



THE EFFECTS OF COUPLESTRESS ON THE PERFORMANCE OF SQUEEZE FILM BETWEEN ELLIPTICAL PLATES LUBRICATED WITH A FERROFLUID

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ABSTRACT

In the present study, the performance of squeeze films between parallel elliptical plates lubricated by non-Newtonian couplestress ferrofluid has been investigated using transverse magnetic field. Based on the Shliomis' ferrohydrodynamic model along with stroke micro-continuum theory the Reynolds' expression was derived for engineering application. Furthermore, the solution for pressure distribution has been obtained in the analysis. Results shows that the non ferrofluid characterized the squeeze film operating with a higher standard of Langevin parameter and volume concentration of magnetic particles enhances the load capacity and maximize the squeeze time of the elliptical plate compared to Newtonian and non-ferrofluid case.

Keywords: couple stress squeeze film, elliptical plates, ferro fluids, non-newtonian fluids.

INTRODUCTION

Lubricants are needed in each apparatus and motor for reducing friction, the measure of wear that happens during machine activity, working temperature, and minimize the heat consumption of the metal surface. Besides, lubricants have numerous properties that can be blended and coordinated to meet machine working necessities. Along with these different synthetic substances that can be added to permit a machine to proficiently run at extreme temperature and pressure. In addition, some researcher's used the ferrofluid as a lubricant to improve the performance of squeeze film characteristics; Shah *et al.* [1] investigated such studies for the parallel plate, the circular film investigated by Deheri *et al.* [2], using the theory of ferrofluid lubrication, the application of the short bearing studied by Tipei[3], and Zhang[4]. The slider porous bearing is studied by Shah *et al.* [5]. In addition, based on Shliomis Ferrohydrodynamic models the squeeze film characteristics lubricated by ferrofluid considered for curved annular plates is analyzed by Shah *et al.* [6]. Lin *et al.* [7] analyzed the squeeze film performance between parallel plates that are lubricated by ferro-fluid stressed by non-Newtonian couples. From the results compared to the Newtonian and non-ferro-fluid case, they found that a greater load capacity and lengthen the reaction time for the squeeze films by using couple stress ferro-fluid. Furthermore, by using the effect of temperature and pressure Daniel *et al.* [8, 9] examined the performance of finite elasto-hydrodynamic journal bearing lubricated by couple stressed ferro-fluid. as a result they found that the obtained temperature distribution in the journal bearing specified the enhancement in the temperature by an increase in magnetism and couple stresses. Also, they concluded that the pressure increments with the expansion in magnetism and couple stresses. The thermal effect on the hydrodynamic general bearing which is lubricated by

magnetic fluid together with couple stress is analyzed by Nada *et al.* [10]. They revealed from the results that the combined effect of couplestresses and magnetic fluid is significantly apparent in the bearing characteristics.

More investigations are necessary to provide further information on the squeeze film.

The combined effect of piezo-viscous coupling stresses and rotational inertia on the squeeze film efficiency of rough circular discs is investigated by Daliri *et al* [11]. According to the results; they found that the squeeze film performance increases with increasing couplestress effects as well as viscosity pressure dependency. Also they revealed that increasing the rotational inertia effects diminishes the squeeze film characteristics. Hanumagowda *et al.* [12] studied the effect of viscosity pressure dependence on the output of the non-Newtonian squeezed film between parallel stepped circular plates, taking into account the effects of surface roughness; they found that the use of couple stress fluid and the dependence on viscosity pressure increases the characteristics of the squeezed film. Nevertheless, several geometry concepts of squeezed films lubricated by couplestress can be found in literature. Such as the porous circular disks investigated by Bujurke *et al.* [13], sphere and flat plate system analyzed by Naduvinamani *et al.* [14], parallel finite plates described by Ramanaiah [15] and the parallel circular discs studied by Lin *et al.* [16]. Recently, Akbar *et al.* [17] investigated the performance of squeeze film between parallel triangular plates with the couple stressed ferrofluid fluid as a lubricant. As evidenced by their exploration, it found that compared to Newtonian fluid, couple stress fluid and ferrofluid enhance the squeezed film performance. Nonetheless, numerous endeavors have been made before.

