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DIFFRACTION OF A GROUNDED CABLE ON A CONDUCTING SPHEROID IN SEAWATER

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ABSTRACT

Based on a rigorous solution of the problem, analytical expressions are obtained for calculating the diffraction of the electromagnetic field of a grounded cable on a conducting spheroid in seawater. The calculations are based on the original method of the authors of the analytical continuation of the exact solution of the problem on direct current in the frequency domain, by decomposing the electric field of the spheroid in multipoles. Moreover, the analysis of the solution allows us to represent the secondary electric field of the spheroid as the field of an equivalent dipole. Graphs of the flow characteristics of a conducting spheroid are given.

Keywords: grounded cable, electrode antenna, secondary electric field, pass-through characteristic, dipole field.

1. INTRODUCTION

Grounded cables or electrode antennas are used in electromagnetic systems to monitor water areas. These systems are designed to detect various underwater objects, and are divided into passive, active and combined systems. Active systems consist of transmitting (generator) and receiving antennas. General antennas create a primary electromagnetic field, which creates secondary electromagnetic sources in moving objects, the fields of which are recorded by magnetic and/or electric receiving antennas [1]. For shallow water areas, the adaptive electric method is the most promising [2]. The essence of this method is to create a field of spreading currents of transmitting electrode antennas in a controlled water volume and register distortions of this field by conducting or non-conducting objects. In the simplest case, it is convenient to create a field of spreading currents in a conducting medium by a system of two metal electrodes connected by a cable. Such a system is called a grounded cable or an electrode antenna. Analytical expressions for calculating the electromagnetic field of a conducting sphere in the field of a grounded cable are obtained by the authors in [3].

The purpose of this article is to derive general analytical expressions for calculating the electromagnetic field of a conducting elongated spheroid located in the field of a grounded cable with an acceptable accuracy for practice based on a strict solution of the problem of electrodynamics. The choice of the spheroid as the object of research is made for three reasons: 1) by changing the ratio of the main semi-axes (eccentricity), it is possible to model bodies of various geometries from a sphere to a disk; 2) the spheroid still has a sufficiently high symmetry, which greatly facilitates the calculations; 3) a spheroid, unlike a cylinder, has a regular solution for the secondary electromagnetic field in spheroidal coordinates.

2. GROUND CABLE FIELD

The electric field of the grounded cable of an electrode antenna is determined by the free charges on the metal electrodes [2]. The field of free charges on the electrodes is conveniently represented by their potentials.

In the case of direct current for metal electrodes in the form of round thin disks, there is an exact solution. It is obtained as the limit transition of the potential of a charged conducting ellipsoid. The potential U at an arbitrary point $p(\rho, z)$ in space from two counterphase-fed thin metal disks with a radius b located at a distance $2l$ from each other (Figure-1) has the form [5]:

$$U = \frac{2U_0}{\pi}\left(\operatorname{arctg}\frac{b}{\sqrt{\eta_1}} - \operatorname{arctg}\frac{b}{\sqrt{\eta_2}}\right), \tag{1}$$

где U_0 – potential of disks; $r_{1(2)} = \sqrt{\rho^2 + (l \mp z)^2}$ - distances from the centers of the first and second disks; $\eta_{1(2)} = \dfrac{2}{r_{1(2)}^2 - b^2 + \sqrt{\left[\left(r_{1(2)}^2 - b^2\right)^2 + 4b^2(l \mp z)^2\right]}}$ - the elliptical coordinate of the observation point from the centers of the first and second disks, here the (-) sign refers to index 1, and the (+) sign refers to index 2.

