



# DIGITAL LEVEL CONTROL FOR THE CE105MV COUPLED TANK SYSTEM

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## ABSTRACT

In this article the design and implementation of a digital level controller for a coupled tank system is presented. The CE105 MV tank system model is developed. Due to the system is not linear, mathematical modeling is performed using the equations that describe the dynamic behavior of the system through its physical characteristics. The controller is implemented using microcontrollers, visualization hardware and instruction's introduction is also performed. The relative gains array (RGA) is used to determine the interaction between the inputs and outputs of the system, in order to choose the best way to control the variables. Finally, a graphical interface in LabVIEW software is performed in order to observe the system behavior in real time. As a result, good monitoring of set points is achieved by the control system implemented.

**Keywords:** multivariable control, matlab, microcontrollers, modeling, coupled tanks.

## 1. INTRODUCTION

The level control in tanks and the flow between them in industrial processes has been widely documented. In these processes the liquid must be stored and pumped to other tanks in such a way that liquid level is controlled and the flow regulated. Control can be defined as the indirect manipulation of magnitudes in a system called plant through another system called control system (Balcells, et al, 1997). Tanks can be configured in cascade or in a coupled way, this is done with some type of connection between them. In addition to this feature, in a coupled tanks system, interaction is allowed by controlling and manipulating one or more variables. However, in many control applications modeling of this type of systems is important so the system dynamics can be fully understood. Good model representation is important for the success of any control strategy. CE105MV coupled tank system has been selected for this work. This training system has been worked in other universities generating important results as mentioned below.

The CE 105 tanks liquid level was controlled by classical control methods using LabVIEW programming language (Pérez, S, 2011). Based on physical equations that describe the system, the transfer function of the system was found.

In the International Conference on Mechatronics, Electronics and Automotive Engineering a Model and Control for Coupled Tanks using LabVIEW was introduced. Model identification is done by sinusoidal signal at different frequencies, as result the bode plot obtained is applied for designing the controller. Since the system is nonlinear the mathematical model is also presented to compare with the experimental model obtained. (Bastida, et al, 2013).

The Proportional-Integral (PI) controller to monitor and control liquid level in an interconnected CE 105 model coupled tank is investigated (Hussein, and Mishra, 2014). The PI SubVI already exists in the LabVIEW library that gives reasonable performance but to get a better system performance and monitor the liquid

levels more accurately, another SubVI is derived from the PI controller mathematical equations.

In the first part of this work the approach of the CE 105 MV tank system mathematical modeling based on the physical and operating characteristics supplied by the manufacturer is carried out. The response of the variables in real time is monitored using a LabVIEW interface. The controller design is implemented with microcontrollers. Finally, the controlled system response with effective follow-up to changes in set points is shown.

## 2. MATERIALS AND METHODS

### 2.1 CE 105 MV Tank System

This work is carried out in the CE105MV multivariable coupled tank system shown in Figure-1. This system is a laboratory module developed for academic purposes for learning and research of control methods for this type of systems.

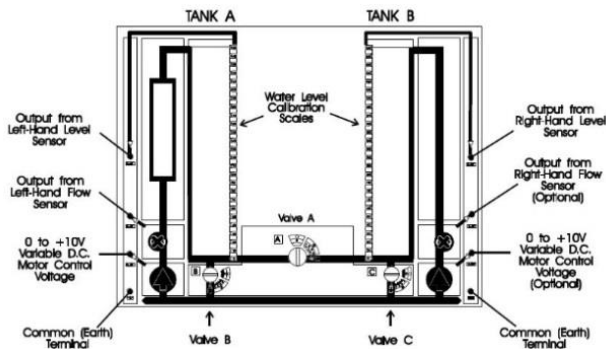


**Figure-1.** CE 105MV tank system.

The CE105MV tank system is comprised of two separate vertical tanks, which are connected through a flow



channel. A rotary valve can be used to vary the cross-sectional channel area, the other two valves are discharged to the reserve tank located below the tanks and thus change the flow characteristics between the tanks. Depending on the above, the system works as SISO (Single Input, Single Output), SISO coupled, SIMO (Single Input, Multiple Output) or MIMO (Multiple Input, Multiple Output). Each tank has its own feed pump. The front schematic diagram of the plant is shown in Figure 2.