The current study is focused on improved characteristics performance on squeezed film between elliptical plates along with the couple stressed ferrofluid as a lubricant,



within the presence of a transverse magnetic field that has not been explored before. In this paper, it is analyzed that non-Newtonian ferrofluid lubricated squeeze film operating with the larger value of parameters such as Langevin parameter and volume concentration of magnetic particles provides a greater efficiency in load capacity and enhance the response time compared to a conventional of Newtonian and non-ferrofluid.

Mathematical Formulation

For an elliptical plate lubricated with a non-Newtonian ferrofluid in the presence of a transverse magnetic field, Figure-1 describes the squeeze film configuration. The upper plate is approaching towards the lower plate with the squeezing velocity $\partial h / dt$.

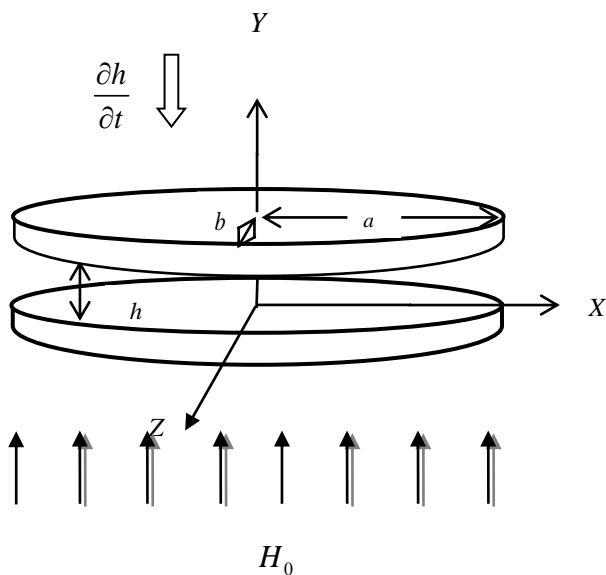


Figure-1. The squeeze film configuration between elliptical plates lubricated with a non-Newtonian in the presence of applied magnetic field.

The following assumptions are made in the present study:

- a) The effect of non-Newtonian solvent on magnetization relaxation was abandoned.
- b) The ferrofluid magnetization inside the ferrofluid was neglected.
- c) The inertia effects are ignored in the film region.

Under these assumptions and following the procedure,

The basic governing equations of non-Newtonian ferrofluid can be expressed as:

Film pressure (p), the velocity components $u, v,$ and w are in the directions x, y and z – respectively.

$$\eta \frac{\partial^2 u}{\partial y^2} + \eta \tau \frac{\partial^2 u}{\partial y^2} - \eta_c \frac{\partial^4 u}{\partial y^4} = \frac{\partial p}{\partial x} \tag{1}$$

Where: η - Suspension viscosity; η_c - New material constant. Responsible for couple stress fluid property

$$\eta \frac{\partial^2 w}{\partial y^2} + \eta \tau \frac{\partial^2 w}{\partial y^2} - \eta_c \frac{\partial^4 w}{\partial y^4} = \frac{\partial p}{\partial z} \tag{2}$$

$$\frac{\partial p}{\partial y} = 0 \tag{3}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{4}$$

$$\text{Where } \tau = \frac{3}{2} \phi \left(\frac{\xi - \tanh \xi}{\xi + \tanh \xi} \right) \tag{5}$$

Represents the rotational viscosity,
 Where: ϕ - Volume concentration of particles.
 ξ - Langevin parameter.

Both parameter measuring the strength of applied magnetic fields.

