LINEAR QUADRATIC REGULATOR DESIGN FOR VEHICLE SUSPENSION SYSTEM

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ABSTRACT

Suspension system is one of the important parts of any vehicle system. The main task of suspension system is to ensure the stability of the vehicle from any road disturbances and provide security and to the passenger. However, most of the vehicle manufacturer is still facing difficulties to fully achieve these objectives. There are various of research that focusing on the techniques to obtain a good stabilization and comfortability of vehicle suspension system. Thus, this work proposed Linear Quadratic Regulator (LQR) to maintain good road handling of the suspension system and provide comfortability of the passenger. The system performance with LQR is then compared with the system with pole-placement controller. The simulation results show that suspension system with LQR managed to maintain the stabilization of the system within the acceptable transient response specification. Besides, the performance of suspension travel, car body acceleration and wheel deflection of the system is improved with LQR in terms of its maximum peak and settling time.

Keywords: suspension system, pole-placement controller, linear quadratic regulator.

INTRODUCTION

Suspension system consists of tires, tire air, springs, shock absorbers and linkages which connects a vehicle to its wheels. This connection allows the relative motion between them. The main objective of suspension system is to support both road holding or handling and ride quality since these two characteristics always inverse proportional to each other. A good suspension system can maintain good road handling and at the same time ensure the comfortable of the passenger. Besides, the system needs to keep the road wheel in contact with the road surface such that all the road or ground forces acting on the vehicle can be detected through the contact patches of the tires. On the other hand, the suspension system will protect the vehicle itself and any cargo or luggage from any damage or wear. Thus, a good and right compromise is involved in tuning a good suspension system.

The important functionality of the vehicle suspension system is to support the vehicle body as well as to provide the comfort driving to the passengers by rejecting the unpleasant vibratory motion induced from the irregular road inputs. Also, the suspension should maintain adequate vertical load to provide the vehicle stability when the car turns, brakes, or accelerates. The need of suspension systems is typically rated by its ability to maintain good road handling and to improve passenger comfort. Active suspension poses the ability to give a better performance of suspension systems by reducing traditional design compromise between handling and ride comfort by directly controlling the suspensions force actuators, which is a closed-loop system. As such, the need of stabilizing function for the system is crucial.

Various techniques are proposed to acquire a good performance of vehicles' suspension system. An adaptive controller is designed for a quarter-size vehicle's active suspension system (Arjon Turnip *et al*, 2015). The parameter of the controller is tuned by using Lyapunov method and the simulation results show that an active suspension system can improve the ride comfort and the road holding compared with the conventional passive suspension systems as well with semi-active suspension. PID controller based on Internal Model Control (IMC) is designed for an active vehicle suspension system (AVSS) of a quarter car model. The designed controller compromises the two conflicting criteria between the passenger ride comfort and road handling (Truong Nguyen Luan Vu *et al*, 2017). Different kinds of road input signals are selected to test the systems. The simulation results indicate that the performance of proposed AVSS has improved in terms of reducing the peak overshoot of sprung mass displacement and sprung mass acceleration in compared with the other traditional passive suspension systems. (Sarot Hlangnamthip *et al,* 2019) proposed the design of PID controller integrated with Cuckoo optimization method. The result of PID controller is compared with PI and PD controller. An Adaptive Neuro Fuzzy Inference System (ANFIS) is designed and implement for an active suspension system to improve the travelling comfort of the passengers (Hari V.M, Lakshmi et.al, 2015). The designed controller is then compared with conventional PID controller. The result shows that the system with ANFIS improves the vehicle ride comfort. H∞ miscellaneous feedback controller (HMIFC) is designed for stabilization of active suspension system (Gang Wang et.al, 2019). The controller is designed based on Lyapunov theory and the linear matrix inequality (LMI) method. Simulation and experimental results prove the effectiveness of the proposed HMIFC compared to traditional H∞ controller. Nonlinear sliding mode controller was proposed for active suspension system in (Hong Ming Chen, 2012). Two road profiles, a sine wave and a single bump, respectively, were given to demonstrate the effectiveness of the proposed control scheme. Simulation results suggest that the proposed control scheme may result in a system with fast-response and high-precision tracking of a virtual sky-hook damper force so that high ride quality and handling performance can be obtained. PI controller is designed to control the dynamic behaviour of vehicle active suspension system with road disturbance is given to the system. Kharitonov theorem is used to adjust the parameters of the controller (Rahul Mittal, 2015). A study which concerns the problem of fuzzy logic controller design for vehicle active suspension system is present in (Lei Cheng et.al, 2018). Three performance requirements which are ride comfort, good road-holding ability and suspension stroke limitation are considered in the design process. Genetic Algorithm is employed to optimize the parameters of membership functions and fuzzy control rules to obtain better control performance. (Vishnu Vidya, *et al*, 2017) comes up with a model reference adaptive control scheme based on neural network for an active suspension system. A quarter car model with 2-DOF is selected for the analysis, which covers the vertical dynamics of vehicle. LQR is used as a benchmark controller and the performance of proposed controller is determined by carrying out computer simulations using MATLAB and SIMULINK. PI controller and Linear Quadratic Regulator controller are designed to stabilize the dynamic responses of active suspension system with parametric uncertainty followed by road irregularities. The system is represented in polytopic form to ensure the robustness of the controller on the parametric uncertainties in the suspension system (Rahul Mittal, 2015).

