COMPARISON BETWEEN GREY MODELS, EXPONENTIAL SMOOTHING MODEL AND HOLT-WINTER MODEL IN PREDICTING THE NUMBER OF PEOPLE INFECTED WITH THE COVID 19 PANDEMIC IN IRAQ

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ABSTRACT

The methods of time series analysis are one of the most important statistical methods used in this field, as the study of a phenomenon over a period of time and identification of its pattern and behavior alongside with the factors affecting, predicting it as well, the future values of the phenomenon under study and contributing to the formulation of future policies and the development of plans could build a service system that addresses the negative aspects of the studied case and raise the level of services provided by the state. The study aims to determine the best model for predicting the levels of infection with the COVID 19 pandemic and the preferred model, among those models was the Grey Verhulst model according to the predictive measures MAPE and RMSE, where the lowest values of these measures were for the GVM (1,1) model which indicates a high predictive efficiency and the possibility of adopting its predictions to determine the future levels of the numbers of people infected with the COVID 19 pandemic. The study also indicated that the rest of the models were statistically acceptable, but they were not of high predictive efficiency compared to the GVM model (1,1).

Keyword: exponential smoothing, grey models, accumulating generation operator AGO, winter additive model.

INTRODUCTION

It is no secret to anyone that the spread of COVID 19 pandemic has negative effects on various aspects of life, especially developing countries which suffer from a notable weakness in the level of medical services, as well as the absence of health system and preventive awareness among the people of these countries including Iraq which makes these countries an easy environment to the spread of the pandemic. As it has taken an awful spread at the beginning of recent year 2021 after the emergence of new strains of the virus, including the British and Indian ones that caused a significant increase in the number of infections. This pandemic which knows no geographical borders has left many negative effects, some of which affected the economic and political conditions, and some of them reflected on the level of education, which declined due to the disruption of educational institutions for reasons related to prevention and social distancing that prevent its spread or limit its transmission, the importance of time series and its prediction in determining the level of infections for future periods may appear, so that these results and the recommendations based on them may be basically reliable in making decisions taken by health authorities in addressing this pandemic and limiting its spread. The important question raised by this study is what is the model that can be adopted in predicting the number of infections with the COVID 19 pandemic in Iraq and on what criteria was this done?

This study is divided into two parts, one of which is theoretical, dealing with the theoretical foundations of the additive holt winter and simple exponential smoothing models, and the grey models, the other part is practical in which data on the numbers of people infected with the COVID 19 pandemic are analyzed. It is in the form of weekly observations starting from the beginning of January 2021 until mid-May with a rate of 22 weekly observations with a number of conclusions and recommendations based on the results.

THEATRICAL SIDE

Grey Model GM (1,1) [5] [7]

The grey prediction model GM (1,1) is one of the most popular models for its ease and high computational efficiency, as it uses a differential equation to describe the variable to be predicted and can be based on small samples to provide accurate and short-term predictions. The steps of building a grey prediction model GM can be summarized (1,1) as follows:

Step 1:

Forming an accumulating series from the original data series using the Accumulating Generation Operator AGO, which is assumed to consist of n non-negative observations. The original data series is described by relationship No. 1 as follows:

$$\mathrm{X}^{(0)} = \left(\mathrm{X}^{0}(1), \mathrm{X}^{(0)}(2), \mathrm{X}^{(0)}(3), \ldots\ldots, \mathrm{X}^{(0)}(\mathrm{n})\right) \tag{1}$$

AGO Accumulating Generation Operator is described by relationship No. 2 and 3 and the resulting series is described by relationship No. 4 as follows:

$$X^{(1)}(1) = X^{(0)}(1) \qquad if\ k = 1 \tag{2}$$

$$X^{(1)}(k) = \sum_{i=1}^{k} X^{(0)}(i) \quad if\ k = 2,3,\ldots,n \tag{3}$$

$$X^{(1)} = \left(X^{(1)}(1), X^{(1)}(2), \ldots, X^{(1)}(n)\right) \tag{4}$$

Step 2:

The next step is to generate the series of means $Z^1(k)$resulting from $X^{(1)}$through the following equation and as follows:

$$Z^{(1)}(k) = 0.5X^{(1)}k) + 0.5X^{(1)}(k-1)) \quad (5)$$

where the resulting series from Equation 5 is described by formula No. 6 where $Z^{(1)}$is the arithmetic mean of the adjacent data:

$$Z^{(1)} = \left(Z^{(1)}(1), Z^{(1)}(2), Z^{(1)}(3), \ldots, Z^{(1)}(n)\right), k = 2,3,\ldots,n \quad (6)$$

Step 3:

Through$X^{(1)}(k)$ و $Z^{(1)}(k)$,the linear grey variance equation of the GM grey prediction model (1, 1) is formed which can be described by formula No. 7.

$$X^{(0)}(k) + \alpha Z^{(1)}(k) = \beta \quad , \ k = 2,3,\ldots,n \quad (7)$$

Where:

α: Grey development coefficient
β: The grey control parameter

Also, the differential equation of the grey prediction model GM(1,1) is of the first degree and with one variable, as follows:

$$\frac{dX^{(1)}(k)}{dk} + \alpha X^{(1)}(k) = \beta \quad (8)$$

Step 4:

The next step is the step of estimating the model parameters α and β, where the LSE least squares method is used, which is described by formula 9 and as follows:

$$[\,\alpha, \beta\,]^T = (B^T B)^{-1} B^T Y \quad (9)$$

Where:

$$Y = \begin{bmatrix} X^{(0)}(2) \\ X^{(0)}(3) \\ \cdot \\ \cdot \\ \cdot \\ X^{(0)}(n) \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -Z^{(1)}(2) & 1 \\ -Z^{(1)}(3) & 1 \\ \cdot & \\ \ldots\ldots & \\ \cdot & \\ \ldots\ldots & \\ \cdot & \\ -Z^{(1)}(n) & 1 \end{bmatrix}$$

Step 5:

In this step, the values of the estimators for the original series are calculated using the formula No. 10 and using the formula No. 11, the inverse cumulative generation factor formula I AGO, we find the estimators of the values for the original series at time k, as follows:

$$\hat{X}^{(1)}(k+1) = \left[X^{(0)}(1) - \frac{\beta}{\alpha}\right] e^{-\alpha k} + \frac{\beta}{\alpha} \quad (10)$$

Where: $\hat{X}^{(1)}(1) = X^{(0)}$ (1)

$$\hat{X}^{(0)}(k) = \hat{X}^{(1)}(k) - \hat{X}^{(1)}(k-1)\,, \text{ if } k = 2,3,\ldots,n \quad (11)$$

The purpose of calculating the values of the estimators for the original series data is to know the amount of the error which represents the difference between the real and estimated values and that the error *ε (k)* and the relative error Δ_k is a basis for determining the model's priority if it is compared to another model. Accordingly, the error and relative error are described by relations 12 and 13, respectively:

$$\varepsilon(k) = X^{(0)}(k) - \hat{X}^{(0)}(K) \quad (12)$$

$$\Delta_k = \frac{|\varepsilon(k)|}{X^{(0)}(k)} \times (\%) \quad (13)$$

The resulting series of values from relationship 11 is described by relationship 14, as follows:

$$\hat{X}^{(0)} = \left(\hat{X}^{(0)}(1), \hat{X}^{(0)}(2), \ldots, \hat{X}^{(0)}(n)\right) \quad (14)$$

As for the predictive values at time $k + h$, they are calculated according to the formula 15:

$$\hat{y}_p^{(0)}(k+h) = \left(y^{(0)}(1) - \frac{\hat{\beta}}{\hat{\alpha}}\right) e^{-\hat{\alpha}(k+h-1)} (1 - e^{\hat{\alpha}}) \quad (15)$$