**Figure-2.** Front diagram of CE105MV coupled tank system.

To obtain the mathematical modeling of system, the physical variables that make up the tank system and the technical data provided by the manufacturer are considered, as shown in Table-1.

**Table-1.** CE105MV technical specifications.

<b>Tank A</b>	Cross Sectional Area = 9350 mm <sup>2</sup>
<b>Tank B</b>	Cross Sectional Area = 9350 mm <sup>2</sup>
<b>Valves A B and C</b>	10 mm Valve Orifice Cross Sectional Area = 78.5 mm <sup>2</sup>
<b>Liquid Level Sensors</b>	0 to 10 V DC Output Corresponding to 0 to 250 mm As indicated on the front panel water level scales.
<b>Pump Flow Sensors</b>	0 to 10 V DC Output Corresponding 0 to 4400 cm <sup>3</sup> / min As indicated on the front panel flowmeter.

## 2.2 Hardware Implemented

To execute the necessary processing actions, PIC16F877A microcontroller is used. Proper performance is achieved due to its main features, such as 14.3 Kbyte flash memory, 368-byte static RAM, 33 I/O pins, 8 A/D channels (10-bit resolution).

To convert a digital signal (bits) to an analog voltage or current signal, the Digital-Analog Converter DAC0808 with a resolution of 8 bits is used.

To amplify signals from sensor circuits and in some cases to attenuate signals, the operational amplifier TL084 was implemented. There are several configurations

for amplifiers such as: no inverter, inverter, adder, subtract, among others.

The information is displayed through the LCD 16x2 LCD screen indicating that it has 2 rows of 16 characters each.

To enter the set point, the 4x4 matrix keyboard is implemented. It is an array of connected buttons in rows and columns that allows sending binary information of 16 possible characters in just 8 connection points.

## 2.3 CE105MV Mathematical Model

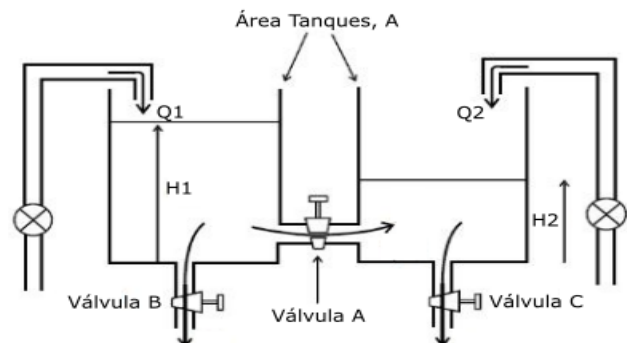
Differential equations of the system dynamic behavior are found based on physical dimensions in Table 1 and the scheme in Figure 3:

For tank 1,

$$Q_1 = A \frac{dH_1}{dt} + Q_a + Q_b \quad (1)$$

For tank 2,

$$Q_2 = A \frac{dH_2}{dt} - Q_a + Q_c \quad (2)$$



**Figure-3.** Multivariable tank system diagram.

Where,

$A$ : Transversal area of the tank.

$V_1, V_2$ : Volume of the fluid in the tank.

$Q_1, Q_2$ : Pump flow

$Q_a, Q_b, Q_c$ : Valve Outflow.

Valves A, B and C behave like a standard edge hole, so the flow through them is related to the flow level in the respective tank, therefore, the following expressions are obtained:

$$Q_a = (C_{da}) * (a_a) * \left( \sqrt{((2) * (g) * (|H_1 - H_2|))} \right)$$

$$Q_b = (C_{db}) * (a_b) * \left( \sqrt{((2) * (g) * (H_1))} \right)$$

$$Q_c = (C_{dc}) * (a_c) * \left( \sqrt{((2) * (g) * (H_2))} \right)$$

Where,

$a_a, a_b, a_c$ : Cross hole area

$C_{da}, C_{db}, C_{dc}$ : Valve discharge coefficient

$g$ : Gravitational constant.

