# DEVELOPMENT OF A COMPUTATIONAL TOOL FOR KINEMATIC ANALYSIS OF PLANAR MECHANISMS WITH ONE DEGREE OF FREEDOM BY MEANS OF NUMERICAL METHODS 

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#### Abstract

This technological development consists of the implementation of numerical methods, vectors, two-dimensional matrices and programming language, whose purpose is to analyze and calculate kinematic variables of position, velocity and acceleration of flat mechanisms with a predefined degree of freedom. For the development of this desktop application, the advances that have been made in this area were reviewed and, based on this; didactic software was proposed that meets the necessary requirements for the kinematic analysis of flat mechanisms with a degree of freedom. An algorithm was developed, and then the instructions coded in a high-level programming language, which allows the processing of usersupplied input parameters through graphical interfaces and dialog boxes. The processed data are dimensional and geometrical properties, location and relative joints of the physical elements belonging to a configuration type of flat mechanisms. This computational tool, which consumes few computer resources, provides kinematic resolution of flat mechanisms with a degree of freedom in processing times of the order of milliseconds, on the other hand, it avoids using geometric and analytical methods, which are tedious and extensive in written equations, therefore, it reduces the risk of misusing a variable or a mathematical sign. The functionality of this application is tested by solving exercises proposed and existing in the literature on the theory of machines and mechanisms.


Keywords: numerical method, machines and mechanisms, programming, degree of freedom, graphical interface.

## INTRODUCTION

Machines are fundamental components found in every industrial process and they in turn are made up of mechanisms that are "devices that transform motion into a desirable pattern, and generally develop very low forces and transmit little power" [1]. It is therefore necessary to develop technological tools to design, analyze and implement efficient, functional, secure and optimal mechanisms.

Machine design, theoretical and applied mechanics have developed methods based on the disciplines of geometry, algebraic and vector mathematics for the resolution of the kinematic variables of planar mechanisms with a degree of freedom, these are known as graphical and analytical methods. These methods present limitations and disadvantages in the operation of solving mechanisms, since they are more tedious, which makes them more susceptible to error and require longerresolution times than those methods based on computerized numerical analysis.

This work guarantees the analysis of geometrically complex mechanisms, minimizing the execution time and the probability of human error when calculating the position, velocity and acceleration variables. At the same time, graphical interfaces are developed toillustrate the critical points where acceleration can cause structural failures such as link rupture or decoupling of the relative joints. This is achieved by coding the Newton-Raphson open numerical method, mathematical operations of vectors, matrices, and the design of graphic interfaces that allow the treatment of input and output parameters of the physical components that are part of the configuration of the type mechanisms.

## MECHANISM GENERALITIES

## Flat Mechanisms

Rigid parts that are configured and connected in such a way that they produce the desired motion in the machineare considered as mechanism [3]. In a mechanism, rigid parts are joined together to form a chain. One of the parts is called the bed plate, because it serves as a frame of reference for the movement of all the other parts. The bed plate is normally a non-moving part. Links are the individual parts of the mechanism and are considered rigid bodies that are connected to other links to transmit motion and forces. Theoretically, a true rigid body doesnot deform during motion. Although in reality there is no rigid body, the links of the mechanisms are designed considering a minimum deformation and are assumed tobe rigid.


Figure-1. Four-bar flat mechanism.

## Mobility

The mobility of a planar mechanism can be classified according to the number of degrees of freedom (DOF) it possesses. The degrees of freedom of a
mechanism is equal to the number of independent parameters (variables) required to uniquely define its position in space at any instant of time.

## Links and Joints

Mechanisms consist of parts connected for the purpose of transmitting motion and force [3]. A linkage is a mechanism where rigid parts are joined together to forma chain.

- Link: is a rigid body (assumed) that has at least two nodes that are points of union with other links.
- Joint: is a connection between two or more links (at their nodes), which allows some movement, or potential movement, between the connected links.


Figure-2. Links of different order.

## MATHEMATICAL MODELING

The kinematics of planar mechanisms deals with the way bodies move. It is the study of the geometry of motion. Kinematic analysis involves the determination of position, displacement, rotation, rapidity, velocity and acceleration of mechanisms without considering external and inertial forces.

## Coordinate System

The crucial point of any method of kinematic analysis of mechanisms is the definition of the coordinates of the mechanism. These coordinates are constituted by a set of non-independent parameters that unequivocal define the position of each and every one of its elements; they are not independent because any set of parameters whose number is greater than the number of degrees of freedom of the mechanism must satisfy certain additional geometric compatibility equations, which are known as constraint equations.

## Natural Coordinates

It is a formulation that is able to completely define the position of a mechanism, avoiding the use of angular variables. Natural coordinates can be considered anevolution of Cartesian coordinates, where the reference points, usually located at the center of mass of each element, are moved to kinematic pairs, so that each element must have at least two basic points [7].


Figure-3. Natural coordinates.