The viscosity of the suspension can be approximated to that of the Einstein formula, according to Shliomis [18, 19].

$$\eta = \eta_0 (1 + 2.5\phi) \tag{6}$$

Where: η_0 - the viscosity of the main liquid.

For the velocity components, the boundary conditions are:

$$(i) u(y=0) = 0, w(y=0) = 0 (\text{No-slip}) \tag{7a}$$

$$\left. \frac{\partial^2 u}{\partial y^2} \right|_{y=0} = 0, \left. \frac{\partial^2 w}{\partial y^2} \right|_{y=0} = 0 (\text{Vanishing couplestress}) \tag{7b}$$

$$(ii) u(y=h) = 0, w(y=h) = 0 (\text{No-slip}) \tag{8a}$$

$$\left. \frac{\partial^2 u}{\partial y^2} \right|_{y=0} = 0, \left. \frac{\partial^2 w}{\partial y^2} \right|_{y=0} = 0 (\text{Vanishing couplestress}) \tag{8b}$$

$$w(y=h) = \frac{dh}{dt} \tag{8c}$$

Expression for the velocity components u and v from the boundary conditions is obtained.



$$u = \frac{1}{2\eta_0 A^2} \left(\frac{\partial p}{\partial x} \right) \left[z^2 - hz + \frac{2l_c^2}{A^2} \left\{ 1 - \frac{\cosh\left(\frac{(2z-h)A}{2l_c}\right)}{\cosh\left(\frac{hA}{2l_c}\right)} \right\} \right] \quad (9)$$

$$w = \frac{1}{2\eta_0 A^2} \left(\frac{\partial p}{\partial z} \right) \left[y^2 - hy + \frac{2l_c^2}{A^2} \left\{ 1 - \frac{\cosh\left(\frac{(2y-h)A}{2l_c}\right)}{\cosh\left(\frac{hA}{2l_c}\right)} \right\} \right] \quad (10)$$

Where:

$$A = \sqrt{(1 + 2.5\phi)(1 + \tau)} \quad (11)$$

Integrating the continuity equation (4) over the film thickness h and using the corresponding boundary conditions, one can obtain the following expression

$$\frac{\partial}{\partial x} \left(\int_0^h u dy \right) + \frac{\partial}{\partial y} \left(\int_0^h w dy \right) = - \frac{\partial h}{\partial t} \quad (12)$$

Substituting equation (9) and (10) in (11), one can modify Reynolds equation as:

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial z^2} = \frac{2\eta_0 A^2 (dh/dt)}{f(l_c, \tau, \phi, h)} \quad (13)$$

$$\text{Where } f(h, l_c, \phi, \tau) = -\frac{h^3}{6} + \frac{2l_c^2}{A^2} \left[h - \frac{2l_c}{A} \tanh\left(\frac{hA}{2l_c}\right) \right] \quad (14)$$

In these equations, $A = A(\phi, \tau)$ which is defined in equations (11), $\tau = \tau(\phi, \xi)$ is defined in (5) and $l_c = \eta_c / \eta_0$ describes the characteristics of length of additives which is responsible for couplestress fluid.

The pressure field boundary conditions are as follows:

$$p(x_1, z_1) = 0 \quad (15)$$

Where

$$\frac{x_1^2}{a^2} + \frac{z_1^2}{b^2} = 1 \quad (16)$$

Solving equation (13) and (15), one obtains the following expression:

$$p = \frac{2\eta_0 A^2 (dh/dt)}{f(l_c, \tau, \phi, h)} k(x_1, z_1) \quad (17)$$

$k(x_1, z_1)$ Is the Poisson equation's solution and is given by

$$k(x_1, z_1) = \frac{a^2 b^2}{2(a^2 + b^2)} \left(1 - \frac{x_1^2}{a^2} - \frac{z_1^2}{b^2} \right) \quad (18)$$