The expression (1) for the electric field potential is very difficult for mathematical analysis. We simplify it by assuming that $r_{1(2)} >> b$. In this case,

$$U = \frac{2U_0 b}{\pi}\left(\frac{1}{r_1} - \frac{1}{r_2}\right). \tag{2}$$

In general, in the case of alternating current, the exact solution for the electric field potential of free charges on two thin antiphase-fed metal disks at a point $p(\rho, z)$ can be obtained by the method of the Fourier-Bessel integral transformation [2]:

$$U = \frac{2U_0}{\pi}\int_{-\infty}^{+\infty}\left[e^{-q(l-z)} - e^{-q(l+z)}\right] J_0(\gamma\rho)\frac{\sin(\gamma b)}{\gamma q}\gamma d\gamma. \tag{3}$$

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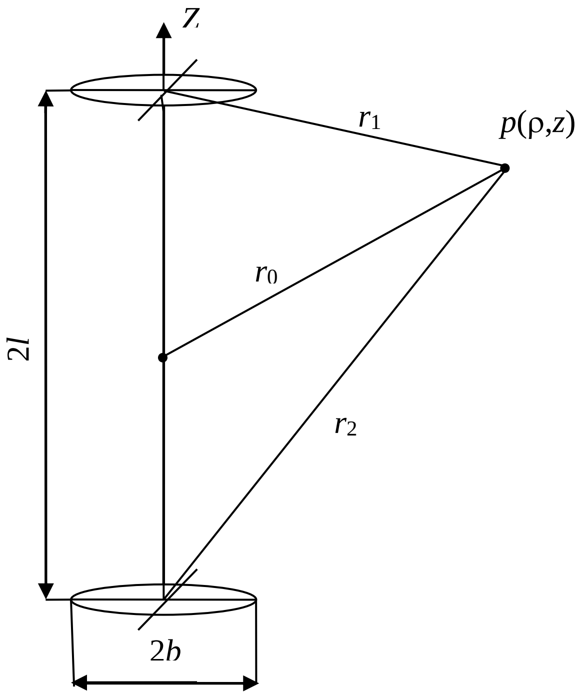

Figure-1. The electrical field of the grounded wire.

The potential (3) at a distance of three times the radius of the disks b, with an acceptable accuracy for practice, has the form:

$$U = \frac{2U_0 b}{\pi}\left(\frac{e^{-ik_3 r_1}}{r_1} - \frac{e^{-ik_3 r_2}}{r_2}\right). \qquad (4)$$

Thus, the potential of an electric antenna at a distance $r >> b$ from its electrodes is equal to the algebraic sum of the potentials of two point sources U_0 with an alternating voltage amplitude and a circular frequency ω. Where $k_3 = \sqrt{-i\omega\mu_0\sigma_3}$ - coefficient of propagation in seawater; σ_3 - specific electrical conductivity of water; $\mu_0 = 4\pi\cdot 10^{-7}$ - Gn/m - magnetic permeability of the vacuum.

3. CALCULATION OF THE SECONDARY FIELD OF A CONDUCTING SPHEROID

If an underwater object enters the field of spreading currents of the transmitting electrode antenna, the conductivity of which differs from the conductivity of the environment, the primary field will be distorted. Let's estimate the value of this distortion using the example of an object in the form of an elongated conducting spheroid. The exact solution of the secondary field of a conducting sphere in the form of a zonal harmonic expansion is well known [4]. In contrast to the sphere, the solution for a spheroid can be represented in terms of spheroidal functions [5].

However, this method of calculation involves a large amount of calculations and is not very informative. It is easier to obtain an exact solution in the case of direct current, and then decompose the resulting solution in multipoles and make an analytical continuation of the solution in the frequency domain [6].

We decompose the primary field (2) by Legendre polynomials [4]:

$$U = \frac{4U_0 b}{\pi r_0}\sum_{n=0}^{\infty} P_{2n+1}(\cos\theta)\left(\frac{l}{r_0}\right)^{2n+1}, \qquad (5)$$

where are $P_{2n+1}(\cos\theta)$ - the odd-order Legendre polynomials; θ - the azimuthal angle in the spherical coordinate system.