## Restriction Equations

There is a close relationship between the mechanism- dependent coordinates and the type of constraint equations that interrelate them. Thus, relative coordinates give rise to closed-loop constraint equations; reference point coordinates give rise to torque constraint equations, and basic coordinates to rigid solid (or element) and torque constraint equations. We will call the set of all constraint equations appearing in the problem $\varphi(q)$ and always write them in the form $\varphi(q)=0 ; \varphi(q)=0$ : Set of constraint equations:
n: $\quad$ Number of dependent coordinates
g : Degrees of freedom of the mechanism
m : Number of constraint equations, where $\mathrm{m}=\mathrm{n}-\mathrm{g}$

## Rigid Solid Constraint Equation

In the plane case, the condition that an element is a rigid solid is generally reduced to the condition that the distances between the basic points belonging to that element are kept constant. In some particular cases, such as when three basic points are aligned, the three conditions of constant distance are not independent, and a new type of condition must be resorted to, which is that of constant area of the triangle formed by the three basic points. Mathematically, these conditions can be formulated in the form:


Figure-4. Constraint equation rigid element defined by2 points in the plane.
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Figure-5. Constraint equations rigid element defined by 3 points in the plane.


$$
\begin{gathered}
\boldsymbol{n}=\left\{x_{1}, y_{1}, x_{2}, y_{2}, x_{3}, y_{3}\right\} \\
\boldsymbol{g}=3 \\
\boldsymbol{n}=6 \\
\boldsymbol{m}=6-3=3 \\
\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}-L_{12}^{2}=0 \\
\left(x_{3}-x_{1}\right)-\frac{L_{13}}{L_{12}}\left(x_{2}-x_{1}\right)=0 \\
\left(y_{3}-y_{1}\right)-\frac{L_{13}}{L_{12}}\left(y_{2}-y_{1}\right)=0
\end{gathered}
$$

Figure-6. Constraint equations rigid element defined by 3 points aligned in the plane.


Figure-7. Constraint equations rigid element defined by 5 points or more.

## Kinematic Torque Constraint Equation

To extract the equations that arise due to the constraintsof the kinematic pairs, we only have to relate to each other the coordinates of the points and vectors that are involved in that kinematic pair.


Área del triángulo $\overline{123}=0$ Área del triángulo $\overline{123}=\frac{\text { base } * \text { altura }}{2}$ Área del triángulo $\overline{123}=\frac{L_{13} * L_{12} \operatorname{sen} \theta}{2}$
$\left|r_{12} \times r_{13}\right|=\left|\left\{\frac{x_{2}-x_{1}}{y_{2}-y_{1}}\right\} \boldsymbol{x}\left\{\frac{x_{3}-x_{1}}{y_{3}-y_{1}}\right\}\right|=\left(x_{2}-x_{1}\right)\left(y_{3}-y_{1}\right)-\left(y_{2}-y_{1}\right)\left(x_{3}-x_{1}\right)$

$$
\begin{gathered}
\left(x_{2}-x_{1}\right)\left(y_{3}-y_{1}\right)-\left(y_{2}-y_{1}\right)\left(x_{3}-x_{1}\right)=\frac{L_{13} * L_{12} \operatorname{sen} \theta}{2}=0 \\
\left(x_{2}-x_{1}\right)\left(y_{3}-y_{1}\right)-\left(y_{2}-y_{1}\right)\left(x_{3}-x_{1}\right)=0
\end{gathered}
$$

Figure-8. Articulated sliding constraint equations. Point 3 aligned with (12) Relative coordinates constraintequation

## Relative Coordinates Constraint Equation



$$
\begin{gathered}
r_{12} \cdot r_{13}-k=0 \\
r_{12} \cdot r_{13}-L_{12} L_{13} \cos \theta=0 \\
\left\{\frac{x_{2}-x_{1}}{y_{2}-y_{1}}\right\} \cdot\left\{\frac{x_{3}-x_{1}}{y_{3}-y_{1}}\right\}-L_{12} L_{13} \cos \theta=0 \\
\left(x_{2}-x_{1}\right)\left(x_{3}-x_{1}\right)+\left(y_{2}-y_{1}\right)\left(y_{3}-y_{1}\right)-L_{12} L_{13} \cos \theta=0
\end{gathered}
$$

Figure-9. Angle constraint equations.
Note: this constraint equation does not apply when $\theta=0^{\circ}$ and $180^{\circ}$, when point 2 moves infinitesimally and perpendicularly to the vector $\overline{\mathcal{B}}$ changes the angle $\theta$ but not the projection o $\bar{f}$ Bon $\bar{B}$

Alternative:

$$
\begin{gathered}
\left|r_{12} \times r_{13}\right|-L_{12} L_{13} \operatorname{sen} \theta=0 \\
\left|r_{12} \times r_{13}\right|=\left(x_{2}-x_{1}\right)\left(y_{3}-y_{1}\right)-\left(y_{2}-y_{1}\right)\left(x_{3}-x_{1}\right) \\
\left(x_{2}-x_{1}\right)\left(y_{3}-y_{1}\right)-\left(y_{2}-y_{1}\right)\left(x_{3}-x_{1}\right)-L_{12} L_{13} \operatorname{sen} \theta=0
\end{gathered}
$$

Note: this constraint equation does not apply when $\theta=90^{\circ}$ and $-90^{\circ}$, when point 3 moves infinitesimally and parallel to the vector $\overline{\overline{12}}$, changes the angle $\theta$ but doesnot change the area of the triangle $\Delta \overline{123}$.

## Modeling Notation

$\mathrm{q}: \quad \mathrm{n} \times 1$ vector of dependent coordinates.
$\mathrm{z}: \quad$ vector g x 1 of independent coordinates (degrees offreedom).
natural coordinates, therefore, in this problem we calculate the positions of the elements of the mechanismas a function of the degree of freedom.
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## Velocity Problem

In the velocity problem we have as data $\mathrm{q}, \mathrm{z}$ therefore,
all the velocities of the points of each element of the mechanism are calculated as a function of the input data.

## Acceleration Problem

In the velocity problem we have as data $\mathrm{q}, \mathrm{q}, \mathrm{z}$, therefore, all the accelerations of the points of each element of the mechanism are calculated as a function of the input data.


Figure-10. Representation of notation on a schematicflat mechanism.