Since an active suspension system is considered as a regulation case, usually state-feedback controller is needed to control the system. In this work, a quarter-car active suspension system is presented as a model in Matlab and Simulink. A Linear Quadratic Regulator (LQR) is proposed to improve the ride comfort, car handling and minimize vibration due to road roughness or uneven road profile. The designed LQR is compared with the Pole Placement controller which is also developed for the same system. The effectiveness of the proposed control scheme is validated by analyzing the amount of generated force (u in Newton) and the transient performance (settling time and overshoot) that reflect to the improvement in ride comfort, car handling and vibration minimization due to road roughness or uneven road profile.

METHODOLOGY

The implementation of the work starts with modelling a quarter-car model for an active suspension system. Then, pole-placement controller is designed for the model followed by designing Linear Quadratic Regulator for the same model.

System Modelling

The equation for a quarter-car model movement is found by including vertical forces on the sprung masses and un-sprung masses. As shown in Figure-1, most of the quarter-car suspension model will represent the mass of the car body as the sprung mass, while the mass of the wheels is represented as the un-sprung mass. In this research, an active quarter-car suspension system is modelled as 2 DOF system in time domain as shown in Figure-2.

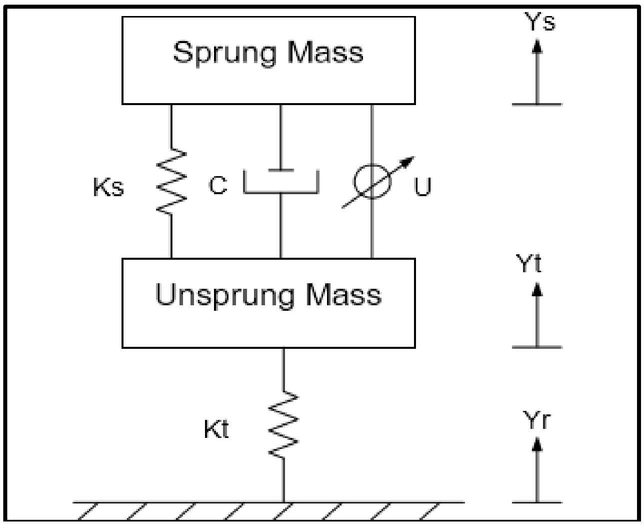

Figure-1. Quarter-car model for an active suspension system.

