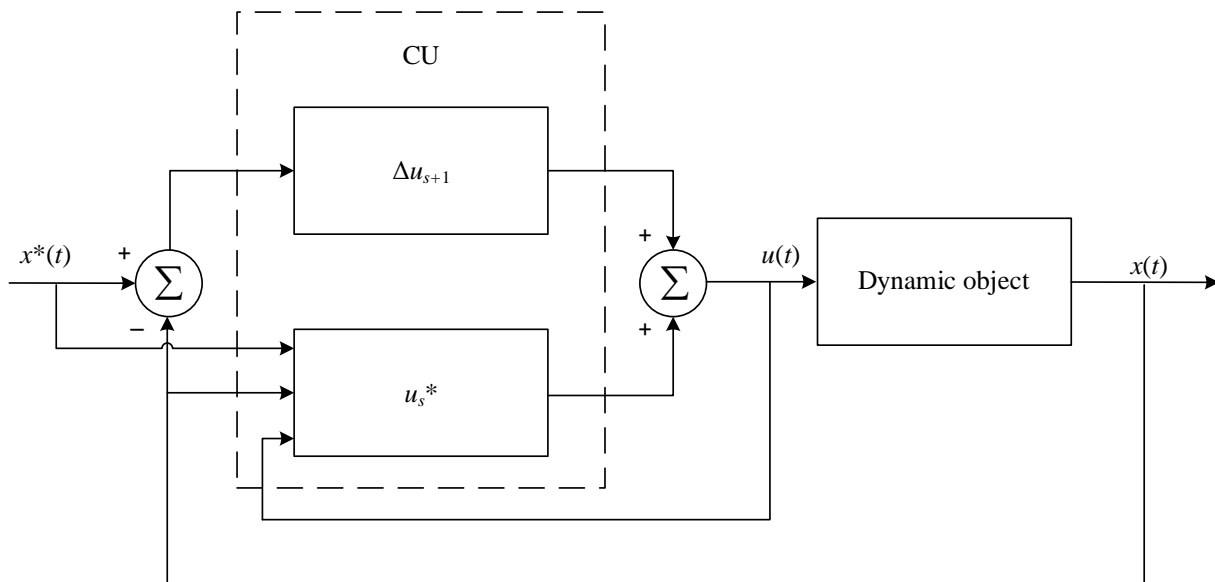


Figure-2. Dual control scheme of a dynamic object.

The dualism of the algorithm (6) is as follows. At the first control cycles, the main role in the formation of control actions is played by the term  $\Delta u_{s+1}$  from formula (6). But with the accumulation of information about the object, the role of the term  $u_s^*$  increases. The developed control system is a kind of hybrid control systems, which were considered, for example, in [10].

In this case, we use the following estimation to get the value  $u_s^*$  from equation (6)

$$u_s^* = \frac{\sum_{i=1}^s u_i \cdot \Phi\left(\frac{x_{s+1}^* - x_i}{c_s}\right) \prod_{j=1}^k \Phi\left(\frac{x_{s-j} - x_{i-j}}{c_s}\right)}{\sum_{i=1}^s \Phi\left(\frac{x_{s+1}^* - x_i}{c_s}\right) \prod_{j=1}^k \Phi\left(\frac{x_{s-j} - x_{i-j}}{c_s}\right)} \quad (7)$$

The control algorithm for nonlinear dynamic systems is constructed as follows. The differential equation degree of the dynamic process model  $k$  is determined on the basis of the rule of selection of essential variables. The value  $k$  is further used in the calculation of control actions in (6), where only selected variables are present.

**COMPUTATIONAL EXPERIMENT**

Consider the possibility of using non-parametric algorithms to control different types of nonlinear dynamic objects. In this experiment, the quality of management was evaluated according to two characteristics:

- a) The regulation time  $t_p$  is the time from the beginning of the control to the moment when the output value

differs from the task of no more than some specified value  $\alpha$ . Usually it is  $\alpha = 0.05x$ .

- b) Relative control error  $W_p$  equals to the total deviation of the actual output of the process from the set point during the whole control time over the set point, expressed in relative values, in%:

$$W_p = \frac{1}{s} \frac{\sum_{i=1}^s |x_i - x_i^*|}{x^*}$$

The value  $W_p$  shows the degree of deviation of the output value from the  $x_i^*$  point, expressed as a percentage.