Grey Verhulst Models [1] [9]

The Grey Verhulst model is one of the types of grey models and it is commonly used. It is meant of studying many phenomena such as the increase in population growth and the growth and reproduction of living organisms and others, as well as being a sequential prediction model. If compared to the grey prediction modelGM (1,1). It is a suitable model for predicting the sequences that show clear exponential growth and describe the monotonous variables. As for the non-monotonous changes, the evolution of the sequences that are in the form of waves should be taken into consideration. From this standpoint, a Grey Verhulst model is created. Suppose that $X^{(0)}$is the original series of observations described by the formula (1) and $X^{(1)}$is the accumulative series described by the formula (4) and which is found using the accumulative generation factor AGO described by the formula (3) and that the series of moving media $Z^{(1)}$ shown by the formula (6) and which was found by equation (5), we can describe the grey Verhulst model whose differential equation is as follows:

$$\frac{dx^{(1)}}{dx} + \alpha x^{(1)} = \beta (x^{(1)})^2 \quad (16)$$

And by solving differential equation No. 16, we get the grey Verhulst model difference equation as follows:

$$X^{(0)}(k) + \alpha Z^{(1)}(k) = \beta(Z^{(1)}(k))^{2} \quad (17)$$

To estimate the model parameters α and β, the formula of the OLS (Ordinary Least Squares method) is used as follows:

$$[\alpha, \beta]^T = (B^T B)^{-1} B^T Y$$

Where:

$$Y = \begin{bmatrix} X^{(0)}(2) \\ X^{(0)}(3) \\ \cdot \\ \cdot \\ \cdot \\ X^{(0)}(n) \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -Z^{(1)}(2) & (Z^{(1)}(2))^2 \\ -Z^{(1)}(3) & (Z^{(1)}(3))^2 \\ & \cdot \\ & \cdots \\ & \cdot \\ & \cdots \\ & \cdot \\ -Z^{(1)}(n) & (Z^{(1)}(n))^2 \end{bmatrix}$$

After estimating the parameters of the model, the estimations of the original values of the series data are calculated using the following formula and at time K:

$$\hat{X}^{(1)}(k+1) = \frac{\alpha X^{(0)}(1)}{\beta X^{(0)}(1) + (\alpha - \beta X^{(0)}) e^{-\alpha k}} \quad (18)$$

Using the Inverse Accumulating Generation Operator (IAGO), we find the predictive values of the original string data $\hat{X}^{(0)}$, as follows:

$$\hat{X}^{(0)}(k) = \hat{X}^{(1)}(k+1) - \hat{X}^{(1)}(k) \quad (19)$$

Prediction Accuracy Criteria [2] [4] [7]

Many criteria depend on determining which models are adopted in predicting the future behavior of the phenomenon under study which depend mainly on the errors of the model, and the most important of these criteria is the Mean Absolute Percentage Error (MAPE) and Root mean square error (RMSE)

1 - Mean Absolute Percentage Error (MAPE):

$$\text{MAPE} = \frac{1}{n} \sum_{k=1}^{n} \frac{|X^{(0)}(k) - \hat{X}^{(0)}(k)|}{X^{(0)}(k)} \times 100 \qquad k = 1, \dots, n \quad (17)$$

4 **- Root mean square error (RMSE):**

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_{k=1}^{n} (X^{(0)}(k) - \hat{X}^{(0)}(k))^2} \quad (18)$$

Single Exponential Smoothing [3] [6] [8]

The process of smoothing data that contains white noise is called exponential smoothing and giving it decreasing weights exponentially with the observations that are far from the current observation X_t. Considering that the current observations contain more accurate information than the previous observations, and if we describe the general formula for it, we suppose the existence of a time series $(X_1, X_2, \dots, X_t)$to estimate the value of the time observation X_{t+1}using the following equation:

$$\hat{X}_{t+1} = \alpha X_t + (1-\alpha) X_{t-1}$$

$\hat{X}_{t+1}$forecasting for period t.
$X_t$actual time series value at time t.
αsmoothing constant ($0 \leq \alpha \leq 1$)

By successive substitution, we get:

$$\hat{X}_{t+1} = \alpha X_t + (1-\alpha) X_{t-1} + \alpha(1-\alpha)^2 X_{t-2} + \alpha(1-\alpha)^3 X_{t-3} + \cdots$$

The values of α are called weights.