On the surface of a spheroid in a conducting medium, the potential is zero, so

$$U(r_0, \theta) = -U_p(\theta, \eta). \qquad (6)$$

In general, we will look for the secondary field of a spheroid in the form of a general solution of the Laplace equation in spheroidal coordinates [7]. At the same time, for certainty, it should be assumed that the initial charge of the spheroid is zero, so the total charge of the spheroid in the external field must also be zero (the external field only separates the charges). The potential U_p from the charges induced on the spheroid at an arbitrary point in space, taking into account the symmetry of the problem and the condition of the secondary field tending to zero at an infinite distance, has the form [7]:

$$U_p = \sum_{n=0}^{\infty} C_n P_n(\cos\theta) Q_n(\eta), \qquad (7)$$

where are C_n - arbitrary constants; $Q_n(\eta)$ - Legendre functions of the second kind.

The coefficients of the expansion of the secondary field of the sphere by spherical harmonics can be found by substituting (5) and (7) in (6), and then multiplying both parts of equation (6) by $P_n(\cos\theta)\sin\theta$, and integrating them by φ from 0 to 2π and by θ from 0 to π. At the same time, we take into account the orthogonality property of Legendre polynomials [7]:

$$\int_0^{\pi} P_n(\cos\theta) P_m(\cos\theta)\sin\theta \, d\theta = 0, \quad \text{at } n \neq m;$$

$$\int_0^{\pi} P_n(\cos\theta) P_m(\cos\theta)\sin\theta \, d\theta = \frac{2}{2n+1}, \quad \text{at } n = m.$$

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It can be seen from expression (5) that the primary field is different from zero only for odd terms of the series, so in expression (7) only odd coefficients C_{2n+1} will remain, the first two of them have the form:

$$C_1 = -\frac{4U_0 bl}{\pi r_0^2}\frac{1}{Q_1(\eta_0)};\ C_3 = -\frac{4U_0 bl}{\pi r_0^4}\frac{1}{Q_3(\eta_0)}, \qquad (8)$$

where function is $Q_1(\eta_0) = \operatorname{arth}(\eta_0) - \eta_0$; $\eta_0 = \sqrt{1-\left(\frac{c}{a}\right)^2}$ - the eccentricity of the spheroid, c and a are the minor and major semi-axes of the elongated spheroid.

Note that the function $Q_1(\eta)$ has the form [4]:

$$Q_1(\eta) = \operatorname{arth}\left(\sqrt{\frac{a^2-c^2}{a^2+S}}\right) - \sqrt{\frac{a^2-c^2}{a^2+S}}, \qquad (9)$$

where S with the orientation of the semi major axis of the spheroid along the Z axis is equal to:

$$S = \frac{\left(R^2 - a^2 + \sqrt{\left(R^2-a^2\right)^2 + 4a^2 Z^2}\right)}{2}, \qquad (10)$$

where $R = \sqrt{x^2+y^2+z^2}$ - distance from the center of the spheroid to the observation point.

The functions $Q_n(\eta)$ for the $n > 2$ real argument at 1 are not defined, so (8) has an exact solution in the form of the first term of the series.

Decomposing the Taylor series in powers of x, and substituting the first four terms of the expansion in (9), we obtain:

$$Q_1 \approx \frac{1}{3}\sqrt{\left(\frac{a^2-c^2}{a^2+S}\right)^3} + \frac{1}{5}\sqrt{\left(\frac{a^2-c^2}{a^2+S}\right)^5} + \frac{1}{7}\sqrt{\left(\frac{a^2-c^2}{a^2+S}\right)^7} + \ldots \qquad (11)$$

Expression (11) is a multiplexing of an arbitrary field [4]. The first term in (11) is dipole, the second is quadrupole, and the third is octupole. As the distance R increases from the spheroid, the fields above the dipole rapidly decay. In practice, it is of interest to detect an object at maximum ranges, so (7), taking into account (8), (10) and (11), will take the form:

$$U_p = -\frac{U_0 bl}{\pi^2 r_0^2}\cos\theta\frac{V_S}{N_z}\left(\frac{1}{R^3}\right), \qquad (12)$$

where $V_S = \frac{4}{3}\pi ac^2$ - is the volume of the spheroid; $N_z = \frac{\left(1-\eta_0^2\right)}{\eta_0^3}\left(\operatorname{arth}\eta_0 - \eta_0\right)$ - the depolarization coefficient along the Z-axis.