Table-1. Types of kinematic problems of mechanisms.

| Problem | Data | Unknowns |
| :---: | :---: | :---: |
| Position | $z$ | $q$ |
| Velocity | $q, \dot{z}$ | $\dot{q}$ |
| Acceleration | $q, \dot{q}, \ddot{z}$ | $\ddot{q}$ |

## NUMERICALS METHODS

Numerical methods are techniques by which mathematical problems can be formulated in such a way that they can be solved using arithmetic operations. Although there are many types of numerical methods, they share a common characteristic: they invariably require a good number of tedious arithmetic calculations 8].

Table-2. Advantages and disadvantages of numerical methods.

| Advantages | disadvantages |
| :---: | :---: |
| They allow us to approach <br> the solutions of equations <br> thatcannot be solved by <br> other methods. | Consume high processing <br> capacity |
| They can handle a large <br> number of equations and <br> variableswithout operating <br> errors. | The speed of convergence <br> depends on the complexityof <br> the problem, most of thetime it <br> is slow. |
| They have a low error <br> rate, which makesthem <br> reliable. |  |

Dependent coordinates: vector $q=$
$x 3$
$\left.{ }_{y_{3}}\right\} \mathbf{I} x_{4} \mathbf{I} \quad \mathbf{I} y_{4} \mathbf{I}$
I $\theta$ J $9 x 1$

Independent coordinates (degrees of freedom):
vector $z=\{\theta\} 1 \times 1$
We note that z is a subset of $\mathrm{q} z \subseteq q$

## Types of Kinematic Problems

In the kinematic analysis of planar mechanisms of one degree of freedom, resolution is given to position, velocity and acceleration.

## Position Problem

In the position problem we have as data z , which are the independent variables of degrees of freedom, and we have as unknowns $q$, which are dependent variables of

## OPEN NUMERICALS METHODS

In the closed methods the root lies within an interval predetermined by a lower and an upper limit. Repeated application of these methods always generates increasingly closer approximations to the root. Such methods are said to be convergent because they progressively approach the root as the calculation proceeds. In contrast, open methods are based on formulas that require only a single starting value x or start with a pair of them, but do not necessarily enclosethe root. These sometimes diverge or move away from the true root as the calculation proceeds. However, when open methods converge, they generally converge muchfaster than closed methods.

## NEWTON RAPHSON METHOD

Among the methods of successive approximations to find some roots of an algebraic or transcendental equation, the Newton-Raphson method is the one with the best efficiency characteristics, since it almost always converges to the solution and does so in a
reducednumber of iterations. This method is applicable in both algebraic and transcendental equations and with it, it is possible to obtain complex roots. Of the formulas for locating roots, the Newton-Raphson formula is perhapsthe most widely used. If the initial value for the root is xi, then a tangent can be drawn from the point [xi,f(xi)] on the curve. Usually, the point where this tangentcrosses the $x$ axis represents an improved approximation of the root (see Figure-11).


Figure-11. Graphical representation of the Newton Raphson method.

The Newton-Raphson method, like all methods of successive approximations, starts from a first approximation and by applying a recurrence formula will approach the root sought, so that the newapproximation is located at the intersection of the tangent to the curve of the function at the point and the abscissa axis.

The first derivative in x is equivalent to the slope:
$f^{\prime}\left(x_{i}\right)=\frac{f\left(x_{i}\right)-0}{x_{i}-x_{i+1}}$
which is rearranged to obtain:
$\boldsymbol{x}_{i+1}=\boldsymbol{x}_{i}-\frac{f\left(\boldsymbol{x}_{i}\right)}{f^{\prime}\left(\boldsymbol{x}_{i}\right)}$

## SYSTEM OF EQUATIONS

A set of two or more equations with more than one unknown that form a mathematical problem that consists of finding the values of the unknowns that satisfy these operations.

## Systems of Linear Equations

A system of linear equations, also known as a linear system of equations or simply linear system, is a set of linear equations, that is, a system of equations where each equation is of first degree.

## Systems of Nonlinear Equations

A system of equations is nonlinear if at least one of its equations is nonlinear (there is a degree greater than one).

## Procedure Newton Raphson Method System of Nonlinear Equations with n Variables

To apply this procedure to a system of $n$ equations withn unknowns, we represent the variable $x$ by x 1 and the variable y by x 2 . The system of two equations is writtenin a more general form:
$\left\{\begin{array}{l}f_{1}\left(x_{1}, x_{2}\right)=0 \\ f_{2}\left(x_{1}, x_{2}\right)=0\end{array}\right.$
Suppose that at stage k of calculation process we start from any point (x1, x2) and move to another very close point $(\mathrm{x} 1+\Delta \mathrm{x} 1, \mathrm{x} 2+\Delta \mathrm{x} 2)$. The values of the functions are f 1 and f 2 at that point are approximately:
$f_{1}\left(x_{1}+\Delta x_{1}, x_{2}+\Delta x_{2}\right) \approx f_{1}\left(x_{1}, x_{2}\right)+\frac{\partial f_{1}}{\partial x_{1}} \Delta x_{1}+\frac{\partial f_{1}}{\partial x_{2}} \Delta x_{2}$
$f_{2}\left(x_{1}+\Delta x_{1}, x_{2}+\Delta x_{2}\right) \approx f_{2}\left(x_{1}, x_{2}\right)+\frac{\partial f_{2}}{\partial x_{1}} \Delta x_{1}+\frac{\partial f_{2}}{\partial x_{2}} \Delta x_{2}$
If the point $(x 1+\Delta x 1, x 2+\Delta x 2)$ is a solution of the system of equations, then

$$
\begin{aligned}
& f_{1}\left(x_{1}, x_{2}\right)+\frac{\partial f_{1}}{\partial x_{1}} \Delta x_{1}+\frac{\partial f_{1}}{\partial x_{2}} \Delta x_{2}=0 \\
& f_{2}\left(x_{1}, x_{2}\right)+\frac{\partial f_{2}}{\partial x_{1}} \Delta x_{1}+\frac{\partial f_{2}}{\partial x_{2}} \Delta x_{2}=0
\end{aligned}
$$