By replacing the above in equations 1 and 2, first-order nonlinear equations are obtained. In order to make them useful for control systems, they must be linearized



considering small variations on the desired operating fluid level in the tanks.

$$\begin{aligned} H_1 &= H'_1 + h_1 \\ H_2 &= H'_2 + h_2 \\ Q_1 &= Q'_1 + q_1 \\ Q_2 &= Q'_2 + q_2 \\ Q_a &= Q'_a + q_a \\ Q_b &= Q'_b + q_b \\ Q_c &= Q'_c + q_c \end{aligned}$$

Where  $H'_1, H'_2, Q'_1, Q'_2, Q'_a, Q'_b, Q'_c$  are constants normal level of operation, and  $h_1, h_2, q_1, q_2, q_a, q_b, q_c$  are small changes at that level. Equations 1 and 2 are approximated by the tangent line at normal operating levels as shown in Figure-4, then:

$$Q'_1 + q_1 = A \frac{dh_1}{dt} + Q'_a + q_a + Q'_b + q_b \quad (3)$$

$$Q'_2 + q_2 = A \frac{dh_2}{dt} + Q'_c + q_c - Q'_a + q_a \quad (4)$$

The slope of the characteristic valve level  $H'$  is represented in the following equation

$$\frac{dQ}{dH} = \frac{q}{h}$$

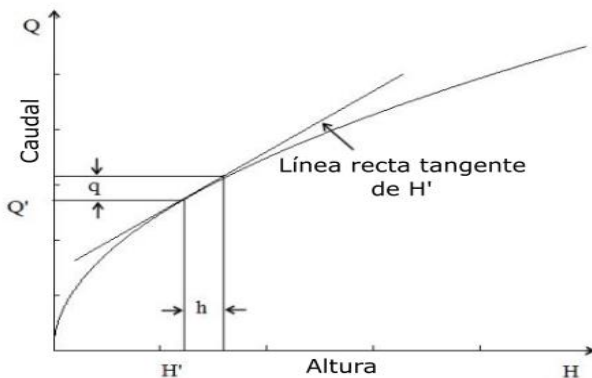


Figure-4. Height versus flow chart.

Equations 3 and 4 are modified using the characteristic slope and equations 5 and 6 are obtained:

$$Q'_1 + q_1 = A \frac{dh_1}{dt} + Q'_a + h_1 \frac{dQ_a}{dH_1} - h_2 \frac{dQ_a}{dH_2} + Q'_b + h_1 \frac{dQ_b}{dH_1} \quad (5)$$

$$Q'_2 + q_2 = A \frac{dh_2}{dt} + Q'_c + h_2 \frac{dQ_c}{dH_2} - Q'_a - h_1 \frac{dQ_a}{dH_1} + h_2 \frac{dQ_a}{dH_2} \quad (6)$$

When the level is constant  $h_1 = h_2 = q_1 = q_2 = q_a = q_b = q_c = 0$ , therefore, equations 5 and 6 are given in steady state for the flow and level. Then the following conditions are defined:

$$\begin{aligned} Q'_1 &= Q'_a + Q'_b \\ Q'_2 &= Q'_c - Q'_a \end{aligned}$$

$$D_1 = \frac{dQ_a}{dH_1}; D_2 = \frac{dQ_a}{dH_2}; D_3 = \frac{dQ_b}{dH_1}; D_4 = \frac{dQ_c}{dH_2}$$

These conditions and the Laplace transform are applied to obtain equations 7 and 8.

$$Q_{1(s)} = SAH_1 + H_1D_3 - H_2D_2 + H_1D_1 \quad (7)$$

$$Q_{2(s)} = SAH_2 + H_2D_4 - H_1D_1 + H_2D_2 \quad (8)$$

Due to MIMO system is 2x2, two flow inputs 2x2,  $(Q_1, Q_2)$  and two flow inputs  $(H_1, H_2)$  are considered. In this way, four transfer functions are obtained, which are achieved by combining each output on a different input.