In the case of alternating current, the dipole field (12) will take the form [4]:

$$U_p = -\frac{U_0 bl}{\pi^2 r_0^2}\cos\theta\frac{V_S}{N_z}\left(\frac{e^{-ik_3 R}}{R^3}\left[1+ik_3 R\right]\right). \qquad (13)$$

We introduce the dipole moment of the equivalent dipole of a spheroid: $p^{(1)} = -\frac{U_0 bl}{\pi^2 r_0^2}\frac{V_S}{N_z}$.

The resulting solution (13) does not take into account the attenuation of the primary field in seawater. To account for this fact, we decompose U in (4) by spherical functions [4]:

$$U = i\frac{2}{\pi}k_3 bU_0\sum_{n=0}^{\infty}(2n+1)j_n(k_3 l)h_n^{(2)}(k_3 r_0)\left[p_n(\cos\theta) - p_n(-\cos\theta)\right], \qquad (14)$$

where is $j_n(x)$ - the spherical Bessel function of the 1-st kind; $h_n^{(2)}(k_3 r_0)$ - the spherical Bessel function of the 3-rd kind; $p_n(x)$ - Legendre polynomials.

Substituting (14) and (7) in (6) and performing the operations performed above, we find the expression for the constant C_1:

$$C_1 = -i\frac{12U_0 bk_3}{\pi}j_1(k_3 l)\frac{h_1^{(2)}(k_3 r_0)}{Q_1(\eta_0)}, \qquad (15)$$

where is $h_1^{(2)}(k_3 r_0) = -\frac{e^{-ik_3 r_0}}{k_3 r_0} + i\frac{e^{-ik_3 r_0}}{(k_3 r_0)^2}$ - the spherical Bessel function of the 3-rd kind; $j_1(k_3 l) = \frac{\sin k_3 l}{(k_3 l)^2} - \frac{\cos k_3 l}{k_3 l}$ - the spherical Bessel function of the 1-st kind.

Decomposing the spherical Bessel function of the 1-st kind into a Taylor series, taking into account that $|k_3 l| < 0{,}2$, we get that $j_1(k_3 l) \approx \frac{k_3 l}{3}$.

Next, using and transform $ik_3 = i\sqrt{-i\omega\mu_0\sigma_3} = (i+1)\sqrt{(\omega\mu_0\sigma_3)/2} = (i+1)k_0$ with this in mind we get:

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$$-ik_3^2 h_1^{(2)}(k_3 r_0) = \frac{e^{-k_0 r_0}}{r_0^2}\left[\left((1+k_0 r_0)\cos k_0 r_0 - k_0 r_0 \sin k_0 r_0\right) - i\left((1+k_0 r_0)\sin k_0 r_0 + k_0 r_0 \cos k_0 r_0\right)\right]. \quad (16)$$

As a result, the expression (15), taking into account (16), will take the form:

$$C_1 = \frac{U_0 bl}{3\pi^2}\frac{V_S}{N_z}(\alpha - i\beta), \quad (17)$$

where $\alpha = \frac{e^{-k_0 r_0}}{r_0^2}\left[(1+k_0 r_0)\cos k_0 r_0 + k_0 r_0 \sin k_0 r_0\right]$;

$$\beta = \frac{e^{-k_0 r_0}}{r_0^2}\left[(1+k_0 r_0)\sin k_0 r_0 - k_0 r_0 \cos k_0 r_0\right].$$

In the case of alternating current, the induced field of the dipole (13) will take the form:

$$U_p = -\frac{U_0 bl}{\pi^2 r_0^2}\cos\theta \frac{V_S}{N_z}(\alpha - i\beta)(\alpha_1 - i\beta_1), \quad (18)$$

where $\alpha_1 = \frac{e^{-k_0 R}}{R^3}\left[(1+k_0 R)\cos k_0 R + k_0 R \sin k_0 R\right]$;

$$\beta_1 = \frac{e^{-k_0 R}}{R^3}\left[(1+k_0 R)\sin k_0 R - k_0 R \cos k_0 R\right].$$

Let us define the secondary field of the conducting spheroid for the scheme of parallel placement of the generator and two receiving electrode antennas of the same length $2l$ (Figure-2). Here, the generator and receiving electrode antennas are oriented along the axis Z. The distance between the generator and receiving electrode antennas is equal to l_1. The object is an elongated spheroid with semi-axes a and c. From the geometry of Figure-2, we have: h - the height of the center of the spheroid above the bottom of the reservoir; r_0 - the distance from the center of the system to the center of the spheroid; $\cos\theta = \frac{z}{r_0}$.