We write the system of equations in matrix form to clear $\Delta \mathrm{x} 1$ and $\Delta \mathrm{x} 2$.

$$
\binom{f_{1}}{f_{2}}+\left(\begin{array}{ll}
\frac{\partial f_{1}}{\partial x_{1}} & \frac{\partial f_{1}}{\partial x_{2}} \\
\frac{\partial f_{2}}{\partial x_{1}} & \frac{\partial f_{2}}{\partial x_{2}}
\end{array}\right)\binom{\Delta x_{1}}{\Delta x_{2}}=0
$$

We call vector x the vector (x1, x2), the vector function F is formed by two elements which are the functions ( $\mathrm{f} 1, \mathrm{f} 2$ ) and the square matrix of dimension two is the Jacobian J. We clear $\Delta x 1$ and $\Delta x 2$ from the system of equations or the vector $\Delta x$.
$\mathbf{F}(\mathbf{x})+\mathbf{J} \Delta \mathbf{x}=\mathbf{0}$
$\Delta \mathbf{x}=-\mathbf{J}^{-1} \mathbf{F}$
$\mathrm{J}-1$ is the inverse matrix of J and $\Delta \mathrm{x}$ is the difference vector between the vector that gives us the coordinates of the new point $x k+1$, knowing the coordinates of the previous point xk

$$
\mathbf{x}_{k+1}=\mathbf{x}_{k}-\mathbf{J}^{-1} \mathbf{F}
$$

For a system of $n$ equations:

$$
\left\{\begin{array}{l}
f_{1}\left(x_{1}, x_{2} \ldots x_{n}\right)=0 \\
f_{2}\left(x_{1}, x_{2} \ldots x_{n}\right)=0 \\
\ldots \\
f_{n}\left(x_{1}, x_{2} \ldots x_{n}\right)=0
\end{array}\right.
$$

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The procedure is written:

$$
\left(\begin{array}{c}
x_{1}^{\prime} \\
x_{2}^{\prime} \\
\ldots \\
x_{n}^{\prime}
\end{array}\right)=\left(\begin{array}{c}
x_{1} \\
x_{2} \\
\ldots \\
x_{n}
\end{array}\right)-\left(\begin{array}{cccc}
\frac{\partial f_{1}}{\partial x_{1}} & \frac{\partial f_{1}}{\partial x_{2}} & \cdots & \frac{\partial f_{1}}{\partial x_{n}} \\
\frac{\partial f_{2}}{\partial x_{1}} & \frac{\partial f_{2}}{\partial x_{2}} & \cdots & \frac{\partial f_{2}}{\partial x_{n}} \\
\ldots & \ldots & \cdots & \ldots \\
\frac{\partial f_{n}}{\partial x_{1}} & \frac{\partial f_{n}}{\partial x_{2}} & \cdots & \frac{\partial f_{n}}{\partial x_{n}}
\end{array}\right)^{-1}\left(\begin{array}{c}
f_{1}\left(x_{1}, x_{2} \ldots x_{n}\right) \\
f_{2}\left(x_{1}, x_{2} \ldots x_{n}\right) \\
\ldots \\
f_{n}\left(x_{1}, x_{2} \ldots x_{n}\right)
\end{array}\right)
$$

## PROGRAMMING FUNDAMENTALS

According to the nature of computer operation, it is said that computers always execute commands in a format that is intelligible to them; these commands are grouped into programs, known as software, which, for study, in turn, is divided into two parts: the internal representation format of the programs, which constitutes the machine language or executable code, and the external presentation format, which is a file or a set of files, which may or may not be in a format that can beseen/read by the user.

## ALGORITHMS

An algorithm is a well-defined, ordered and finite list of operations, which allows finding the solution to a given problem. Given an initial state and an input, it is through successive and well-defined steps that a final state is reached, in which a solution (if there are several) or the solution (if it is unique) is obtained.
An algorithm can be expressed in:

- Natural language
- Pseudocode
- Flowcharts
- Programs


## JAVA LANGUAGE

Java is a programming language developed by Sun Microsystems. Java was introduced in the second half of 1995 and has since become a very popular programming language. Java is a highly valued language because Java programs can run on various operating system platforms such as Windows, Mac OS, Linux or Solaris. James Gosling, the head of the Java development team, realized the promise of a platform-independent language. The goal was to design a language that would allow a single application to be programmed once and then run on different machines and operating systems. To achieve portability of Java programs, a runtime environment is used for the compiled programs. This environment is called Java Runtime Environment (JRE). It is free and available for all major operating systems. This ensures that the same Java program can run on Windows, Mac OS, Linux or Solaris [9].


Figure-12. Java Runtime Environment (JRE). Source: [9].

## INTERACTIVE DEVELOPMENTS (IDE)

An integrated development environment is a computer application that provides comprehensive services to facilitate software development for the developer or programmer. Typically, an IDE consists of a source code editor, automatic build tools and a debugger.

## IDE NetBeans

Program that serves as an IDE that allows programmingin different languages, it is ideal for working with the JAVA development language (and all its derivatives), it also offers an excellent environment for programming in PHP.

## CHARACTERIZATION OF FLATMECHANISMS WITH ONE DEGREE OF FREEDOM TO OBTAIN SOFTWARE INPUT PARAMETERS

Conditions and identification of the type of links with their respective lengths, kinematic pairs, fixed and mobile positions in order to configure a type of mechanism, which is then performed geometric analysis, velocities and accelerations.