The value of the error is calculated using the following formula:

$$\varepsilon_t = X_t - \hat{X}_t$$

The error equation indicates that its value at time t is the difference between the real value and the estimated value at the same time, and the effect of the value of α on the amount of smoothing can be observed when its value is large and close to 1. The new prediction for it contains a large adjustment for the error in the previous prediction, and on the contrary, if its value approaches 0, the adjustment is slight.

Holt Winter's Additive Method [2] [8]

Winter's additive method is considered the most appropriate method for forecasting time series in general, and it uses the seasonal component, the general trend component, and the seasonal component at each time point with three weights to smoothing. In order to refine the data and through the use of the regression model with time as an independent variable, the initial values of the weights are determined, the seasonal vehicle values are determined using Illusory variables, and the regression model is estimated for the series data after removing the general trend vehicle. Its equations can be described as follows:

$$L_t = \alpha (X_t - S_{t-p}) + (1-\alpha)[L_{t-1} + T_{t-1}]$$
$$T_t = \gamma[L_t - L_{t-1}] + (1-\gamma) T_{t-1}$$
$$S_t = \delta(X_t - L_t) - (1-\delta) S_{t-p}$$
$$\hat{X}_t = L_{t-1} + T_{t-1} + S_{t-p}$$

Where as:

S_t seasonal, L_t length of time series, T_t Trend Linear Model, (α, γ, β) smoothing constant, X_t Observation the time series at the time t.

Practical Side

The Single Exponential Smoothing mode, Holt Winters Additive model, Grey Model GM(1,1) and Grey Verhulst model were constructed and the results were analyzed based on data on the numbers of people infected with the COVID 19 pandemic in Iraq, which are data in the form of weekly observations shown in Table-1 and as follows:

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ARPN Journal of Engineering and Applied Sciences

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Table-1. Time series data for people infected with the COVID 19 pandemic.

year	month	week	Injuries	year	month	week	Injuries
2021	January	1	5841	2021	March	12	35461
2021	January	2	5370	2021	March	13	39718
2021	January	3	5290	2021	April	14	41726
2021	January	4	5547	2021	April	15	52903
2021	January	5	6044	2021	April	16	51769
2021	February	6	9643	2021	April	17	52908
2021	February	7	16387	2021	April	18	42140
2021	February	8	25151	2021	May	19	43629
2021	February	9	27286	2021	May	20	37629
2021	March	10	31928	2021	May	21	25298
2021	March	11	32069				

Single Exponential Smoothing Mode

A single exponential smoothing model was built and the estimated values of the time series data for the numbers of people infected with the COVID 19 pandemic were calculated, and the values of errors and relative errors were calculated, and the results were as follows:

Table-2. Estimates and smoothing values for time series data and errors for single exponential smoothing model.

Time	COVID 19	Smooth	Predict	Error
1	5841	5841.0	5841.0	0.0
2	5370	5699.7	5841.0	-471.0
3	5290	5576.8	5699.7	-409.7
4	5547	5567.9	5576.8	-29.8
5	6044	5710.7	5567.9	476.1
6	9643	6890.4	5710.7	3932.3
7	16387	9739.4	6890.4	9496.6
8	25151	14362.9	9739.4	15411.6
9	27286	18239.8	14362.9	12923.1
10	31928	22346.3	18239.8	13688.2
11	32069	25263.1	22346.3	9722.7
12	35461	28322.5	25263.1	10197.9
13	39718	31741.1	28322.5	11395.5
14	41726	34736.6	31741.1	9984.9
15	52903	40186.5	34736.6	18166.4
16	51769	43661.3	40186.5	11582.5
17	52908	46435.3	43661.3	9246.7
18	42140	45146.7	46435.3	-4295.3
19	43629	44691.4	45146.7	-1517.7
20	37629	42572.7	44691.4	-7062.4
21	25298	37390.3	42572.7	-17274.7