We choose a non-gradient multidimensional optimization method of Nelder-Mead as an optimization algorithm, since this method is effective at a low speed of calculation of the minimized function. A region of possible values of bandwidth parameter ( $c_s \in [0.01, 4]$ ) of the kernel function was set to select the initial vertices of a deformable polyhedron.  $n + k + 1$  points were chosen arbitrarily, where  $n$  is the number of input variables,  $k$  is the order of the difference equation, which form the simplex  $n + k$ .

Let the difference equation of a dynamic object contain power functions, the equation of the object is described as:

$$x_t = 0,2 \cdot x_{t-1}^{1,2} - 0,3 \cdot x_{t-2} + 0,4 \cdot x_{t-3} + 1,5u_t \quad (8)$$

Set the sampling rate:



$$h = \frac{T}{s} = 0.1,$$

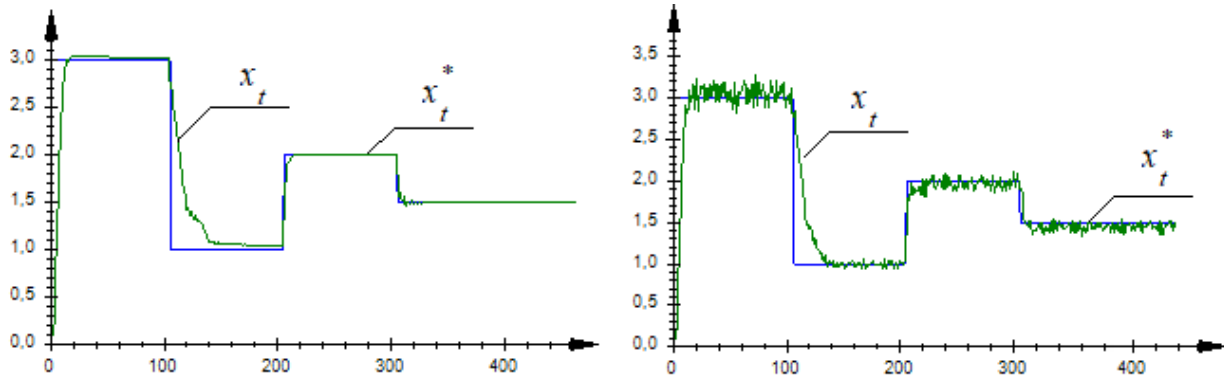
where  $T$  is the system operation time,  $s$  is the sample size.

The additional noise signal  $\xi_i$  is normally distributed  $M(\xi) = 0, D(\xi) < \infty$ , where  $D(\xi)$  is varied within the region of  $0 < D(\xi) < 1$  in very small steps. The output signal is calculated by:

$$x_i = x(t_i) + c \cdot \xi_i,$$

where  $x(t_i)$  is the output of the system (disregarding the effect of noise). The constant  $c$  is determined the level (intensity) of the noise.

The results of object control (8) with active accumulation of information are presented in Figure-3: a) in the absence of noise, b) in case of noise ( $\xi = 7\%$ )