## FOUR-BAR MECHANISM



Figure-13. Nomenclature of four-bar mechanisms with relative revolute joints.
$\boldsymbol{O}_{\mathbf{2} \boldsymbol{x}}$ : First fixed point of the mechanism bed plate in horizontal direction.
$\boldsymbol{O}_{2 \boldsymbol{y}} \boldsymbol{y}$. First fixed point of the mechanism bed plate in vertical direction.
$\boldsymbol{O}_{4} \boldsymbol{x}$ : Second fixed point of the mechanism bed plate inhorizontal direction.
O4y: Second fixed point of the mechanism bed plate invertical direction.
A: Mobile articulation of the mechanism.
B: Mobile articulation of the mechanism.
$\mathbf{L 2}$ : Length of the first link of the mechanism (crank ordriver).
L3: Length of the second link of the mechanism (connecting rod or coupler).
L4: Length of the third link of the mechanism (oscillatoror driven).

## Definition of parameters:

$\boldsymbol{O}_{2 \boldsymbol{x}}$ : First fixed point of the mechanism bed plate in horizontal direction.
$\boldsymbol{O}_{2} \boldsymbol{y}$ : First fixed point of the mechanism bed plate in vertical direction.
A: Mobile articulation of the mechanism.
B: Mobile articulation of the mechanism.
L2: Length of the first link of the mechanism (crank ordriver).
L3: Length of the second link of the mechanism

Length
of the(connecting rod or coupler).
L4: Mechanism collar link (oscillator or driven).
fourth link according to the fixed points of themechanism (bedplate).
$\boldsymbol{\theta}$ : Mechanism input parameter (degree of freedom).
Mathematical modelling of four-barmechanism joints
Modeling and constraint equations in natural coordinates of the one-degree free four-bar mechanism.
$\boldsymbol{\theta}:$ Mechanism input parameter (degree of freedom).

## Mathematical modelling of crank-slidemechanism joints

Modeling and constraint equations in natural coordinates of the one-degree-of-freedom crank-slide mechanism.


Figura-14. Schematic representation of the four-bar mechanism.
Source: Authors.
Constraint equations of the crank-slide mechanism.
$\left(x_{1-} x_{A}\right)^{2}+\left(y_{1}-y_{A}\right)^{2}-L_{2}^{2}$
$(x-x)^{2}+(y-y)^{2}-L^{2}$
$\boldsymbol{\varphi}(\boldsymbol{q})=$
$\left(x_{2}-x_{1}\right)^{2}$
$+\left(y_{2}-y_{1}\right)^{2}$
$-L^{2}$
$=0$
$\varphi(q)=$
2121
$y_{2}-k$
$3=0$
$\left(x_{3}-x_{B}\right)^{2}+\left(y_{3}-y_{B}\right)^{2}-L_{4}{ }^{2}$
\} x _ { 1 } - x _ { A } - L _ { 2 } * \operatorname { c o s } ( \theta ) \mathbf { J }
Iterative calculation of the position of the four-barmechanism: $\varphi_{q} \Delta q=-\varphi(q)$

Iterative calculation of four-bar mechanism speed: $\varphi_{q} \dot{q}=0$
Calculation of accelerationof four-bar mechanism: $\varphi_{q} \ddot{q}=$ $-\varphi \dot{q} \dot{q}$

## CRANK MECHANISM-SLIDE



Figure-15. Nomenclature of crank mechanisms deslizadera.
Source: Authors.
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Figure-16. Schematic representation of the crank-slide mechanism.
Source: Authores.

## $\mathbf{I}_{1}-x_{A}-L_{2} * \cos (\theta) \mathbf{J}$

Iterative calculation of crank-slide mechanism position: $\varphi_{q} \Delta q=-\varphi(q)$

Iterative calculation of the speed of the crankslidemechanism: $\varphi_{q} q^{\circ}=0$

Iterative calculation of acceleration of the crank-slidemechanism: $\varphi_{q} \ddot{q}=-\varphi_{q} \dot{q}$

## SIX-LINK MECHANISM WITH TRIAD



Figure-17. Nomenclature of six-link mechanisms with triad.
Source: Autohrs.
Definition of parameters:
$\boldsymbol{O}_{2 \boldsymbol{x}}$ : First fixed point of the mechanism bed plate in horizontal direction.
$\boldsymbol{O}_{2} \boldsymbol{y}$ : First fixed point of the mechanism bed plate in vertical direction.
$\boldsymbol{O}_{4 \boldsymbol{x}}$ : Second fixed point of the mechanism bed plate inhorizontal direction.
O4y: Second fixed point of the mechanism bed plate invertical direction.
$\boldsymbol{O}_{\mathbf{6} \boldsymbol{x}}$ : Fixed point of the mechanism bed plate in horizontal direction.
$\boldsymbol{O}_{6} \mathbf{y}$ : Fixed point of the mechanism bed plate in verticaldirection.
A: Mobile articulation of the mechanism.
B: Mobile articulation of the mechanism.
C: Mobile articulation of the mechanism.
D: Mobile articulation of the mechanism.
L2: Length of the first link of the mechanism (crank ordriver).
L3: Length of the second link of the mechanism (coupler).
$\boldsymbol{L 4 B C}$ : Mechanism link (1 side triad).
L4CO4: Mechanism link (1 side triad).
L4BO4: Mechanism link (3 side triad).
L5: Length of the fifth link of the mechanism (coupler).
L6: Longitud del sexto eslabón del mecanismo (oscilador).
$\boldsymbol{\theta}$ : Mechanism input parameter (degree of freedom).

## Mathematical Modeling of Six-Link Mechanism Joints with Triad

Modeling and constraint equations in natural coordinates of the one-degree-of-freedom six-link mechanism with triad.


Figure-18. Schematic representation of a six-link mechanism with triad.