Mathematical modelling of an active quarter-car suspension system begins by deriving the un-sprung mass, M_1 and sprung mass, M_2.

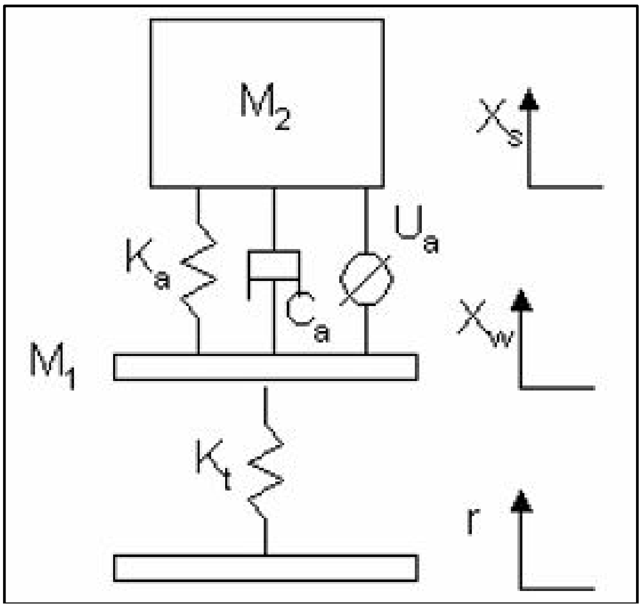

Figure-2. Quarter-car model for an active suspension.

Based on Figure-2, the representation of the system is given as

$$\ddot{X}_s = -\frac{C_a}{M_2}\left(\dot{X}_s - \dot{X}_w\right) - \frac{K_a}{M_2}\left(X_s - X_w\right) + \frac{U_a}{M_2} \quad (1)$$

$$\ddot{X}_w = \frac{C_a}{M_1}\left(\dot{X}_s - \dot{X}_w\right) + \frac{K_a}{M_1}\left(X_s - X_w\right) - K_t\left(X_w - r\right) - \frac{U_a}{M_1} \quad (2)$$

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where;

M_1	=	mass of the wheel/unsprung mass (kg)
M_2	=	mass of the car body/sprung mass (kg)
r	=	road profile
X_w	=	wheel displacement (m)
X_s	=	car body displacement (m)
K_a	=	stiffness of car body spring (Nm/s)
K_t	=	stiffness of tire (N/m)
C_a	=	damper (Ns/m)
U_a	=	force actuator

Let the state variables are:

$$X_1 = X_s - X_w$$
$$X_2 = \dot{X}_s$$
$$X_3 = X_w - r$$
$$X_4 = \dot{X}_w \tag{3}$$

Therefore, in state space equation, equation (1) can be written as:

$$\dot{X} = AX(t) + BU_a(t) + F\dot{r}(t) \tag{4}$$

so,

$$\dot{X}_1 = \dot{X}_s - \dot{X}_w = X_2 - X_4$$
$$\dot{X}_2 = \ddot{X}_s$$
$$\dot{X}_3 = \dot{X}_w - \dot{r} = X_4 - \dot{r}$$

$$\dot{X}_4 = \ddot{X}_w \tag{5}$$

where;

$X_1 = X_s - X_w$	=	suspension travel
$\dot{X}_s$	=	car body velocity
$X_2 = \ddot{X}_s$	=	car body acceleration
$X_3 = X_w - r$	=	wheel deflection
$\dot{X}_w$	=	wheel velocity
$X_4 = \ddot{X}_w$	=	wheel acceleration

From equation (1) and (2), rewrite the equation (4) in matrix form based on the variable in equation (3) and (5). Therefore, the mathematical modelling of the system is obtained as follows in equation (6) :

$$\dot{X} = AX(t) + BU_a(t) + F\dot{r}(t) \tag{6}$$

The parameter of the suspension system is shown in Table-1. Then the numerical system can be deducted as in equation (7).