**Figure-3.** Control results of nonlinear dynamic object (8)

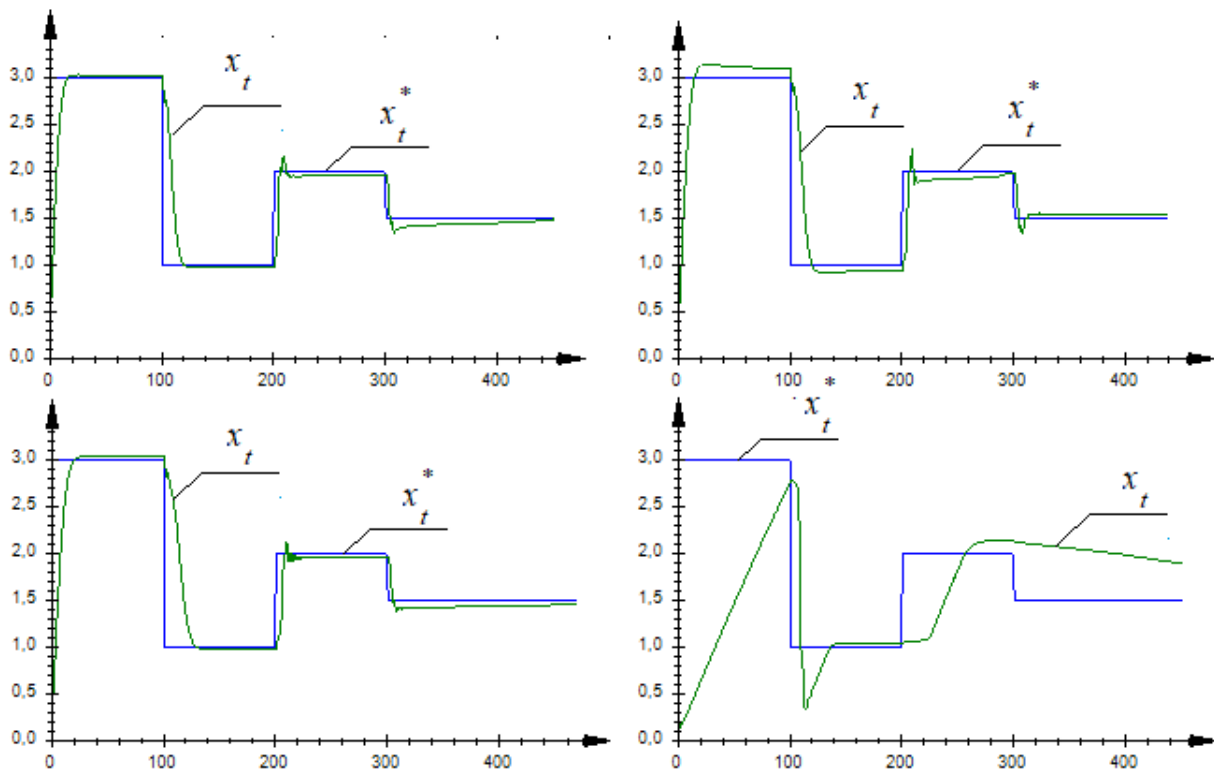
a) in the absence of noise,  $W_p = 0,143, t_p = 10,3$

б) noise is  $\xi = 7\%$ ,  $W_p = 0,165, t_p = 15,9$

Thus, from an analysis of Figure-3 and the values of  $W_p$  and  $t_p$ , we can conclude that the nonparametric control algorithm (6) successfully copes with the task of regulating nonlinear dynamic systems even in the case of

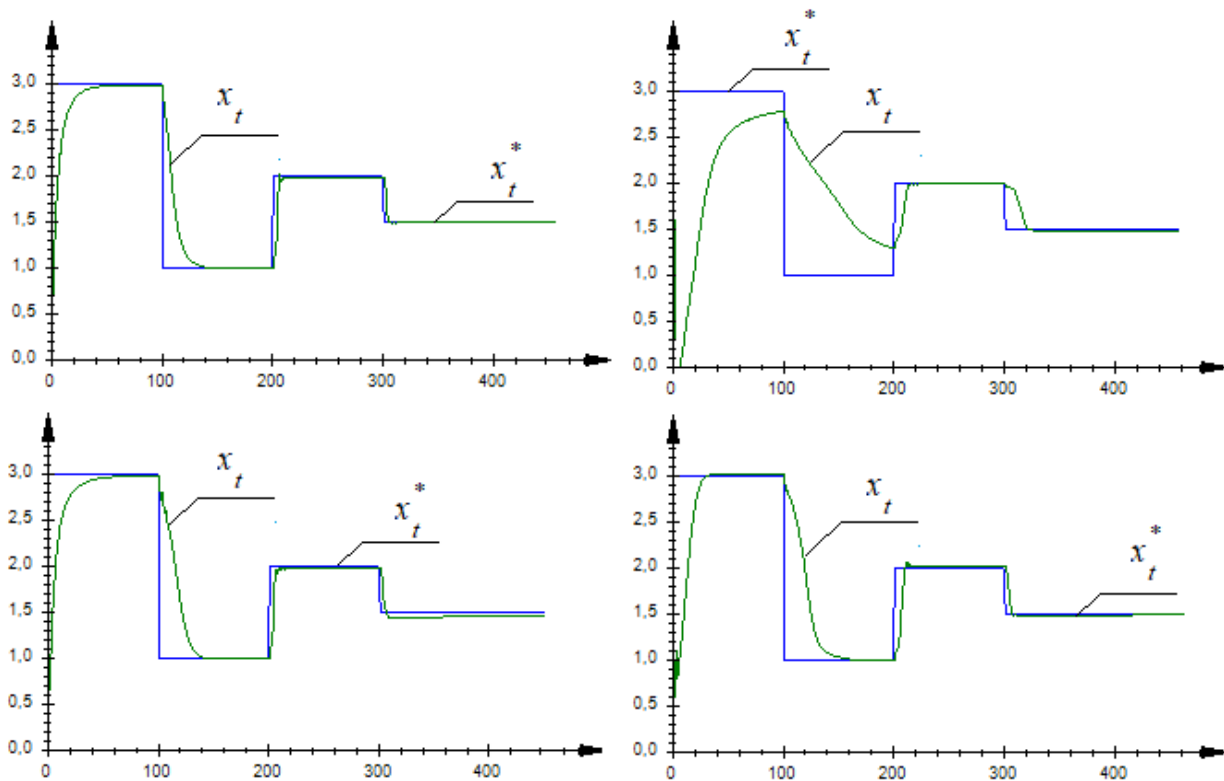
active accumulation of information, as well as when external interference affects the object of study.