Using the expression (17) in equation (18), the pressure distribution for elliptical plates can be expressed as follow:

$$p = \frac{\eta_0 A^2 (dh/dt)}{f(l_c, \tau, \phi, h)} \left\{ \frac{a^2 b^2}{(a^2 + b^2)} \left(1 - \frac{x_1^2}{a^2} - \frac{z_1^2}{b^2} \right) \right\} \quad (19)$$

Present the non-dimensional and parameter variables as follows:

$$x^e = \frac{x}{a}, y^e = \frac{y}{a}, p^* = \frac{ph_0^3}{\eta_0^2 \pi ab (-dh/dt)}, h^* = \frac{h}{h_0},$$

$$l^* = \frac{l_c}{h_0}, f^*(l^*, \tau, \phi, h^*) = \frac{f(l_c, \tau, \phi, h)}{h_0^3}$$

The non-dimensional pressure distribution expression can be obtained as:

$$p^* = \frac{ph_0^3}{\eta_0 (-dh/dt) \pi ab} = - \frac{A^2}{\pi f^*(l^*, \tau, \phi, h^*)} \left(\frac{a^*}{a^{*2} + 1} \right) (1 - x^{*2} - z^{*2}) \quad (20)$$

Where $a^* = a/b$

$$f^*(l^*, \tau, \phi, h^*) = -\frac{h^{*3}}{6} + \frac{2l^{*2}}{A^2} \left[h^* - \frac{2l^*}{A} \tanh\left(\frac{Ah^*}{2l^*}\right) \right] \quad (21)$$

For the enhancement of the load carrying capacity integrating the film pressure over the film region is given by:

$$W = \int_{-a}^a \int_{-\frac{b}{a}\sqrt{a^2-x_1^2}}^{\frac{b}{a}\sqrt{a^2-x_1^2}} p dz_1 dx_1 \quad (22)$$

Substituting the pressure expression presented in equation (19) and performing integration, one can obtain the expression of load carrying capacity for an elliptical plate after non-dimensional sing as:

$$w^* = \frac{Wh_0^3}{\eta_0 (dh/dt) \pi^2 a^2 b^2} = - \frac{A^2}{2\pi f^*(l^*, \tau, \phi, h^*)} \left(\frac{a^*}{a^{*2} + 1} \right) \quad (23)$$

To observe the film height varying with the time elapsed, dimensionless response time is introduced.

$$t^* = \frac{Wh_0^2 t}{\eta_0 \pi^2 a^2 b^2} \quad (24)$$



The relationship between the time versus height can be expressed in the non-dimensional form as follows:

$$\frac{dh^*}{dt^*} = \frac{2\pi f(l_c, \tau, \phi, h)}{A^2} \left(\frac{a^{*2} + 1}{a^*} \right) \quad (25)$$

Integrating equation (24) with the initial condition (i.e., $h^*(t^* = 0) = 1$) the approaching time for the upper plate can be expressed as:

$$t^* = \frac{A^2}{2\pi} \left(\frac{a^*}{a^{*2} + 1} \right) \int_1^{h_1^*} \frac{dh^*}{f^*(l^*, \tau, \phi, h^*)} \quad (26)$$

Where $h_1^* = h_1 / h_0$

RESULTS AND DISCUSSIONS

From the above derivation, the characteristics of the squeeze film are evaluated by the three parameters for elliptical plates lubricated with non-Newtonian couplestress ferrofluid: the particle volume concentration ϕ and the Langevin parameter ξ characterises the strength of the magnetic fields applied and the parameter of the couple stress characterizes the couple stress effects.

Figure-2. illustrates the variation of non-dimensional film pressure p^* for elliptical plates with the dimensionless coordinate x^* for various standards of ϕ and ξ for both Newtonian and non-Newtonian case ξ below the film height $h^* = 0.4$ and aspect ratio $a^* = 2$. The results of the non-Newtonian case are found to dispense higher squeeze film pressure in comparison to the Newtonian lubricant situation. It is seen that the squeeze film pressure increase with increasing the values of ϕ and ξ . In the presence of transverse magnetic field with the influence of ferro-fluid and volume concentration ϕ . Enhance the pressure is more prevalent than compared to Newtonian non-ferrofluid situation.