$$\begin{aligned} \frac{H_{1(s)}}{Q_{1(s)}} &= \frac{(SA + D_4 + D_2)K_pK_h}{(S^2A^2 + SA(D_1 + D_2 + D_3 + D_4) + (D_3D_4 + D_3D_2 + D_1D_4))} \\ \frac{H_{1(s)}}{Q_{2(s)}} &= \frac{(D_2)K_pK_h}{(S^2A^2 + SA(D_1 + D_2 + D_3 + D_4) + (D_3D_4 + D_3D_2 + D_1D_4))} \\ \frac{H_{2(s)}}{Q_{1(s)}} &= \frac{(D_1)K_pK_h}{(S^2A^2 + SA(D_1 + D_2 + D_3 + D_4) + (D_3D_4 + D_3D_2 + D_1D_4))} \\ \frac{H_{2(s)}}{Q_{2(s)}} &= \frac{(SA + D_3 + D_1)K_pK_h}{(S^2A^2 + SA(D_1 + D_2 + D_3 + D_4) + (D_3D_4 + D_3D_2 + D_1D_4))} \end{aligned}$$

The level sensor gain  $K_h = 40 \text{ V/m}$  and pump gain  $K_p = 6,66 \times 10^{-6} \frac{\text{m}^3}{\text{seg}} \text{ V}$  must be considered for greater modeling accuracy. All parameters are considered at a chosen operating point of 6.5 volts input and the following transfer functions are obtained:

$$\begin{aligned} G_{11(s)} &= \frac{H_{1(s)}}{Q_{1(s)}} = \frac{(S \ 0.02849 + 0.008529)}{(S^2 + S \ 0.6271 + 0.01177)} \\ G_{12(s)} &= \frac{H_{1(s)}}{Q_{2(s)}} = \frac{(0.008372)}{(S^2 + S \ 0.5988 + 0.01167)} \\ G_{21(s)} &= \frac{H_{2(s)}}{Q_{1(s)}} = \frac{(0.008372)}{(S^2 + S \ 0.5983 + 0.01152)} \\ G_{22(s)} &= \frac{H_{2(s)}}{Q_{2(s)}} = \frac{(S \ 0.02849 + 0.00853)}{(S^2 + S \ 0.6272 + 0.0118)} \end{aligned}$$

## 2.4 Relative Gain Array

RGA (Relative Gain Array) is considered a very important tool in the analysis of multivariable systems. Not only in the selection of manipulated and controlled variables pairs, but it has also been used to predict the behavior of controlled responses (Espinosa, 2003). As the system model is known, the RGA is calculated analytically.

From equations 7 and 8, and taking into account variables  $A, D_1, D_2, D_3, D_4$  in the two resulting equations, equations 9 and 10 are achieved:

$$H_1 = \frac{(106,9519S + 32,0284)}{(S^2 + 0,5885 S + 0,0032)} Q_1 + \frac{30,8845}{(S^2 + 0,5885 S + 0,0032)} Q_2 \quad (9)$$



$$H_2 = \frac{30,8845}{(S^2+0,5885 S+0,0032)} Q_1 + \frac{(106,9519S+32,0284)}{(S^2+0,5885 S+0,0032)} Q_2 \quad (10)$$

Now, equations 9 and 10 are compared with expressions 11 and 12 of RGA theory.

$$y_1 = K_{11}u_1 + K_{12}u_2 \quad (11)$$

$$y_2 = K_{21}u_1 + K_{22}u_2 \quad (12)$$

Solving,

$$\lambda_{11} = \frac{1}{1 - \frac{K_{12}K_{21}}{K_{11}K_{22}}}$$

Considering that,

$$\lambda_{12} = 1 - \lambda_{11}$$

$$\lambda_{21} = \lambda_{12}$$

$$\lambda_{22} = \lambda_{11}$$

Besides,

$$\Lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix}$$

Where,

$K_{ij}$ : Static gain of transfer function  $G_{ij}$

$u_i$ : System input

$y_j$ : System output

$\lambda_{ij}$ : RGA Parameter

$i$ : Controlled variable

$j$ : Manipulated Variable

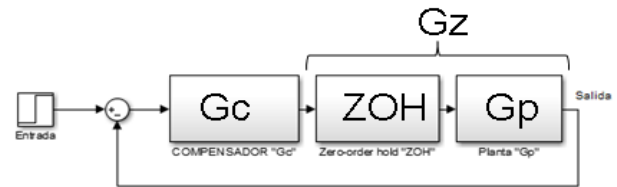
The RGA for the system is obtained

$$\Lambda = \begin{bmatrix} 14,2542 & -13,2542 \\ -13,2542 & 14,2542 \end{bmatrix}$$