Below (Figures 4-5) the control results for various options for describing dynamic systems are presented.



**Figure-4.** Control results of nonlinear dynamic object

- a)  $x_t = 0,2 \cdot x_{t-1} - 0,3 \cdot x_{t-2}^{1,2} + 0,4 \cdot x_{t-3} + 1,5u_t$ ,  $W_p = 0,088$ ,  $t_p = 12,2$   
 б)  $x_t = 0,2 \cdot x_{t-1} - 0,3 \cdot x_{t-2} + 0,4 \cdot x_{t-3}^{1,2} + 1,5u_t$ ,  $W_p = 0,113$ ,  $t_p = 14,3$   
 в)  $x_t = 0,2 \cdot x_{t-1}^{1,5} - 0,3 \cdot x_{t-2} + 0,4 \cdot x_{t-3} + 1,5u_t$ ,  $W_p = 0,115$ ,  $t_p = 23,1$   
 г)  $x_t = 0,2 \cdot x_{t-1}^2 - 0,3 \cdot x_{t-2} + 0,4 \cdot x_{t-3} + 1,5u_t$ ,  $W_p = 0,66$ ,  $t_p = 148$



**Figure-5.** Control results of nonlinear dynamic object

- a)  $x_t = 0,2 \cdot x_{t-1} - 0,3 \cdot x_{t-2} + 0,4 \cdot \sqrt{x_{t-3}} + 1,5u_t$ ,  $W_p = 0,082$ ,  $t_p = 13,6$   
 б)  $x_t = 0,2 \cdot \frac{1}{x_{t-1}} - 0,3 \cdot x_{t-2} + 0,4 \cdot x_{t-3} + 1,5u_t$ ,  $W_p = 0,456$ ,  $t_p = 102,4$   
 в)  $x_t = 0,2 \cdot \sin(x_{t-1}) - 0,3 \cdot x_{t-2} + 0,4 \cdot x_{t-3} + 1,5u_t$ ,  $W_p = 0,105$ ,  $t_p = 25,1$   
 р)  $x_t = 0,2 \cdot x_{t-1} - 0,3 \cdot \log_{10}(x_{t-2}) + 0,4 \cdot x_{t-3} + 1,5u_t$ ,  $W_p = 0,128$ ,  $t_p = 28,5$

The following conclusions can be made. If the difference equation of a dynamic object contains degrees to the second order, the control algorithm (6) successfully copes with the task of regulating a nonlinear dynamic object. The relative control error  $W_p$  increases if a power-law function is introduced for a variable, the coefficient at which has a greater effect on the output quantity, and with an increase in the degree, the quality of control deteriorates significantly. If the degree of a variable in the difference equation is 2 or more, then controlling a nonlinear dynamic object using the control algorithm (6) is impossible. If the difference equation of the object contains fractional degrees, the control algorithm (6) also copes with the task of bringing the output variable to the desired value. From the results of this computational experiment, it can be seen that even when each variable is raised to a power, the quality of control is quite high. It can also be noted that if trigonometric or logarithmic functions are contained in the difference equation of a dynamic object, then the application of control algorithm (6) is possible. A significant deterioration in the quality of control is observed in the presence of an exponential

function in the equation of the object. At negative degrees, the quality of control is also significantly reduced.

## CONCLUSIONS

A nonparametric dual control algorithm of nonlinear dynamic objects is developed. The main feature is the use of information about the order of the difference equation of a dynamic object when calculating control actions. This approach is associated with determining the optimal blur coefficients of a kernel function when using a nonparametric model. In this case, the model of a dynamic object is a non-parametric estimation of the regression function from observations, in which, in addition to the input actions, the output values of the object at previous time instants are also taken into account. As a result of numerical studies, the effectiveness of the developed nonparametric algorithm was confirmed. The experiments were carried out under conditions of external noise, with master actions of various kinds.



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