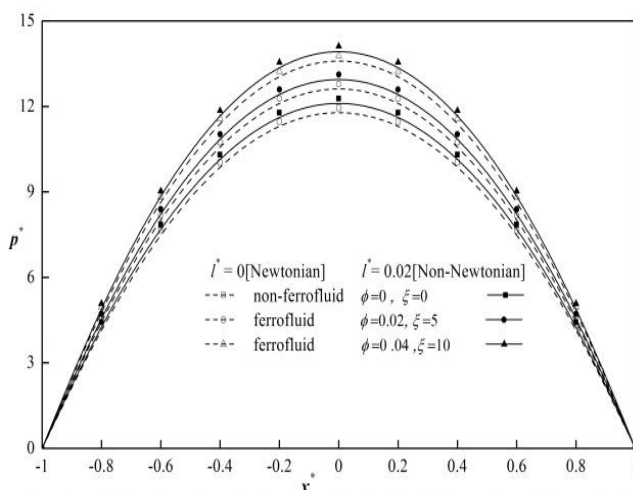


Figure 2: Variation of non-dimensional Pressure p^* with x^* for different values of l^*, ϕ and ξ with $h = 0.4, a^* = 2, y^* = 0$.

Figure-3. delineate the variance of load capacity w^* versus the couplestress parameter l^* for the altered values of ϕ and ξ below the film height $h^* = 0.4, h^* = 0.6$ at $a^* = 2$. The combined effect of non-Newtonian ferrofluid on the load carrying capacity is more pronounced for the squeeze film operating at a smaller film height $h^* = 0.4$ compared to the condition under the film height h^* . In the presence of transverse magnetic field, the effect of couplestress ferro-fluid is seen to significantly increase the load carrying ability compared to the non-ferrofluid case.

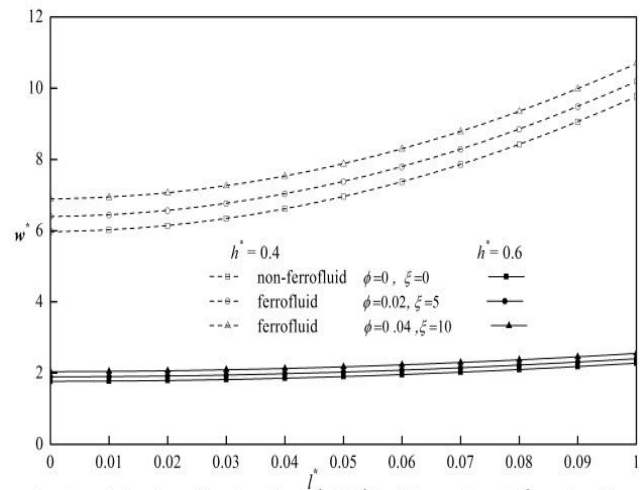


Figure3: Variation of non-dimensional load w^* with l^* for different values of h^*, ϕ and ξ with $a^* = 2$.

Figure-4. represents the non-dimensional load capacity w^* as a function of non-dimensional lubricant film thickness h^* for various values ϕ and ξ couple stress parameter l^* it found that the smaller values h^* the higher range of w^* are obtained. Further, an increase the value ϕ and ξ ($\phi = 0.02, \xi = 5; \phi = 0.04, \xi = 10$) it is concluded that the effect of non-Newtonian ferrofluid based lubricants enhances the load carrying capacity value compared to the non-ferrofluid instance.