With the value of the smoothing constant equal to α = 0.3 and the values of each of MAPE = 27, MAD = 7966 and MSD = 97379204, the following figure shows the spread of the real and estimated values with respect to time.

 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

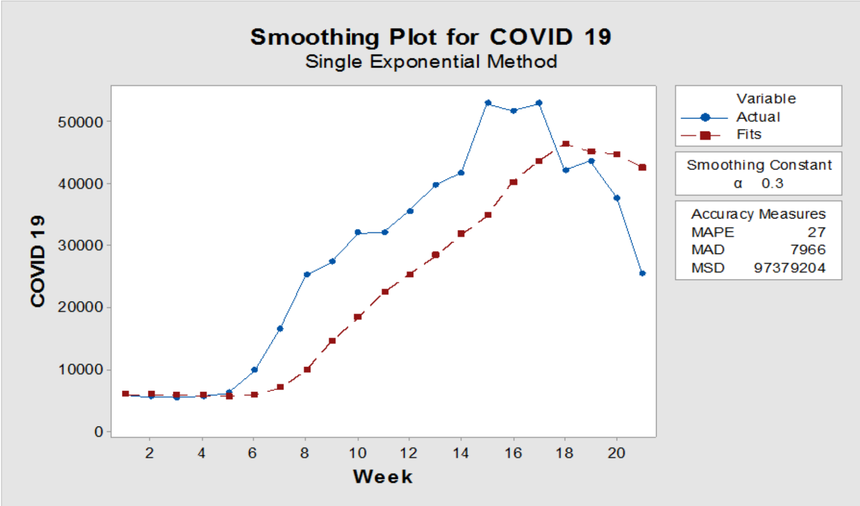

Figure-1. Time series values of the single exponential smoothing model for the real and estimated values.

WINTER AGGREGATION MODEL:

In order to determine the most efficient model for predicting the levels of infection with the COVID 19 pandemic, the winter aggregation model was built and the estimated values and smooth values were calculated for the time series data with errors. The results were shown in Table-3 as follows:

Table-3.

Time	COVID 19	Smooth	Predict	Error
1	5841	5422.3	5326.1	514.9
2	5370	6234.7	6353.5	-983.5
3	5290	5372.2	5080.3	209.7
4	5547	5502.9	5298.6	248.4
5	6044	5117.6	5017.0	1027.0
6	9643	5790.5	6118.8	3524.2
7	16387	7493.3	9293.2	7093.8
8	25151	13200.0	17962.1	7188.9
9	27286	21722.3	29486.4	-2200.4
10	31928	29531.0	36376.2	-4448.2
11	32069	33026.2	38013.9	-5944.9
12	35461	35955.6	38460.8	-2999.8
13	39718	35358.2	36610.8	3107.2
14	41726	39040.7	41590.8	135.2
15	52903	41259.4	43866.0	9037.0
16	51769	48063.7	54443.9	-2674.9
17	52908	55067.3	60330.5	-7422.5
18	42140	54846.6	57010.3	-14870.3
19	43629	51400.4	47354.5	-3725.5
20	37629	41634.4	36032.9	1596.1
21	25298	39971.7	35036.7	-9738.7

The values of the smooth constants were equal to α (level) = 0.418 and γ (trend) = 0.999 and 0.411 = δ (seasonal, respectively. The efficiency measures were MAPE = 16, MAD = 4223, and MSD = 31988996.

