Determining the average reservoir pressure from horizontal well flow tests using the pressure and pressure derivative plot

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Abstract
Average reservoir pressure is a parameter of importance for design, assessment, evaluation, and exploitation of hydrocarbon reservoirs. By excellence, pressure buildup tests are intended for obtaining a measurement of this property. Recently, drawdown tests have been shown to provide the average reservoir pressure by mathematical manipulations of pressure behavior and material balance equations. In this paper, an extension of the TDS Technique is used on drawdown tests for the determination of an expression to obtain an estimation of the average reservoir pressure for horizontal wells in homogeneous and anisotropic formations using an approximation of the pressure behavior in hydraulically-fractured vertical wells. The equation was successfully tested with synthetic examples.

Keywords: formation pressure, shape factor, pressure derivative, pseudosteady-state

1. INTRODUCTION
Several researches on average reservoir pressure were conducted after the middle of the 20th century. Arari (1987) introduced a practical methodology to estimate this property for bounded and constant-pressure boundary reservoirs. A novel, practical, accurate and revolutionary well test methodology was introduced by Tiab (1995). This is known as Direct Synthesis Technique (TDS) which uses unique features found on the pressure and pressure derivative plot from which analytical equations for reservoir characterization are developed. A compilation of the advances in this technique are provided by Escobar (2015, 2019), plus a state-of-the-art on TDS Technique presented by Escobar, Jongkittinarukorn and Hernandez (2018).

Chacon, Djebrouni and Tiab (2004) applied the TDS Technique to develop expressions for the estimation of the average reservoir pressure in such systems as circular and rectangular homogenous reservoirs and hydraulically fractured wells in homogeneous reservoirs. Molina et al (2005) extended the TDS Technique on naturally fractured reservoirs for the determination of the average reservoir pressure. Escobar, Ibáñez and Montealegre-M. (2007) followed the philosophy of the TDS Technique homogeneous and heterogeneous reservoirs being operated under multi-rate conditions. Escobar, Cantillo and Santos (2011) used the hydraulically-fractured well pressure solution to develop an expression to estimate the average reservoir pressure in horizontal wells also under multi-rate testing.

Agarwal (2010) performed a mathematical manipulation of the flow and material balance equations to determine for the first time the average reservoir pressure from flow tests. Escobar, Palomino and Jongkittinarukorn (2019) used Agarwal’s idea combined with the TDS Technique to find expressions for average reservoir pressure and shape factors in vertical wells in homogeneous and naturally fractured formations and in hydraulically fractured wells in homogeneous reservoirs. This work followed the previous study of Escobar et al. (2019) to find the average reservoir pressure in horizontal wells in homogeneous and anisotropic formations by using the fractured well solution as performed by Chacon et al. (2004) and Escobar et al. (2011).

2. MATHEMATICAL FORMULATION
The dimensionless pressure and pressure derivative for oil phase are given by:

\[ P_D = \frac{kh \Delta P}{141.2 q B \mu} \]  \hspace{1cm} (1)

\[ t_D * P_D' = \frac{kh(t \Delta P')}{141.2 q B \mu} \]  \hspace{1cm} (2)

As performed by Chacon et al. (2004) and Escobar et al. (2011), consider only one wing of the infinite conductivity fracture system, the following analogies can be made:

\[ x_f \approx L \] \hspace{1cm} (3)

\[ X_r \approx h \] \hspace{1cm} (4)

\[ P_D = \frac{\bar{K}L_w \Delta P}{141.2 q \mu B} \] \hspace{1cm} (5)

\[ t_D * P_D' = \frac{\bar{K}L_w(t \Delta P')}{141.2 q B \mu} \] \hspace{1cm} (6)
The dimensionless pseudo-pressure and pseudo-pressure derivative for gas phase are given by:

\[ m(P)_D = \frac{hk\left[m(P) - m(P)\right]}{1422.52 q_s T} \quad (7) \]

\[ t^* \Delta m(P)_D = \frac{hk\left[t^* \Delta m(P)\right]}{1422.52 q_s T} \quad (8) \]

By the same token:

\[ m(P)_D = \frac{kL_w\left[m(P) - m(P)\right]}{1422.52 q_s T} \quad (9) \]

\[ t^* \Delta m(P)_D = \frac{kL_w\left[t^* \Delta m(P)\right]}{1422.52 q_s T} \quad (10) \]

The dimensionless time based upon area and effective horizontal wellbore length are, respectively, given by:

\[ t_{DA} = \frac{0.0002637kt}{\phi \mu c, A} \quad (11) \]

\[ t_D = \frac{0.0002637kt}{\phi \mu c, L_w} \quad (12) \]

Agarwal (2010) started with the material balance expression for a single phase fluid in a closed reservoir:

\[ 5.615qB \frac{t}{24} = Ahc(P_i - \bar{P}) \quad (13) \]