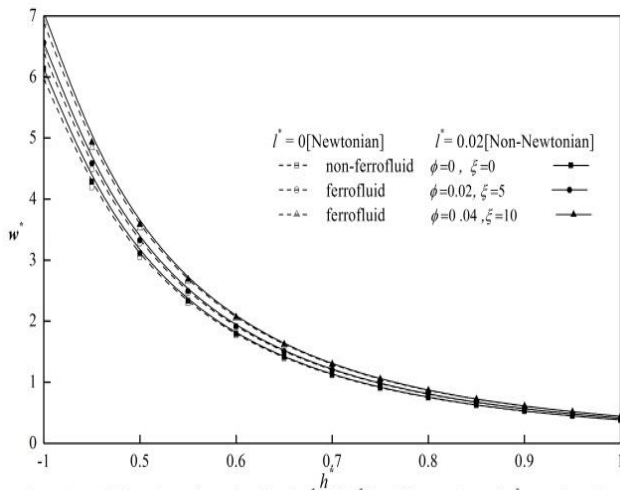


Figure 4: Variation of non-dimensional load w^* with h^* for different values of l^* , ϕ and ξ with $a^*=2$.

Figure-5. describes the variation of non-dimensional load capacity w^* versus volume concentration ϕ , for various standards of l^* , ϕ and film height $h^* = 0.4$. In the presence of magnetic field ($\xi = 5, 10$). in comparison to the Newtonian lubricants case ($l^* = 0$). It has been observed that value of the load carrying capacity w^* is found to increase with the increasing volume concentration value ϕ , when a non-Newtonian couple stress fluid ($l^* = 0.02$) is included. Further, enhancement of load carrying capacity w^* is obtained.

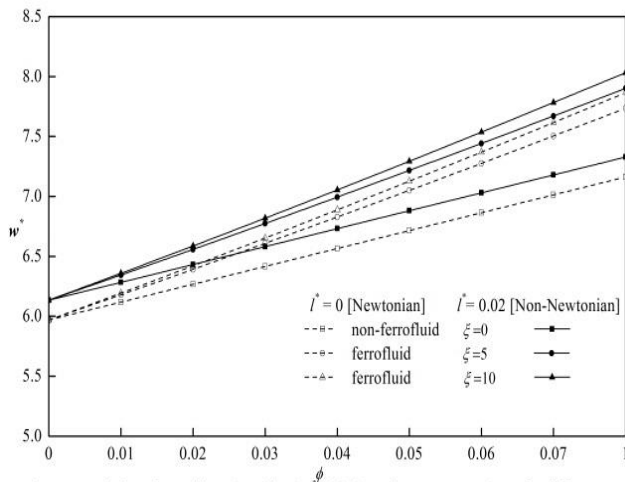


Figure5: Variation of non-dimensional load w^* with the volume concentration ϕ for different values of l^* and ξ at $h^*=0.4, a^*=2$.

Figure-6. illustrates the Non-dimensional response time t^* varies with non-dimensional lubricants film thickness h^* at altered values ϕ, ξ and couple stress parameter l^* . It is seen that the estimation of response time t^* is obtained concerning a diminishing in the estimation of the h^* . However, the influence of the conventional magnetic field along with increasing the

values ϕ and ξ ($\phi = 0.02, \xi = 5; \phi = 0.04, \xi = 10$) the non-Newtonian ferro-fluid case provide a lengthen the response time t^* in the case of the Newtonian non-ferro fluid without magnetic field ($l^*; \phi; \xi = 0$). Also, augmentation of the response time t^* are gotten for the elliptical squeeze film.

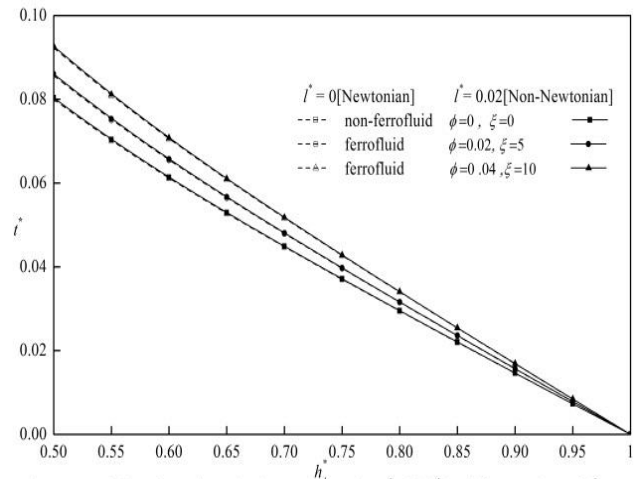


Figure 6: Variation of non-dimensional approaching time t^* with h^* for different values of l^* , ϕ and ξ with aspect ratio $a^*=2$.