Grey Prediction Model GM (1,1):

The grey prediction model GM (1,1) was constructed in order to predict the levels of infection with the COVID 19 pandemic in Iraq, and the results were as follows:

The $X^{(0)}$ time series observations of the number of people infected with the COVID 19 pandemic are described as follows:

$$X^{(0)} = (5841,5370\,, \dots \dots ,25298)$$

By doing accumulative generation, the accumulative series $X^{(1)}$is generated as follows:

$$X^{(1)}(1) = X^{(0)}(2) = 5841$$
$$X^{(1)}(2) = X^{(0)}(1) + X^{(0)}(2) = 11211$$
$$\vdots$$
$$X^{(1)}(18) = X^{(0)}(1) + X^{(0)}(2) \dots \dots + X^{(0)}(21) = 593737$$

The moving average matrix B was as follows:

$$B = \begin{bmatrix} -8526 & 1 \\ -13856 & 1 \\ . & \\ . & \\ . & \\ -581088 & 1 \end{bmatrix}$$

The Y vector can be described by the following formula:

Y= [11211, 16501… 593737]

To estimate the parameters of the model β α, the least squares formula is used, as follows:

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ARPN Journal of Engineering and Applied Sciences

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$$\begin{bmatrix}\hat{\alpha}\\ \hat{\beta}\end{bmatrix} = (B^T B)^{-1} B^T Y = \begin{bmatrix}-0.06070504\\ 15834.65\end{bmatrix}$$

The estimated values of $\hat{X}^{(1)}(k+1)$were calculated for the series observations, model errors and relative errors, and the results are shown in Table-4 as follows:

Table-4. Estimated values of time series data, errors and relative errors of the GM(1,1) model.

Month	Week	Actual data	forecasting	Error$\varepsilon(k)$	Relative error Δ_k
January	1	5841	5841.00	0	0
January	2	5370	16690.71	210.81	3.92
January	3	5290	17735.30	235.26	4.44
January	4	5547	18845.28	239.74	4.32
January	5	6044	20024.72	231.32	3.82
February	6	9643	21277.97	120.66	1.25
February	7	16387	22609.66	37.97	0.231
February	8	25151	24024.70	4.48	0.017
February	9	27286	25528.29	6.44	0.023
March	10	31928	27125.99	15.04	0.047
March	11	32069	28823.69	10.12	0.031
March	12	35461	30627.63	13.63	0.038
March	13	39718	32544.47	18.06	0.045
April	14	41726	34581.28	17.12	0.041
April	15	52903	36745.57	30.54	0.057
April	16	51769	39045.31	24.58	0.047
April	17	52908	41488.98	21.58	0.041
April	18	42140	44085.58	4.62	0.011
May	19	43629	46844.70	7.37	0.016
May	20	37629	49776.49	32.28	0.085
May	21	25298	52891.78	109.07	0.431

The grey differential equation of the GM (1,1) grey prediction model can be described by the following formula:

$$\frac{dx^{(1)}(t)}{dt} - 0.06070504X^{(1)}(t) = 15834.65$$

Figure-2. The spread of time series data values, their estimated values and the errors of the GM(1,1) model.

Grey Verhulst Model GVM (1,1) Model

The GVM(1,1) model is applied after calculating the value of the variable $-Z^1(k)$ in the same way in the GM(1,1) model and the model parameters are estimated using the least squares method after calculating the B matrix of the Grey Verhulst model, as follows:

$$B = \begin{bmatrix} -8526 & 72692676 \\ -13856 & 191988736 \\ & . \\ & . \\ & . \\ -581088 & 3.376632637 \times 10^{11} \end{bmatrix}$$

$$\begin{bmatrix} \hat{\alpha} \\ \hat{\beta} \end{bmatrix} = (B^T B)^{-1} B^T Y = \begin{bmatrix} -0.2874761 \\ -0.00000163172 \end{bmatrix}$$

At the time k, the values of $\hat{X}^1(k+1)$ are calculated and the results are shown in the following table:

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Table-5. Estimated values of time series data, errors and relative errors of the GVM (1,1) model.