Table-1. Parameter of quarter-car suspension system.

Parameters	Numerical value
M_1	59 kg
M_2	290 kg
K_a	16812 N/m
K_t	190000 /m
C_a	1000 Ns/m

$$\begin{bmatrix}\dot{X}_1\\ \dot{X}_2\\ \dot{X}_3\\ \dot{X}_4\end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -\frac{K_a}{M_2} & -\frac{C_a}{M_2} & 0 & \frac{C_a}{M_2} \\ 0 & 0 & 0 & 1 \\ \frac{K_a}{M_1} & \frac{C_a}{M_1} & -\frac{K_t}{M_1} & -\frac{C_a}{M_1} \end{bmatrix} \begin{bmatrix}X_1\\ X_2\\ X_3\\ X_4\end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M_2} \\ 0 \\ -\frac{1}{M_1}\end{bmatrix} U_a + \begin{bmatrix}0\\0\\-1\\0\end{bmatrix}\dot{r} \tag{7}$$

Controller Design

The proposed controller scheme for an active quarter-car suspension system is depicted in Figure-3.

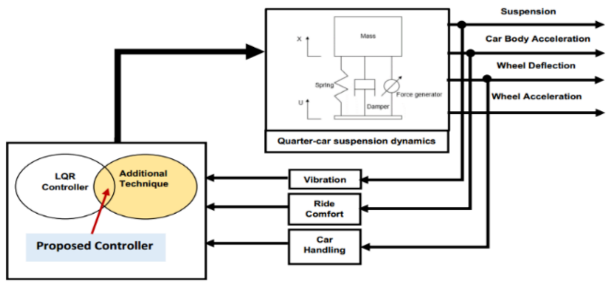

Figure-3. Proposed control scheme.

Two controllers are proposed for the system which are Pole-Placement Controller and Linear Quadratic Regulator.

By using Pole-Placement Method, the controller gain is obtained as

$$K = [-0.1526 \quad -0.0046 \quad 1.4516 \quad -0.0160] \tag{8}$$

Another controller that is also designed for the system is Linear Quadratic Regulator (LQR).

Let the system represented by state-space equation:

$$\dot{X} = AX(t) + BU_a(t) \tag{9}$$

with control law,

$$U_a(t) = -KX(t) \tag{10}$$

the control law can be designed to fulfill the minimization of the linear quadratic performance index in equation (11).

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$$J = \int_0^{\infty} (X^t Q X + U_a{}^t R U_a) dt \quad , \forall X \neq 0 \tag{11}$$

with X is the estimated states, Q and R are square positive definite matrices. Term $X^t Q X$ solves the state regulation problem. Term $U_a{}^t R U_a$ minimizes the energy of the control signal $U_a(t)$ so that the control input is bounded between $U_{min} \leq U_a(t) \leq U_{max}$.

The value of matrix Q and weight factor R that has been used in designing the LQR Controller as shown in equation (12) and (13). Hence, equation (14) shows the value of feedback gain K, optimum P and E that has been obtained in simulation works.

$$Q = \begin{bmatrix} 1000 & 0 & 0 & 0 \\ 0 & 1000 & 0 & 0 \\ 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 1000 \end{bmatrix} \tag{12}$$

$$R = 0.001 \tag{13}$$

$$K = [295 \quad 2578 \quad -30120.9 \quad -2220] \tag{14}$$

RESULTS AND ANALYSIS

To observe the efficacy of the LQR and pole placement method, two different road profile was injected into an active quarter-car suspension system. Figure-4 represents a single bump road profile. The mathematical expression of road profile 1 is shown in equation (15).

$$r(t) = 0.15u(t-9) - 0.15u(t-10) \tag{15}$$

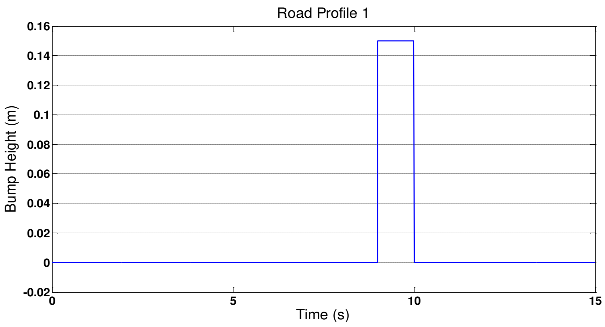

Figure-4. The step input signal graph for road profile 1.