The RGA means the interaction of transfer functions  $G_{12(s)}$  and  $G_{21(s)}$  it is too strong therefore it will be complex to make a good compensation. Consequently, it is decided to implement the control to  $G_{11(s)}$  and  $G_{22(s)}$  of which it will be shown in the next section that their mathematical models respond with a similar dynamic. For this reason, a single compensator was designed, however, they are implemented independently for each transfer function.

## 2.5 Digital Controller

Figure-5 shows the scheme of the proposed control system.



**Figure-5.** Control scheme.

To know the system temporal response without a controller, a MATLAB script based on the mathematical model is created. It is observed that there is no overshoot and the setting time is approximately 200 seconds. The design parameters are chosen to achieve a controlled system with a shorter setting time of 50 seconds and an overshoot with less than 20%.

### 2.5.1 Pole assignment

The desired dominant pole is found based on the chosen design parameters (Ogata, 2010):

$$Pdd = |Z| \cos \angle Z + j (|Z| \sin \angle Z) \quad (13)$$

The magnitude of Z is given by:

$$|Z| = e^{-\zeta W_n T} \quad (14)$$

Where,

$\zeta$ : Damping factor.

$W_n$ : Natural frequency not damped.

$T$ : Sampling time (0.1 seconds)

The angle of Z in radians is given by equation 15

$$\angle Z = (W_n \sqrt{1 - \zeta^2}) T \quad (15)$$

$\zeta$  It is directly related to overshoot, which is given by:

$$\zeta = \frac{|L_n(Sp)|}{\sqrt{\pi^2 + (L_n(Sp) * L_n(Sp))}}$$

Where,

$Sp$ : overshoot

So,

$$\zeta = 0,4559$$

$W_n$  is related to the setting time as follows:

$$t_{s(2\%)} = \frac{4}{\zeta W_n}$$

The 2% criterion is used (Ogata, 1996). And the  $W_n$  is obtained

$$W_n = 0,1755 \frac{\text{radianes}}{\text{segundos}}$$

Replacing in equation 14 and 15 is obtained:





$$|Z| = 0,992$$

$$\angle Z = 0,0156 \text{ radianes}$$

Finally, the desired dominant pole is obtained:

$$P_{dd} = 0,9919 + j(0,0155)$$

The transfer function  $G_{11}$  is discretized, resulting:

$$G_{Z(z)} = \frac{0,0028072(Z - 0,9705)}{(Z - 0,9995)(Z - 0,9424)}$$

The phase input of the zeros and poles of the system is calculated.

$$\theta_1 = \tan^{-1} \frac{0,0155}{0,9919 - 0,9424} = 17,39^\circ$$

$$\theta_2 = \tan^{-1} \frac{0,0155}{0,9919 - 0,9995} = -63,9^\circ$$

$$\phi_1 = \tan^{-1} \frac{0,0155}{0,9919 - 0,9705} = 35,9^\circ$$

And the compensator phase input is found

$$\theta_{Polo} = 90^\circ + \tan^{-1} \frac{1 - 0,9919}{0,01555} = 117,6^\circ$$

$$\theta_{cero} = 180^\circ + 35,9^\circ - 117,6^\circ - 17,39^\circ + 63,9^\circ$$

$$\theta_{cero} = 144,81^\circ$$

That means,

$$\alpha = 0,9919 + \frac{0,0155}{\tan 144,81} = 0,9699$$

Where  $\alpha$  is the location of the controller zero in the  $z$  plane in the second quadrant on the real axis. The transfer function of the compensator is given by:

$$G_c = \frac{K_c (Z - 0,9699)}{(Z - 1)}$$

Where  $K_c$  is the gain that the compensator must contribute, which is applying the module condition:

$$K_c \cong 8$$

Finally, the controller transfer function is defined by:

$$G_c = \frac{8 (Z - 0,9699)}{(Z - 1)}$$

To be implemented in a digital controller,  $G_c$  should be written in equations in difference, as follows:

$$(u_{(K)}) - (u_{(K-1)}) = (8e_{(K)}) - (7,7592 e_{(K-1)})$$

### 3. RESULTS AND DISCUSSIONS

#### 3.1 Controller Implemented Hardware

The printed circuit board (PCB) is designed and implemented as shown in Figure-6.

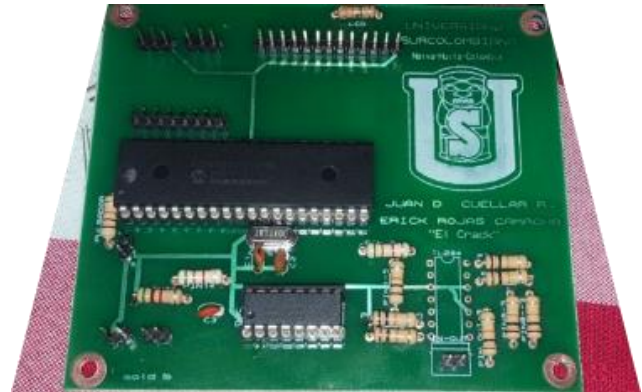


Figure-6. PCB scheme implemented.

As a final product, the hardware is performed and contains the necessary connection terminals, the keypad for entering instructions and on the base are the instructions for use and recommendations for optimal operation as shown in Figure-7.



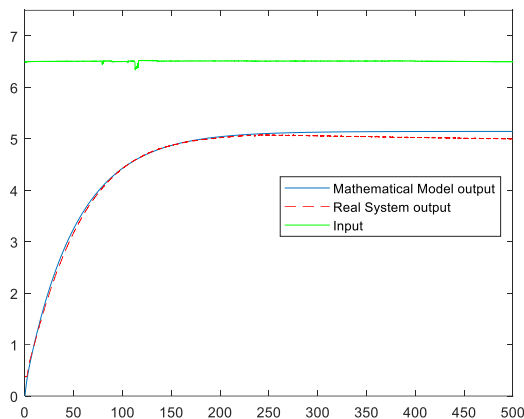
Figure-7. External appearance of controllers.

Finally, the control circuits are done both functionally and physically, each one comes with its respective cables for the supply voltages connections, and the outputs and inputs of the plant. In addition, the instructions for use and recommendations for optimal operation are made.

#### 3.2 Mathematical Model Validation

##### 3.2.1 Mathematical model response for G11 coupled tanks

Figure-8 shows the response of the mathematical model G11 to a chosen input of 6.5 volts. The model response is compared with the actual output of the system to the same input.

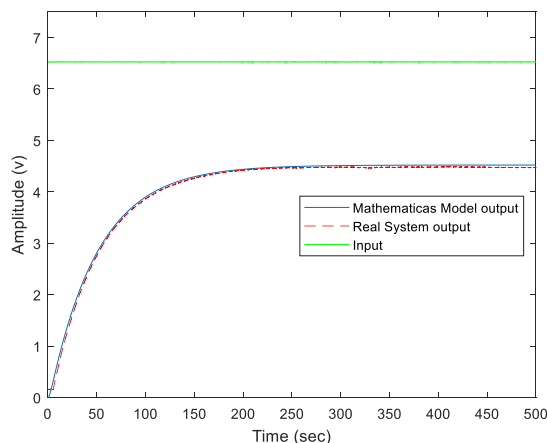


**Figure-8.** Response of G11 mathematical model, coupled tanks system.

A good approximation of the mathematical model output with the system real output is evidenced. A small difference in steady state that does not affect the system dynamics is observed.