Month	Week	Actual data	forecasting	Error$\varepsilon(k)$	Relative error Δ_k
January	1	5841	5841.00	0	0
January	2	5370	6249.44	216.38	4.029
January	3	5290	5698.44	7.72	0.146
January	4	5547	5919.53	6.72	0.121
January	5	6044	6406.37	6.00	0.099
February	6	9643	10015.27	3.86	0.040
February	7	16387	16779.93	2.40	0.014
February	8	25151	25713.20	2.24	0.0089
February	9	27286	28042.70	2.77	0.010
March	10	31928	32743.56	2.55	0.008
March	11	32069	32770.41	2.19	0.007
March	12	35461	36048.51	1.66	0.005
March	13	39718	40187.66	1.18	0.003
April	14	41726	42094.61	0.88	0.003
April	15	52903	53184.97	0.53	0.001
April	16	51769	51983.42	0.41	0.0008
April	17	52908	53060.29	0.29	0.00054
April	18	42140	42256.27	0.28	0.00066
May	19	43629	43716.12	0.20	0.00045
May	20	37629	37700.01	0.19	0.0005
May	21	25298	25351.01	0.21	0.0008

The Grey Verhulst Model GVM equation (1,1), which is a linear regression equation with a dependent variable and two independent variables and it is in the following form:

$$\frac{dx^{(1)}}{dt} - 0.2874761X^{(1)} = -0.00000163172\,(X^{(1)})^2$$

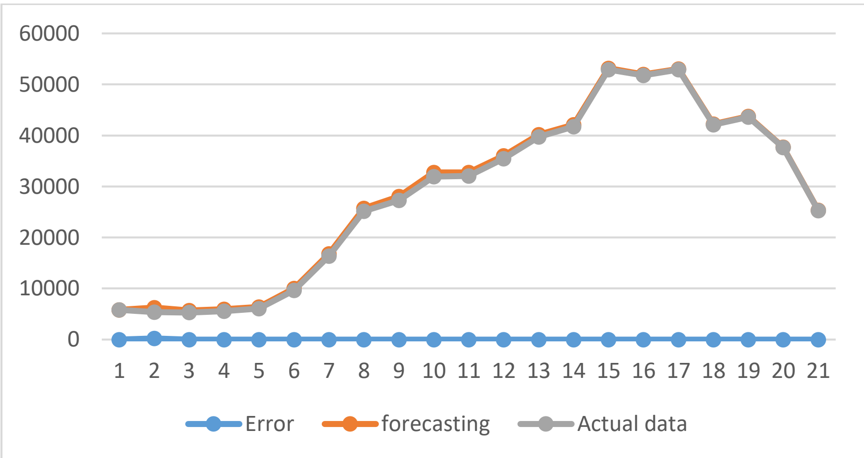

Figure-3. The spread of the time series data values, their estimated values and the errors of the GVM (1,1) model.

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Table-6. Standards for prediction accuracy of models.

Models	MAPE	MSE	RMSE
ESM	27	118755106.2	10897.5
HWAM	16	3244099.48	5695.7
GM(1,1)	66.22	117614025	10845
GVM(1,1)	2.79	210681	459

Table-7. Future predictions for the number of people infected with the COVID 19 pandemic.

week	Month	GVM (1,1)
22	May	25385
23	June	25410
24	June	25430
25	June	25444
26	June	25455