Road profile 2 shown in Figure-5 mimics a 2 bumps road and represented as the step input signal in equation (16).

$$r(t) = \big(0.15u(t-2) - 0.15u(t-3)\big) + \big(0.1u(t-9) - 0.1u(t-10)\big) \tag{16}$$

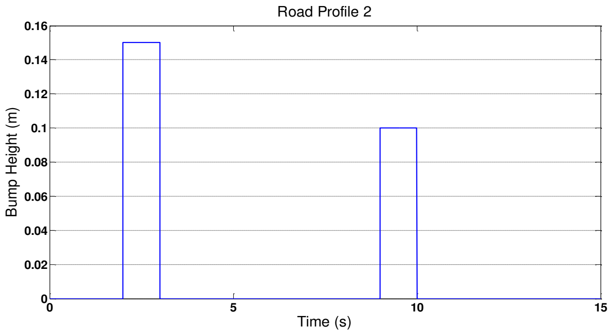

Figure-5. The step input signal graph for road profile 2.

To observe the effectiveness of LQR and pole-placement method, simulation is conducted in MATLAB with SIMULINK toolbox. Figure-6 shows the force generated by the controllers when the suspension is subjected to 0.1 initial conditions. It can be shown that LQR produces less energy as compared to the pole placement method. Figure-7 and Figure-8 show the comparison of force (in Newton) generated by both LQR and pole-placement method when the suspension system is perturbed by road profile 1 and road profile 2 respectively. Table-2 shows the quantitative comparison of both controllers in term of control law (i.e. energy in Newton) and transient (i.e. settling time in seconds). For bump 1 in road profile 1, LQR require 66.32 N to stabilize the system, which is 98% reduction from the energy produced by pole-placement (i.e. 3341 N). In bump 2, LQR produces 44.21 N as compared to 2227 N by pole-placement method. In term of transient performance, LQR quite sluggish in settling time as compared to pole-placement method.

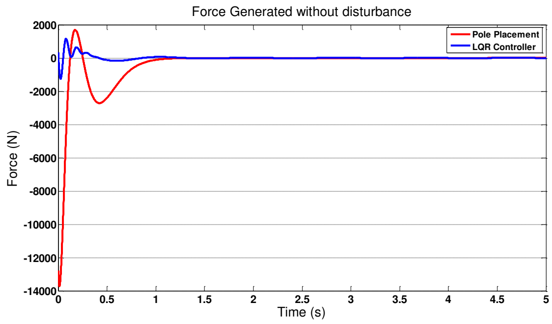

Figure-6. The force generated without disturbance.

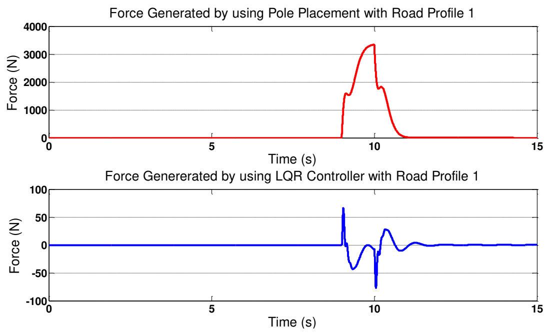

Figure-7. The force generated with road profile 1.