Fuente: Authors
Constraint equations of six-link mechanism with triad.
$\left(x_{1}-x_{A}\right)^{2}+\left(y_{1}-y_{A}\right)^{2}-L_{2}{ }^{2}$
coordinates of the one-degree-of-freedom fast-return mechanism.
」
$\left(x_{2}-x_{1}\right)^{2}$
$+\left(y_{2}-y_{1}\right)^{2} 1-L_{3}{ }^{2}$
$\left(x_{3}-x_{2}\right)^{2}+\left(y_{3}-y_{2}\right)^{2}-L 4 B C^{2}$
$\mathbf{I} \quad \mathbf{I}$
$(x-x)^{2}+(y-y)^{2}-L^{2} \varphi(q)=b 2$
$b 24 B 04=0\left(\left(x_{3}-x_{b}\right)^{2}+\left(y_{3}-y_{b}\right)^{2}-L_{4 C O 4}{ }^{2}\right\}\left(x_{4}-\right.$
$\left.x_{3}\right)^{2}+\left(y_{4}-y_{3}\right)^{2}-L_{5}{ }^{2}$
I $\left(x_{4}-x_{C}\right)^{2}+\left(y_{4}-y_{C}\right)^{2}-L_{6}{ }^{2} \mathbf{I}$
$\boldsymbol{I} x_{1}-x_{A}-L_{2} * \cos (\theta)$ 】
Iterative calculation of the position of the six-link mechanism with triad: $\varphi_{q} \Delta q=-\varphi(q)$
Iterative speed calculation of six-link mechanism with triad: $\varphi_{q} \dot{q}=0$

Iterative calculation acceleration six-link mechanismwith triad: $\varphi_{q} \ddot{q}=-\varphi_{\dot{q}} \dot{q}$
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## QUICK RETURN MECHANISM



Figure-19. Nomenclature of fast return mechanisms.
Source: Autohrs.
$\boldsymbol{O}_{2 \boldsymbol{x}}$ : First fixed point of the mechanism bed plate in horizontal direction.
$\boldsymbol{O}_{2 \boldsymbol{y}} \boldsymbol{y}$ : First fixed point of the mechanism bed plate in vertical direction.
$\boldsymbol{O}_{4 \boldsymbol{x}}$ : Second fixed point of the mechanism bed plate inhorizontal direction.
O4y: Second fixed point of the mechanism bed plate invertical direction.
$\boldsymbol{O}_{6} \boldsymbol{x}$ : Fixed point of the mechanism bed plate in horizontal direction.
$\boldsymbol{O}_{6} \boldsymbol{y}$ : Fixed point of the mechanism bed plate in verticaldirection.
A: Mobile articulation of the mechanism.
B: Mobile articulation of the mechanism.
$\boldsymbol{C}: \quad$ Mobile articulation of the mechanism.
$\boldsymbol{L}_{\mathbf{2}}$ : Length of the first link of the mechanism (crank ordriver).
L3: Collar link.
L4: Link length (oscillator).
L5: Length of the fifth link of the mechanism (coupler).
L6: Output collar link (oscillator).
$\boldsymbol{\theta}$ : Mechanism input parameter (degree of freedom).

## Mathematical Modeling of Joints Fast Return Mechanism

Modeling and constraint equations in natural coordinates of the one-degree-of-freedom fast-return mechanism.

Constraint equations of six-link mechanism with triad.
$\left(x_{1}-x_{A}\right)^{2}+\left(y_{1}-y_{A}\right)^{2}-L_{2}{ }^{2}$
$\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2} 1-L_{3}{ }^{2}$
$\left(x_{3}-x_{2}\right)^{2}+\left(y_{3}-y_{2}\right)^{2}-L 4 B C^{2}$
II
$(x-x)^{2}+(y-y)^{2}-L^{2} \varphi(q)=b 2$
$b 24 B O 4=0$

$$
\begin{aligned}
& \left(\left(x_{3}-x_{b}\right)^{2}+\left(y_{3}-y_{b}\right)^{2}-L_{4} \operatorname{Co4}_{4}^{2} \mathbf{\}}\left(x_{4}-x_{3}\right)^{2}+\left(y_{4}-y_{3}\right)^{2}-\right. \\
& L_{5}^{2} \\
& \mathrm{I}\left(x_{4}-x_{C}\right)^{2}+\left(y_{4}-y_{C}\right)^{2}-L_{6}^{2} \mathbf{I} \\
& \mathbf{I} x_{1}-x_{A}-L_{2} * \cos (\theta) \mathbf{J}
\end{aligned}
$$



Figure-20. Schematic of fast return mechanism.
Source: Authors
Iterative calculation of the position of the sixlinkmechanism with triad: $\varphi_{q} \Delta q=-\varphi(q)$

Iterative speed calculation of six-link mechanism withtriad: $\varphi_{q} \dot{q}=0$

Fast return mechanism constraint equations

$$
\begin{aligned}
& (x 1-x b)^{2}+(y 1-y b)^{2}-L 2^{2} \\
& (x-x)^{2}+(y-y)^{2}-L^{2}
\end{aligned}
$$

$\boldsymbol{\theta}: \quad$ Mechanism input parameter (degree of freedom).
I $2 a$
$2 a 4$ I
Fast return mechanism constraint equations

$$
\begin{aligned}
& (x 1-x b)^{2}+(y 1-y b)^{2}-L 2^{2} \\
& (x-x)^{2}+(y-y)^{2}-L^{2}
\end{aligned}
$$

$\boldsymbol{\theta}$ : Mechanism input parameter (degree of freedom).
I $2 a$
$2 a 4$ I