STATISTICAL ANALYSIS

The results of the analysis indicate that there is a high predictive power of the GVM (1,1) model, according to the values of the predictive accuracy criteria shown in Table-6 where the value of MAPE = 2.79 and the value of RMSE = 459 which is the lowest value if compared with its counterparts In the rest of the models, this indicates a high predictive efficiency. This is followed by the Holt winter additive model where the values of the predictive accuracy criteria were MAPE=16 and RMSE=5695.7 which indicates a good prediction followed by the single exponential smoothing model whose predictive efficiency was acceptable where the values of the MAPE and RMSE predictive accuracy criteria were equal to 27 and 10897.5 respectively. The results of the GM (1,1) grey prediction model also indicated a weak predictive ability compared to other models as the measures of predictive efficiency amounted to 66.22 for MAPE and for RMSE was equal to 18045 which is higher than their counterparts in the previous models. The models are generally acceptable from a statistical point of view, but the most efficient model whose predictions can be adopted in making decisions is the GVM (1,1) Grey Verhulst model. The following figure shows the comparison between the relative error of the models.

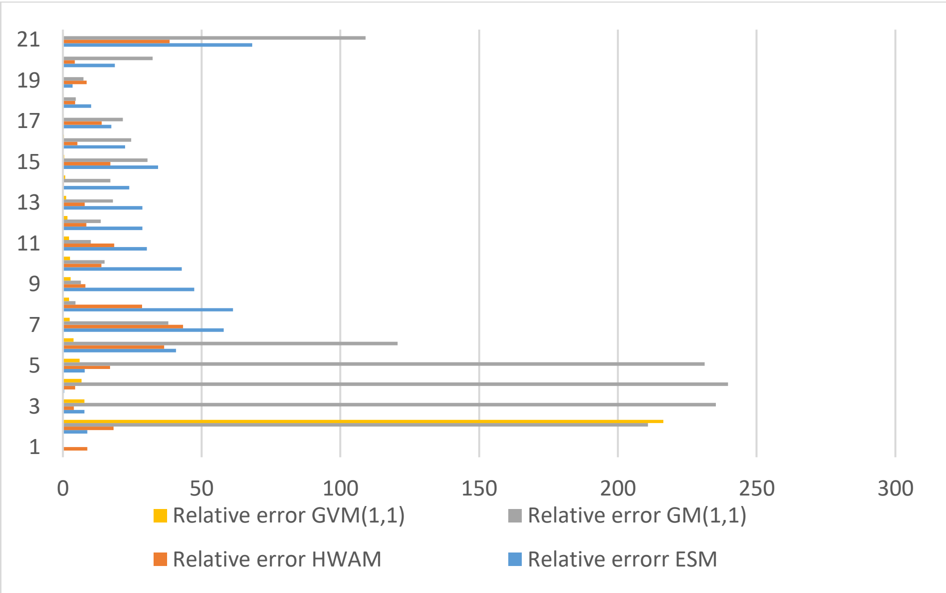

Figure-4. A comparison between the relative error indicators of the models.

CONCLUSIONS

a) The mathematical structure of the Grey Verhulst model and the estimation of parameters by the least squares method led to highly efficient prediction results, especially that the majority of relative errors values are close to zero, this makes the possibility of adopting its results in strategic planning and decision-making.

b) This study used the applied aspect using statistical programs (SPSS, Minitab, Eviews, R) to obtain results that can be judged through the values of the efficiency measures (MAPE, RMSE) on the quality of the model and its ability to predict future levels of the phenomenon under study.

c) The results of the analysis showed a high predictive efficiency of the GVM prediction model (1.1); according to the efficiency criteria MAPE which reached a value of 2.79 and the value of RMSE was

495 which are lower than its counterparts in other models.

d) The Holt winter additive prediction model had a good predictive ability as the MAPE and RMSE values were equal to 16 and 5695.7 which is lower than their counterparts in the GVM model.

e) The efficiency criteria values also indicated an acceptable predictive strength in the exponential smoothing model and weak in the GM prediction model (1,1) according to the MAPE and RMSE criteria, where each of them reached the MAPE=27 and RMSE=10897.5 and for the GM model (1,1). The value of MAPE = 66.22 and RMSE = 10845 which is greater than the previous ones which indicates a weak efficiency in the field.

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