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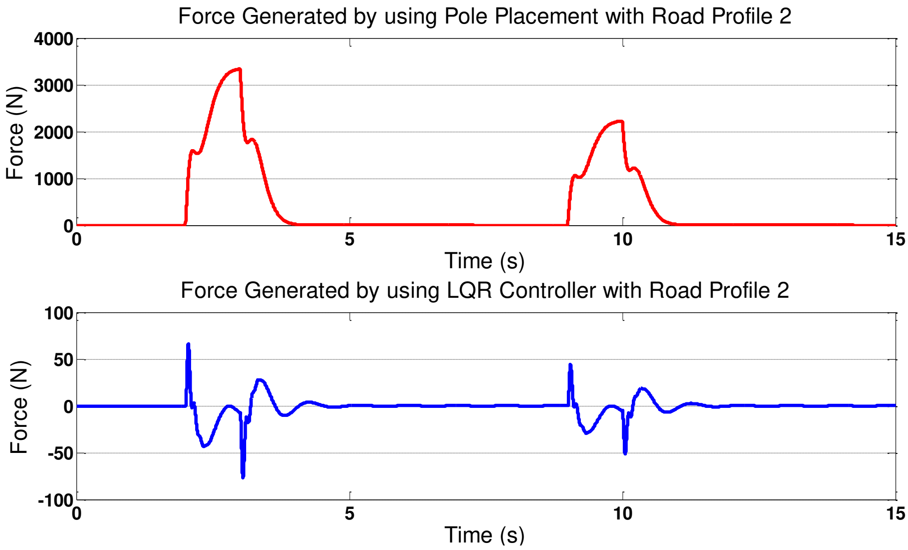

Figure-8. The force generated with road profile 2.

Table-2. Comparison of performance between pole placement and LQR controller.

Elements		**Maximum Peak (N)**		**Percent of reduction (%)**	**Settling Time (s)**	
		Pole Placement	**LQR Controller**		**Pole Placement**	**LQR Controller**
Without Disturbance		1689	1162	31.2	2.588	2.172
Road Profile 1		3341	66.32	98.0	2.520	3.430
Road Profile 2	Bump 1	3341	66.32	98.0	2.286	3.400
	Bump 2	2227	44.21	98.0	2.690	3.410

Analysis on LQR Based on Road Disturbance

Early analysis illustrates that LQR produce good performance compared to pole-placement controller when disturbances in terms of road profile 1 and 2 are injected to the active suspension system. Thus, additional analysis on LQR itself is provided in this section to observe its efficacy on suspension travel, car body acceleration and wheel deflection of the system. The performance of LQR on the suspension system can be analyzed by comparing the performance of suspension travel, car body acceleration and wheel deflection for the system without controller and the system with LQR controller. Suspension travel represents the performance of the vibration due to road roughness. Car body acceleration represents the performance of ride comfort and wheel deflection represents the performance of car handling.

Figure-9, Figure-10 and Figure-11 respectively shows the performance comparison of suspension travel, car body acceleration and wheel deflection of the suspension system when perturbed by road profile 1. While, Figure-12, Figure-13 and Figure-14 show the performance comparison of suspension travel, car body acceleration and wheel deflection of the suspension system when perturbed by road profile 2.

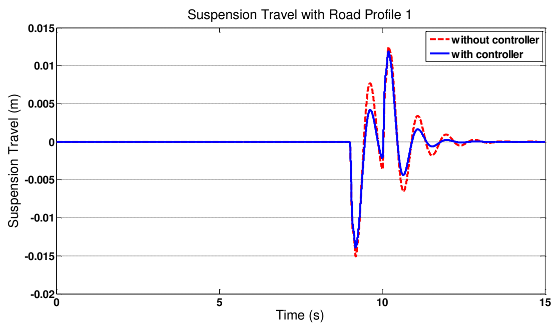

Figure-9. The suspension travel with road profile 1.