## Mathematical Modelling of Eight-Link Mechanism Joints with a Triad

$\varphi(q)=\left((x 3-x 2)^{2}+(y 3-y 2)^{2}-L 5^{2}=0(x 1-x a)(y 2-\right.$
$y a)-(y 1-y a)(x 2-x a)\}$
Modeling and constraint equations in natural
www.arpnjournals.com
$\mathbf{I}(y 3-$ posinicialy-tan(inclinacion)(x3-posinicialx) $\mathbf{I}$
$\mathbf{I} x_{1}-x_{b}-L_{2} * \cos (\theta) \mathbf{J}$
Iterative calculation of position quick return mechanism: $\varphi_{q} \Delta q=-\varphi(q)$

Iterative calculation of the speed of the fast returnmechanism: $\varphi_{q} \dot{q}=0$
Iterative computation acceleration fast return mechanism: $\varphi_{q} \ddot{q}=-\varphi_{q} \dot{q}$

## MECANISMO OCHO ESLABONES CON UNA TRIADAS

Coordinates of the one-degree-of-freedom mechanismwith eight links and a triad.


Figure-21. Schematic representation of an eight-bar mechanism with a triad. Source: Autohrs

Eight-bar mechanism constraint equations with a triad.
$\varphi(q)$


Figura-22. Nomenclatura de mecanismos de retorno rápido.
Fuente: Autores.
Definition of parameters:
$\left.\mathbf{I}(x-x)^{2}+\mathbf{I}-y\right)^{2} L^{2}=23$
$234 C B$
$234 C B$
$\boldsymbol{O}_{2 \boldsymbol{x}}$ : First fixed point of the mechanism bed plate in ( horizontal direction. ${ }^{\text {I }}$
$\boldsymbol{O}_{2 \boldsymbol{y}}$ : First fixed point of the mechanism bed plate in ${ }^{I}$
$\left.\left(x_{3}-x_{B}\right)^{2}+\left(y_{3}-y_{B}\right)^{2}-L_{5}{ }^{2}\right\}$
$\left(x_{4}-x_{c}\right)^{2}+\left(y_{4}-y_{c}\right)^{2}-L_{6}{ }^{2} \mathbf{I}$
$(x-x)^{2}+(y-y)^{2}-L^{2}{ }^{\mathbf{I}}$
I52527 I
vertical direction. $\mathbf{I}\left(y_{5}\right.$-posinicialy-tan (inclinacion) ( $x_{5}$-posinicialx) $\mathbf{I}$
$\boldsymbol{O}_{4} \boldsymbol{x}$ : Second fixed point of the mechanism bed plate inl horizontal direction. $=0$
$\boldsymbol{O}_{4} \boldsymbol{y}$ : Second fixed point of the mechanism bed plate in
$x_{1}-x_{A}-L_{2} * \cos (\theta)$ J
vertical direction.
$\boldsymbol{O}_{6} \boldsymbol{x}$ : Fixed point of the mechanism bed plate in horizontal direction.
O6y: Fixed point of the mechanism bed plate in verticaldirection.
A: Mobile articulation of the mechanism.
B: Mobile articulation of the mechanism.
C: Mobile articulation of the mechanism.
D: Mobile articulation of the mechanism.
E: Mobile articulation of the mechanism.
$\mathbf{L 2 : ~ L}_{2} \quad$ Length of the first link of the mechanism (crank ordriver).
L3: Length of the second link of the mechanism (coupler).
L4BD: Mechanism link (1 side triad).
L4DC: Mechanism link (2 side triad).
L4CB: Mechanism link (3 side triad).
$L_{5}$ : Length of the fifth link of the mechanism (oscillator).
$\boldsymbol{L}_{\mathbf{6}}$ : Length of the sixth link of the mechanism (oscillator).
L7: Length of the second link of the mechanism (coupler).
L8: Mechanism output collar link (oscillator).
Iterative calculation of position quick return mechanism: $\varphi_{q} \Delta q=-\varphi(q)$

Iterative calculation of the speed of the fast returnmechanism: $\varphi_{q} \dot{q}=0$

Iterative computation acceleration fast return mechanism: $\varphi_{q} \ddot{q}=-\varphi \dot{q} \dot{q}$

## DEVELOPED GRAPHICAL INTERFACE

The methodology used for the development of the graphical interface was the "framework", which is used to structure, plan and control the development process in information systems. The approach to interface development was a waterfall model, which is a sequential process in which the development steps are viewed downward, through the phases of needs analysis, design, implementation, testing and integration. The interface was developed using NetBeans integrated development
environment (IDE). The interface was divided into four main sections or panels: Presentation, Input Parameters, Functions and Output Parameters, presented below.

## INTERFACE INTERACTIVITY

The main presentation of the software shows the different configurations of flat mechanism types to analyze, where the end user selects the type of mechanism to evaluate and calculate. See Figure-22.


Figure-23. Mechanism selection window. Source: Autores

The "Input Parameters" panel shown in Figure-23 presents an illustrative image of a flat four-barmechanism, showing the nomenclature used to identify links, angles, and other parameters to help the userbecome familiar with the interface.

This panel also contains the edit boxes for entering inputparameters (link lengths, angles, etc.). At this point of use of the interface, data entry is done by keyboard.


Figure-24. Input parameter blocks.
Source: Authors

## INTERFACE FEATURES

The main window consists of three workspaces, where you can view the graphical representation of the previously selected mechanism; the results graph spaceand the numerical results of position, velocity and acceleration.

The animation speed can be started, paused or resumedwith the Play and Pause buttons.


Figure-25. Main window.
Source: Authors

## RESULTS

## Validation of Results

To verify the position kinematics results, the software was fed with data from several textbook problems, especially from the author [1], well known in academia. Table-3 presents the results of [1] and the software for aspecific problem. There is agreement in the results obtained. Similarly, the results of velocity and acceleration were validated against solved problems of the same reference, presented in Tables 4 and 5, showing practically the same result in each case.