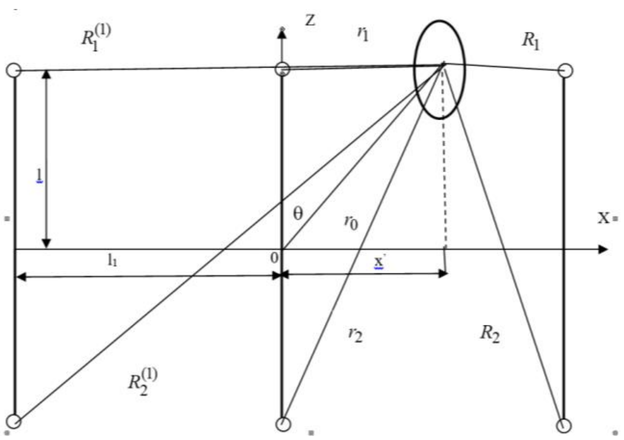

Figure-2. Location of the spheroid, generator and receiving electrode antennas in space.

The most likely movement of the underwater object under study will be parallel to the antennas. From the geometry of Figure-2 we have $r_0 = \sqrt{z^2 + x^2 + h^2}$ - the distance from the center of the spheroid to the center of the generator electrode antenna, and $R_{1(2)} = \sqrt{(z \mp l)^2 + h^2 + (l_1 - x)^2}$ - the distance to the corresponding electrodes of the right receiving electrode antenna, and $R_{1(2)}^{(1)} = \sqrt{(z+l)^2 + h^2 + (l_1 + x)^2}$ - the distance to the corresponding electrodes of the left receiving electrode antenna.

In practice, it is necessary that the signal output of the receiving antenna in the absence of the object is equal to zero. To do this, you need to subtract the signals on the electrodes of the first and second receiving electrode antennas and add the result. For certainty, we take the real part of the secondary field of the spheroid (18) as the passing characteristic of the spheroid. As a result, after elementary transformations of complex numbers, we get:

$$\operatorname{Re}U_p(z) = -\frac{U_0 bl}{3\pi^2}\frac{V_S}{N_z}\left[\left(\alpha\alpha_1^- - \beta\beta_1^-\right) + \left(\alpha\alpha_1^{-(1)} - \beta\beta_1^{-(1)}\right)\right], \quad (19)$$

where

$$\alpha_1^- = \frac{e^{-x_1}}{R_1^3}\left[(1+x_1)\cos x_1 + x_1 \sin x_1\right]\cos\theta_1 - \frac{e^{-x_2}}{R_2^3}\left[(1+x_2)\cos x_2 + x_2 \sin x_2\right]\cos\theta_2;$$

$$\beta_1^- = \frac{e^{-x_1}}{R_1^3}\left[(1+x_1)\sin x_1 - x_1 \cos x_1\right]\cos\theta_1 - \frac{e^{-x_2}}{R_2^3}\left[(1+x_2)\sin x_2 - x_2 \cos x_2\right]\cos\theta_2;$$

$x_0 = k_0 r_0$, $x_1 = k_0 R_1$, $x_2 = k_0 R_2$ - the dimensionless distances expressed in wavelengths in water; $\cos\theta_{1(2)} = -\frac{z \mp l}{R_{1(2)}}$ - are the cosines of the angles between the axis Z and the radii of the vectors $R_{1(2)}$ drawn from the center of the spheroid to the first and second electrodes of the left receiving electrode antenna.