Figure-7. describes the non-dimensional response time t^* is defined as a function of the Langevin Parameter ξ for different values l^* and ϕ . It is observed that the response time t^* does not fluctuate for the non-ferrofluid squeeze film with the Langevin parameter ξ . Nevertheless, under the influence of a conventional magnetic field it is concluded that increasing the value of the ξ ($\xi = 5; 10$), ferro-fluid squeeze film will lead to an improvement in response time.

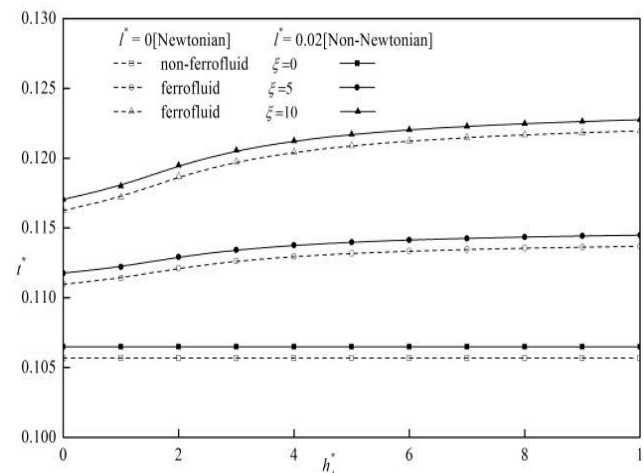


Figure 7: Variation of non-dimensional approaching time t^* with the Langevin parameter ξ for different values of l^* and ϕ at the aspect ratio $a^*=2$.



CONCLUSIONS

The characteristics of the geometry of the squeeze film between parallel elliptical plates lubricated with a non-Newtonian ferrofluid are analyzed in this paper in the presence of transverse magnetic field. The Reynolds form coupled stress ferrofluid squeeze film is derived for engineering application based on the Shilomis Ferro hydrodynamics model together with the Stokes micro continuum theory. In particular, the squeeze film properties are studied by using a combination of couplestresses-ferrofluid operating with a higher Langevin parameter value and volume concentration of magnetic particles. As a result, squeeze film features such as load carrying capacity and elliptical plate response time have been found to increase significantly compared to the Newtonian lubricant case.

NOMENCLATURE

a	Length of semi major axis
b	Length of semi minor axis
H_0	Applied magnetic field in the y - direction
h_0	Lubricant film thickness at initial point
x, y, z	Cartesian coordinates
x^*	a/b Aspect ratio
h	Film thickness
h_1^*	Dimensionless film thickness after time Δt
l_c	Characterizes length of the additives (η_c / η_0)
ϕ	Volume concentration of the particles
ξ	Langevin Parameter
l^*	Non-dimensional couple stress parameter (l_c / h_0)
u, v, w	Velocity components in film region
p	Non-dimensional pressure $\left\{ \frac{ph_0^3}{\eta_0 (dh/dt)\pi ab} \right\}$
t	Mean time of approach
t^*	Non-dimensional time of approach
V	Squeezing velocity $\left(-\frac{dh}{dt} \right)$
w^*	Non-dimensional load carrying capacity $\left(= \frac{wh_0^3}{\eta_0 (dh/dt)\pi^2 a^2 b^2} \right)$
η_c	Couple stress fluid index
K_B	Boltzmann constant

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