Figure-26. Comparison of articulation position. Source: Authors


Figure-27. Comparison of articulation position.
Source: Authors
www.arpnjournals.com

Table-3. Position analysis validation.

| Table <br> P4-1 | Data for the problem 4-6 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Fila | L1 <br> $[\mathbf{c m}]$ | L2 <br> [cm] | $\mathbf{L 3}$ <br> $[\mathbf{c m}]$ | L4 <br> $[\mathbf{c m}]$ | Ө |
| a | 6 | 2 | 7 | 9 | 30 |
| Solutions to the problem 4-6 |  |  |  |  |  |
| Ө3 <br> open | Ө4 <br> open | Ө3 <br> close | Ө4 <br> close |  |  |
| 88,8 | 117,3 | $-115,2$ | $-143,6$ |  |  |
| Problem solutions using the interface |  |  |  |  |  |
| 88,83 | 117,28 | 244,78 | 216,34 |  |  |

Table-4. Speed analysis validation.

| TableS4-1 | Data for the problem 4-6 <br> W2 $=\mathbf{1 0}$ rad/s |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Fila | L 1 <br> $[\mathrm{~cm}]$ | L 2 <br> $[\mathrm{~cm}]$ | L 3 <br> $[\mathrm{~cm}]$ | L 4 <br> $[\mathrm{~cm}]$ | $\Theta$ |
| a | 6 | 2 | 7 | 9 | 30 |
| Solutions to the problem 4-6 |  |  |  |  |  |
| Ө3 <br> open | Ө4 <br> open | Ө3 <br> close | Ө4 <br> close |  |  |
| 6 | 4 | 0,66 | 2,66 |  |  |
| Problem solutions using the interface |  |  |  |  |  |
| 5,99 | 3,9 | 0,66 | 2,62 |  |  |

Table-5. Acceleration analysis validation.

| $\begin{gathered} \text { Table } \\ \text { S4-1 } \\ \hline \end{gathered}$ | Data for the problem 4-6 w2 = $10 \mathrm{rad} / \mathrm{s}$;$\alpha=0$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Fila | $\begin{gathered} \hline \mathrm{L} 1 \\ {[\mathrm{~cm}]} \\ \hline \end{gathered}$ | $\begin{gathered} \mathrm{L} 2 \\ {[\mathrm{~cm}]} \\ \hline \end{gathered}$ | $\begin{gathered} \mathrm{L} 3 \\ {[\mathrm{~cm}]} \end{gathered}$ | $\begin{gathered} \mathrm{L} 4 \\ {[\mathrm{~cm}]} \end{gathered}$ | Ө |
| a | 6 | 2 | 7 | 9 | 30 |
| Solutions to the problem 4-6 |  |  |  |  |  |
| Ө3 <br> open | Ө4 <br> open | $\begin{gathered} \text { Ө3 } \\ \text { close } \end{gathered}$ |  | Ө4 lose |  |
| 26,1 | 53,3 | 77,9 |  | 0,7 |  |
| Problem solutions using the interface |  |  |  |  |  |
| 26,082 | 53,331 | 77,91 |  | 0,66 |  |

Validation of results, SAM 8.0 - ACIMEP 1.0 A study mechanism is presented to compare the results of Acimep 1.0 with the SAM 8.0 software, free demo version. The input data are summarized in Table-6.

Table-6. Mechanism data (Esl: Link).

| Data of the mechanism under study |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Esl <br> $\mathbf{1}$ | Esl <br> $\mathbf{2}$ | Esl <br> $\mathbf{3}$ | Esl <br> $\mathbf{4}$ | $\mathbf{0 2 x}$ | $\mathbf{0 2 y}$ | $\mathbf{O 4 x}$ | $\mathbf{O 4 y}$ |
| 360 | 60 | 400 | 180 | 0 | 0 | 360 | 0 |
| $\Theta$ | w | $\alpha$ |  |  |  |  |  |
| 70 | 2 | 1 |  |  |  |  |  |



Figure-28. Y-axis graphs of joint position.
Source: Authors


Figura-29. X-axis graphs of joint position.
Source: Authors
Application case of fast return cutting machine tools


Figura-30. X-axis graphs of joint position.
Source: Authors
www.arpnjournals.com


Figure-31. Mechanism Acimep 1.0
Source: Acimep 1.0

## CONCLUSIONS

The development and application of a software tool for teaching and learning kinematics and mobility of one- degree-of-freedom planar mechanisms was reported. The need was born due to the high cost of specialized software licenses and the limitation of the available student versions. The design of the interface is didactic so that it is easy to use by the users. The results obtainedusing the developed interface was validated against theresults of other authors and with the results of other similar software.

From the point of view of interactivity, the proposed interface allows the modification of the lengths and orientation of the elements of the mechanism, numerically through the keyboard.

The development of the graphical interface becomes a link between the theoretical and practical concepts of mechanism analysis, virtually showing their behavior and allowing the observation of the phenomena in a versatile way. Version 1.0 of the Acimep 1.0 software works correctly for defined exercises, not "invented" values that physically are not possible to analyze the mechanism. For a 2.0 version, the software should include a validation module that restricts the entry of out-of-range values that cause the systems of equationsto malfunction.
A user's manual for the tool was developed to ensure easy learning and proper use of the software by the user, and thus provide a powerful, free and useful tool for students, teachers and designers who require this software.

The methodology applied in this work, directed to the fulfilment of each of the proposed objectives, thus achieving Acimep 1.0 software that by means of numerical methods performs the kinematic analysis of flat mechanisms with one degree of freedom.

A 2.0 version may include the management of databases to store the history of the analysis of the
mechanisms performed, expansion of the systems of equations to broaden the scope of analysis of more mechanisms.

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