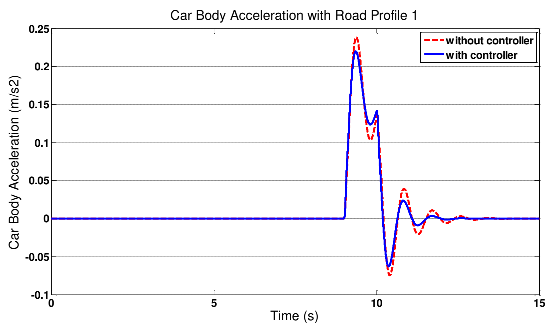

Figure-10. The car body acceleration with road profile 1.

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Figure-11. The wheel deflection with road profile 1.

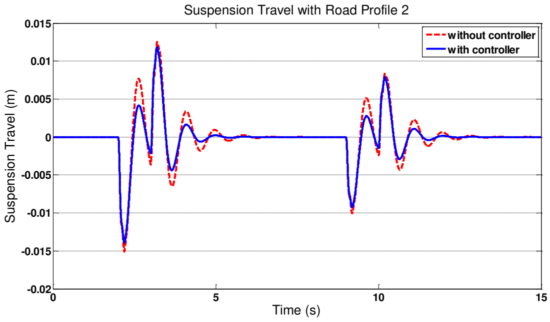

Figure-12. The suspension travel with road profile 2.

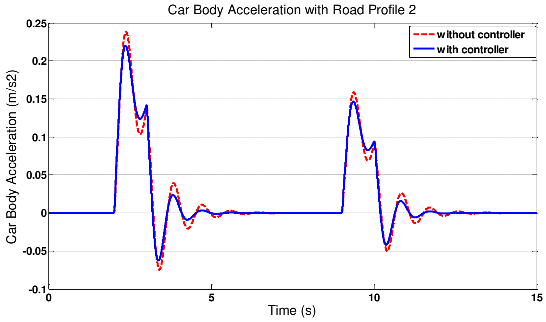

Figure-13. The car body acceleration with road profile 2.

Figure-14. The wheel deflection with road profile 2.

From these six figures which are Figure-9 until Figure-14, it can be observed that the maximum peak performance of the system without controller is higher than the system with controller. All the results of the transient performance of the suspension system with road profile 1 and road profile 2 has been analyzed and tabulated in Table-3, Table-4 and Table-5.

Table-3. Comparison of suspension system performance for road profile 1.

Elements	Maximum Peak		Settling Time (s)	
	Without Controller	With Controller	Without Controller	With controller
Suspension Travel (m)	0.0125	0.0119	5.819	3.710
Car Body Acceleration (m/s2)	0.2383	0.2199	5.980	3.632
Wheel Deflection (m)	0.0025	0.0027	6.140	3.510

Table-4. Comparison of suspension system performance for road profile 2.

Elements	Maximum Peak				Settling Time (s)			
	Without Controller		With Controller		Without Controller		With controller	
	Bump 1	Bump 2	Bump 1	Bump 2	Bump 1	Bump 2	Bump 1	Bump 2
Suspension Travel (m)	0.0125	0.0083	0.0119	0.0079	5.819	5.820	3.710	3.752
Car Body Acceleration (m/s2)	0.2383	0.1589	0.2199	0.1466	5.980	5.580	3.632	3.720
Wheel Deflection (m)	0.0025	0.0016	0.0027	0.0018	6.140	5.730	3.510	3.610

ARPN Journal of Engineering and Applied Sciences

Table-5. The percent of system improvement for road profile 1 and road profile 2.

Elements	Percent of improvement (%)					
	Maximum Peak			Settling Time (s)		
	Road Profile 1	Road Profile 2		Road Profile 1	Road Profile 2	
		Bump 1	Bump 2		Bump 1	Bump 2
Suspension Travel (m)	4.80	4.80	4.82	36.2	36.2	35.5
Car Body Acceleration (m/s2)	7.72	7.72	7.74	39.3	39.3	33.3
Wheel Deflection (m)	0	0	0	45.9	45.9	37.0

By comparing the performance of the system without controller and the system with controller for road profile 1 and road profile 2 in Table-3, Table-4 and Table-5, it can be perceived that the LQR Controller performances produce lower amplitude and faster settling time compared with the system without controller. After implementing the LQR Controller, the performance of suspension travel, car body acceleration, and wheel deflection have been improved as compared to the system without controller.