Which is also:

\[ P_{Dmb}(t_{DA}) = \frac{\bar{h}(P_i - \bar{P})}{141.2 q_B \mu} = 2\pi t_{DM} \quad (14) \]

The governing equation for the pseudosteady-state pressure behavior for a well in a hydraulically fractured well in a homogeneous reservoir were given by Russell and Truit (1964) is:

\[ P_D(t_{DA}) = 2\pi t_{DA} + 0.5 \ln \left( \frac{x_i}{x_f} \right)^2 \left( \frac{2.2459}{C_A} \right) \quad (15) \]

Considering only one wing of the infinite conductivity fracture system, the following analogies can be made:

\[ P_D = 2\pi t_{DA} + \frac{1}{2} \ln \left( \frac{h_c}{L_w} \right)^2 \left( \frac{2.2458}{C_A} \right) \quad (16) \]

Agarwal (2010) started with the material balance expression for a single phase fluid in a closed reservoir:

\[ 5.615qB \frac{t}{24} = Ahc(P_i - \bar{P}) \quad (13) \]

Which Cartesian and logarithmic derivatives are:

\[ P_D(t_{DA}) = 2\pi t_{DA} \quad (17) \]

\[ t_D * P_D(t_{DA}) = 2\pi t_{DA} \quad (18) \]

It is seen by Equation (17) that a log-log plot of \( P_D^* \) versus \( t_{DA} \) provides a horizontal line intercepting the pressure derivative axis at a value of \( 2\pi \). Agarwal (2010) found that the pseudosteady state period, \( t_{pps} \), starts when the arithmetic pressure derivative becomes flat at which time correspond the actual well-flowing pressure value.

Comparison of Equation (18) with Equation (14) provides a resemblance. Agarwal (2010) observed that during the pseudosteady-state flow period:

\[ t_D * P_D(t_{DA}) = P_{Dmb}(t_{DA}) = 2\pi t_{DA} \quad (19) \]

In a flow test, the pressure drop, \( \Delta P \) is defined as \( P_i - P_{wf} \). Subtracting and adding the average reservoir pressure to this, \( \bar{P} \), gives:

\[ P_i - P_{wf} = (P_i - \bar{P}) + (\bar{P} - P_{wf}) \quad (20) \]
or:

$$P_D(t_{DA}) = P_{Dmb}(t_{DA}) + \bar{P}_D(t_{DA})$$

(21)

Here, the procedure presented by Chacon et al. (2004) to obtain an expression for estimation of the average reservoir pressure for a vertical well with an infinite-conductivity fracture is employed. For convenience, let us start with the dimensionless pressure equation for both a horizontal and a vertical well, respectively:

$$P_D(t_{DA}) = \frac{k h_i (P_i - P_{wf})}{141.2 q B \mu}$$

(22)

$$P_{Dmb}(t_{DA}) = \frac{k h_i (P - \bar{P})}{141.2 q B \mu}$$

(23)

$$\bar{P}_D(t_{DA}) = \frac{k h_i (\bar{P} - P_{wf})}{141.2 q B \mu}$$

(24)

According to Agarwal (2010), solving for the dimensionless average reservoir pressure from Equation (17), leads to:

$$\bar{P}_D(t_{DA}) = P_D(t_{DA}) - P_{Dmb}(t_{DA})$$

(25)

Combination of Equation (16), (19) and (25) yields:

$$\bar{P}_D(t_{DA}) = 2 \pi t_{DA} + 0.5 \left[ \ln \left( \frac{h_i}{L_w} \right)^2 \frac{2.2458}{C_A} \right] - 2 \pi t_{DA}$$

(26)

Dividing Equation (26) by Equation (18) gives,

$$\frac{\bar{P}_D(t_{DA})}{t_D \ast \overline{P}_D} = \frac{1}{2} \ln \left[ \left( \frac{h_i}{L_w} \right)^2 \frac{2.2458}{C_A} \right]$$

(27)

Combination of equations (24), (27), (6) and (11) and solving for the average reservoir pressure gives:

$$P = P_{wf} + \frac{301.77(t \ast \Delta P)_{Pwf} \phi(\mu c_i) A}{k_{it_{Pwf}}} \left[ \ln \left( \frac{h_i}{L_w} \right)^2 \frac{2.2458}{C_A} \right]$$

(28)

For gas wells, the product $\mu c_i$ is evaluated at initial conditions. Following a similar procedure as for oil wells, it is obtained:

$$m(P) = m(P_{wf}) + \frac{301.77(t \ast \Delta m(P)_{Pwf} \phi(\mu c_i) A}{k_{it_{Pwf}}} \left[ \ln \left( \frac{h_i}{L_w} \right)^2 \frac{2.2458}{C_A} \right]$$

(29)

The Dietz shape factors $C_A$ can be determined by adapting the expressions provided by Chacon et al. (2004).

$$C_A = \frac{2.2458}{r_w^2} \left( \frac{h_i}{L_w} \right)^2 e^{\frac{E_{ref}}{301.77 \mu c_i A \left[ (\Delta P)_{Pwf} - (t \ast \Delta P)_{Pwf} \right]} - 1}$$

(30)

Notice in Equation (30) that the ratio of the pressure derivatives is replaced by the ratio of the pseudopressure derivatives when dealing with gas wells.