The functions $\alpha_1^{-(1)}$ and $\beta_1^{-(1)}$ are defined from the functions α_1^- and β_1^- by replacing them with $R_{1(2)}$, and $R_{1(2)}^{(1)}$, the corresponding cosines are: $\cos\theta_{1(2)}^{(1)} = -\frac{z \mp l}{R_{1(2)}^{(1)}}$.

3. EXPERIMENTAL RESULTS AND DISCUSSIONS

Figure-3 shows an example of calculating the actual parts of the pass characteristics of (19) $\operatorname{Re}U_p(z)$

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for the depth $h = 3{,}5$ m, and $f = 4 \cdot 10^3$ Hz, $\sigma = 1$ (Om m)$^{-1}$, $b = 0{,}125$ m, $U_0 = 20$ V, $l = 5$ m, $l_1 = 4{,}5$ m and four variants of the passage of the underwater object through the water area under study: $x = 4{,}5; 3; 1{,}5; 0$ m corresponding to curves 1, 2, 3, 4 (Figure-3). Curve 1 corresponds to the passing characteristic of the underwater object moving over the receiving antenna, and curve 4 - over the generator antenna. From the curves in Figure-3, it can be seen, that the shape of the transmission characteristics practically does not change along the studied water basin, and the values of the signal voltages in the observed maxima are close. This makes it easier to pre-process the measured signals and highlight them against the background noise. The instrument system for exploring the underwater basin includes a central generator electrode antenna and two side receiving electrode antennas (Figure-2). The receiving electrode antennas are connected to the inputs of two low-noise differential amplifiers, the signals from which are summed. This is due to the need to obtain almost identical flow characteristics along the studied water basin.

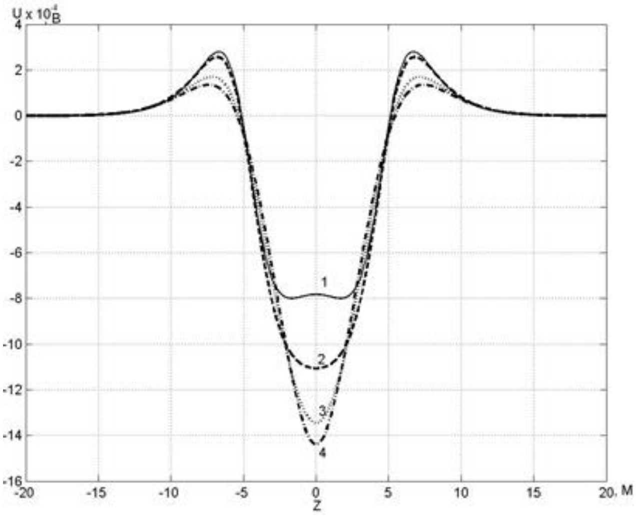

Figure-3. Example of calculating the actual parts of the pass characteristics of (19).

The transmission characteristics of the conducting spheroid are symmetrical with respect to the center of the transmitting electrode antenna, and their maxima are located at $z = 0$. One module provides a width of the studied water basin of about 15 meters with a depth of up to 6 meters. Such antenna systems will be most effective in shallow water basin, where the use of ultrasound is almost impossible due to the numerous reflections of the ultrasonic beam from the bottom and surface of the basin.

3. CONCLUSIONS

Based on a rigorous solution of the electrodynamics problem, it is proved, that the field of a short electrode antenna is determined by the field of spreading currents from the electrodes. Simple analytical expressions of this field in elementary functions are obtained, which make it possible to perform calculations with acceptable accuracy for practice. For the secondary field of a conducting spheroid excited by the field of a grounded cable, an exact solution for direct current is obtained. The solution for alternating current is derived from the solution in the case of direct current by decomposing it into multipoles and replacing the multipoles for direct current with the multipoles for alternating current. This solution was obtained for the first time using the original method. As an example, the calculation of the secondary (diffraction) field of a metal spheroid excited by the field of the transmitting electric antenna for the configuration as in Figure-2. The transmission characteristics are obtained, which are the same in shape at any point of the studied water basin, which is important when processing the received signal. A good agreement between the theoretical calculations and the results of the experimental data is revealed.

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