Suspension travel and car body acceleration performance for the system with LQR Controller for the two types of road profile reduced the amplitude and settling time as compared to the system without controller. Wheel deflection performance of the system with controller also improve even though the amplitude of the system is slightly higher as compared to the amplitude of the system without controller. However, the performances show that the settling time of the wheel deflection for the system with controller is very fast as compared to the system without controller.

CONCLUSIONS

Based on the simulation results of this research, it can be concluded that LQR produces better performance on the active suspension system compared to pole-placement controller. By giving different types of disturbances in terms of different type of road profile, LQR managed to maintain the stabilization of the system within the acceptable transient response specification. Besides, the performance of suspension travel, car body acceleration and wheel deflection of the system is improved with LQR in terms of its maximum peak and settling time.

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REFERENCES

Arjon Turnip, Iwan R. Setiawan, M. Faizal Amri, Tua Agustinus Tamba. 2015. Controller Design for Active Suspension System based on Skyhook Reference Model. International Conference on Technology, Informatics, Management, Engineering & Environment (TIME-E) North Sumatra, Indonesia. September 7-9, 2015. 147-151.

Truong Nguyen Luan Vu, Do Van Dung, Nguyen Van Trang, and Phan Tan Hai. 2017. Analytical Design of PID Controller for Enhancing Ride Comfort of Active Vehicle Suspension System. International Conference on System Science and Engineering (ICSSE). Ho Chi Minh City. July 21-23, 2017. 305-308.

Sarot Hlangnamthip, Chaiyo Thammarat, Deacha Puangdownreong. 2019. CS-Based PID Controller Design for Maglev Vehicle Suspension System. International Electrical Engineering Congress, 6-8 March 2019, Hua Hin Thailand. 1-4.

Hari V.M., Lakshmi P., R. Kalaivani. 2015. Design and Implementation of Adaptive Neuro Fuzzy Inference System for an Experimental Active Suspension System. International Conference on Robotics, Automation, Control and Embedded Systems (RACE). 18-20 February 2015. Chennai India. 1-4.

Gang Wang and Zhijin Zhou. 2019. Design and Implementation of $H\infty$ Miscellaneous Information Feedback Control for Vehicle Suspension System. Journal of Shock and Vibration. 2019: 1-15.

Hong Ming Chen. 2012. Design of a Nonlinear Controller for a Quarter Vehicle Active Suspension System. International Symposium on Computer, Consumer and Control. July 2012. Taichung Taiwan.512-515.

Rahul Mittal; Manisha Bhandari. 2015. Design of Robust PI Controller for Vehicle Active Suspension System with Hydraulic Actuator. International Conference on Communications and Signal Processing (ICCSP), 2-4 April 2015. India. 1-5.

Lei Cheng, Wenping Xue, Guohai Liu, Kangji Li. 2018. Fuzzy Controller Design for Vehicle Active Suspensions Based On Genetic Algorithm. International Conference on

Information, Cybernetics, and Computational Social Systems (ICCSS). 16-19 August 2018. China. 348-352.

Vishnu Vidya, Meher Madhu Dharmana. 2017. Model Reference Based Intelligent Control of an Active Suspension System for Vehicles. International Conference on Circuit, Power and Computing Technologies (ICCPCT). 20-21 April 2017. India. 1-5.

Rahul Mittal. 2015. Robust PI and LQR Controller Design for Active Suspension System with Parametric Uncertainty. International Conference on Signal Processing, Computing and Control (ISPCC). 24-26 Sept 2015. India. 108-113.