3. EXAMPLES

Estimate the average reservoir pressure for the two following simulated examples. The examples were run for different reservoir geometries and the average reservoir pressure was estimated by material balance using a commercial well test interpretation software and reported in Table-1.

3.1. Synthetic Example 1

Figure-2 contains synthetic data of pressure and pressure derivative versus time of a horizontal well generated using data from the second column of Table-1. From that plot, the following information was read:

$$t_{Pwf} = 720.1 \text{ psi} \quad (\Delta P)_{Pwf} = 92.45 \text{ psi} \quad (t \ast \Delta P)_{Pwf} = 33.2 \text{ psi}$$

Notice that the reading is performed as indicated by Agarwal (2010) when the arithmetic pressure derivative becomes flat at late pseudosteady state.

Using Equation (28) the shape factor was estimated to be 0.772 and the resulting average reservoir was 4871.3 psi found with Equation (28). A commercial well testing software provided a value of 4912.9 psi from material balance.
3.2 Synthetic Example 2

A drawdown test of a horizontal in a homogeneous and anisotropic reservoir was generated with data from the third column of Table-1. The pressure and pressure derivative versus time data are reported in Figure-3. From that plot, the following information was read:

$$t_{pwf} = 845.1 \text{ psi}$$

$$\Delta P_{pwf} = 30.41 \text{ psi}$$

$$t^* \Delta P^* = 10.78 \text{ psi}$$

Again, Equations (30) and (28) provided respective values of $$C_A = 0.437$$ and $$P = 3956.3 \text{ psi}$$ while the commercial software provided 3988.1 psi using material balance.

4. COMMENTS ON THE RESULTS

Absolute deviation errors of 0.85 and 0.8 % were found in examples 1 and 2 on the estimation of the average reservoir pressure as compared to material balance with a commercial well testing software. This indicates that the proposed equations and, also, the TDS methodology work well. This was expected since this work is an extension of the one presented by Escobar et al. (2019).

5. CONCLUSIONS

A new expression to estimate the average reservoir pressure from pressure drawdown tests using the TDS Technique is presented for horizontal wells and successfully compared to material balance providing errors lower than 1 %. The governing equation of a hydraulically fractured vertical well was adapted for a horizontal well as performed by Chacon et al (2004) and Escobar et al (2011).

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REFERENCES


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**Nomenclature**

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<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>( A )</td>
<td>Drainage area, ( Ac )</td>
</tr>
<tr>
<td>( C_d )</td>
<td>Dietz shape factor</td>
</tr>
<tr>
<td>( c_i )</td>
<td>Compressibility, ( 1/\text{psi} )</td>
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<tr>
<td>( h )</td>
<td>Reservoir thickness, ( \text{ft} )</td>
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<tr>
<td>( h_x )</td>
<td>Reservoir length along horizontal well</td>
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<tr>
<td>( k )</td>
<td>Areal permeability, ( \text{md} )</td>
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<tr>
<td>( k_v )</td>
<td>Vertical permeability, ( \text{md} )</td>
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<td>( m(P) )</td>
<td>Pseudopressure function, ( \text{psi}^2/\text{cp} )</td>
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<tr>
<td>( P )</td>
<td>Pressure, ( \text{psi} )</td>
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<tr>
<td>( \bar{P} )</td>
<td>Average reservoir pressure, ( \text{psi} )</td>
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<tr>
<td>( P_i )</td>
<td>Initial reservoir pressure, ( \text{psi} )</td>
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<td>( P_{wf} )</td>
<td>Well-flowing pressure, ( \text{psi} )</td>
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<td>( q )</td>
<td>Oil flow rate, ( \text{BPD} )</td>
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<td>( r_w )</td>
<td>Well radius, ( \text{ft} )</td>
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<td>Reservoir temperature, °R</td>
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<td>( t )</td>
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<td>( x_e )</td>
<td>Half-reservoir length (vertical wells), ( \text{psi} )</td>
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<tr>
<td>( x_f )</td>
<td>Half-fracture length, ( \text{ft} )</td>
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<td>( Z )</td>
<td>Gas compressibility factor</td>
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**Greek**

<table>
<thead>
<tr>
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<tr>
<td>( \Delta )</td>
<td>Change, drop</td>
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<tr>
<td>( \phi )</td>
<td>Porosity, fraction</td>
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<tr>
<td>( \mu )</td>
<td>Viscosity, ( \text{cp} )</td>
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**Suffixes**

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