### 3.2.2 Mathematical model response for G12 coupled tanks

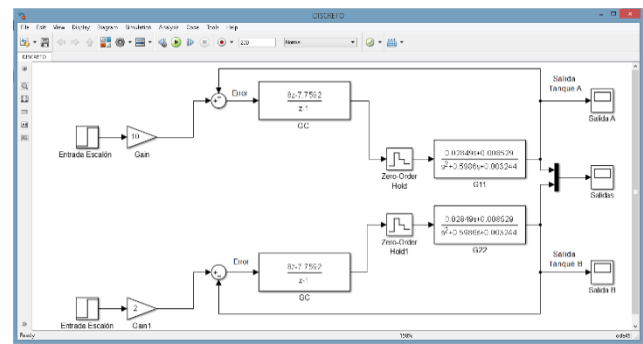
Figure-9 shows the response of G12 mathematical modeling system and the system real output to a 6.5 volt input as mentioned. This model shows an improvement in accuracy with regard to G11 model, especially by reducing the steady state error.



**Figure-9.** Response of G12 mathematical model, coupled tanks system.

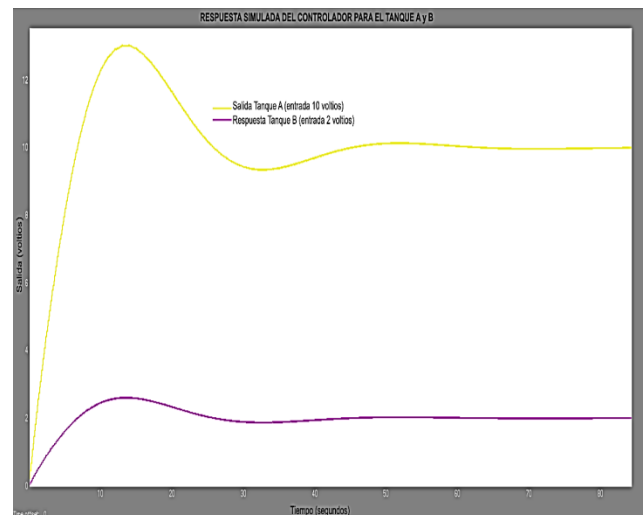
### 3.3 Digital Controller Validation

SIMULINK software is used to verify that the parameters chosen in the controller design are achieved as presented in Figure-10.



**Figure-10.** Block diagram in Simulink of the implemented controller.

In Figure-10 a unit step for each tank is used. The gain adjustment is performed independently for each tank and thus show that they act independently. The error signal obtained from the comparator is entered into  $G_c$  transfer function which is the compensator transfer function in discrete time. The “Zero-Order-Hold” block is responsible for digitizing the signal of  $G_{11}$  and  $G_{22}$  transfer functions. In block  $G_{11}$  the plant transfer function that relates the output and input of tank “A” is defined. In block  $G_{22}$  the plant transfer function that relates the output and input of tank “B” is defined. The signal that comes out of these blocks is the compensated plant response, which is visualized on the scopes as shown in Figure-11.



**Figure-11.** Time response of controllers.

The yellow line represents the compensated output of tank 1 and the violet line represents the output for tank 2.

In Figure-11 it is observed that now the response has an overshoot between 20 and 30%. In fact, this behavior is explained because by reducing the setting time an increase in the impulse overload is presented. However, this overshoot result is in a range within the proposed design parameters.

In the design parameters a settling time of 50 seconds was proposed. In the controlled system response, it was observed that as the set point increases, this time becomes a little longer, behavior that is explained because



more water must be used to fill the tanks and this implies more time. However, it was evidenced during controller tests that, at the maximum set point, the stabilization time was 60 seconds. This response is optimal and efficient.

#### 4. CONCLUSIONS

The mathematical modeling of the tank system is complex, physical characteristics must be considered and some will not be exact but approximate; however, it enabled the representation of the system dynamics.

The use of computational tools such as MatLab allowed the simulation of the controller designed performance based on the mathematical model achieved. The answer is close to the desired design parameters.

It was evidenced that the system response for low level set point is very accurate, however, as this set point increases, the error increases. This is explained because the system is not linear, and the applied control technique is classical.

Finally, the control system manages to keep track of the set points by improving the overshoot characteristics and setting time.

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